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A Robust Mathematical Growth Model

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Abstract

In this paper, we proposed a robust growth model compared it among the different kinds of growth model such as logistic, Malthusian, Gompertz. Our proposed model showed the art of state performance compared to others. As the compared data, we used the manpower, remittance earners of Bangladesh and Overseas Employment (Female Workers). Beside, this research accumulates some of the basic information manpower and remittance earners to develop a mathematical growth model which show the picture our economic condition in future. Our proposed growth model is the best performance in showing the manpower and remittance earner growth rate curves in Bangladesh

Keywords: Gompertz Model, Logistic Model, Manpower, Proposed Model.

Introduction

Migration is a significant feature of globalization. During the decade both Bangladesh and international migration of labor has increased tremendously. Skill Man-power has become important source of employment and plays a vital role in decreasing poverty in Bangladesh. Labor migration has become an important factor for economy in respect of employment generation, GDP growth, poverty reduction. Manpower export has been increasing since 1976 and still now. Number of migrant workers was 6087 in 1976 but at present it stands at 10 million and the flow of total remittances to Bangladesh stands at US $ 16566 million in 2013 (BMET-2013) [4]. Remittances have now become a biggest single source of foreign exchange earnings in Bangladesh. It is 25.36 percent of GDP and 75.5 percent of total export earnings of the country Remittance contributes towards increasing the income of the remittance receiving households and the standard of living [5]. It increases investment in human capital, household consumption and also stimulates the savings and investment. At the household level remittances are used for meeting basic needs and other family expenses. Remittances have both direct and indirect impacts on micro and macro level economics. It not only benefited the remittance receiving families but also contributes to the growth of output and national income [3]. It helps to support payment of imported capital goods and raw materials for industries. The direct contributions of remittances to national income have grown rapidly in the past decade. Remittances have contributed to increase foreign exchange reserve of Bangladesh. Now foreign exchange reserve of Bangladesh is over US $ 33 billion which is more than 7 times higher than the foreign exchange reserve of the year 2019. We have a large unemployed labor force. In 2010 unemployment was 2.6 million and at present it stands at more than 3 million [6]. And more than 10 million including unpaid family helpers Unemployment is a chronic problem in Bangladesh and it is possible to solve this problem to a great extent by exporting manpower. Manpower export is an instrument for increasing foreign exchange earnings and thereby increasing the national income and growth. Remittance has become a dominant variable for economic development of Bangladesh. Recognizing the importance of remittance and migration, the policy makers and the researchers become more attentive to this particular issue. So, we have selected such a topic of national importance for study.

Literature Review

Soon after independence Bangladesh was branded as a ‘bottomless basket’ by foreign pundits. In 2017 the country has transformed into a lower-middle income country [3]. International migration and the remittances sent by the Bangladeshi labor migrants have played a critical role in this breakthrough. The impact of overseas employment and concomitant remittance flow contributed significantly in the success of alleviating poverty in the country as well. Realizing its importance successive governments in Bangladesh have paid attention to migration management [7]. Since 2010 the Refugee and Migratory Movements Research Unit (RMMRU) has been publishing annual reports highlighting the achievements and challenges that migration sector faces. This report on 2019 is a continuation of that process. The Government of Bangladesh has a lot to learn from the experience of India and Pakistan [4]. What Bangladesh is recently experiencing may be the early signs of what was experienced by India before 1992 and by Pakistan in the 1990s. Such a secular decline or stagnation in remittance inflows must not be allowed to happen in the case of Bangladesh. The Bangladesh Bank and the Government must work on both demand and supply sides to restore the lost momentum in remittance inflows. We suggest using remittances for the development of micro-enterprise and focusing on structural changes of remittances in Bangladesh. We empirically examine the macroeconomic determinants of remittance inflows in Bangladesh. Bangladesh maintained a static position of 13^{th} from 1995 to 1997, and moved to 8^{th} position from to 1998 to 2002, attesting to the stability of remittance flows to Bangladesh as compared to global flows [13]. We also mentioned that remittance inflow position was very high and hence likely to be sustainable. Demand side policies should aim at alleviating or addressing the concerns which are contributing to the increased capital flight. On the supply side, efforts must be made to reduce the spread between the interbank and parallel/hundi markets for foreign exchange. Liberalization of Bangladesh's exchange regime-which is still very restrictive compared with India and Pakistan-should help alleviate demand pressure and also contribute to the reduction of spread between the interbank and parallel markets for foreign exchange [6]. Above all, by making Bangladesh more livable for the future generations through rapid economic and infrastructure development, good governance, and application of rule of law will create better environment for our future generations and improve the climate for the private sector to invest at home [14]. This is what India has done-through economic reforms and strong institutions and political process–and Bangladesh should also strive for. Capital flight is a manifestation of the private sector's lack of confidence in our country/economy and economic policy [12]. The recorded lower than targeted levels of private sector investment in Bangladesh for almost a decade and the corresponding increase in capital flight away from Bangladesh should also be seen in this light. Policy makers must pay attention to this emerging disturbing outcome and undertake necessary corrective actions [15].

Background Study

Gompertz Model

A Gompertz function named after Benjamin Gompertz, is a sigmoid function .It is a type of mathematical model for a time series, where growth is slowest at the start and end of a time period .The right-hand or future value asymptote of the function is approached much more gradually by the curve than the left-hand or lower valued asymptote, the contrast to the simple logistic function in which both asymptotes are approached by the curve symmetrically. It is a special case of the generalized growth function.

The derivative of Gompertz function is defined as,

y(t) = $ke^{\ln(\frac{y_0}{k})e^{-bt}}$

For k > 0 and b is not zero. Where parameters are the same as for the Logistic model. The Gompertz and Logistic models are very similar but the inflection point for the Logistic model is at the 50% of the curve, as a symmetric model, while the point for the Gompertz model is at the 37% of the long-term saturation levels, a symmetric model.

Malthusian Growth Model

The simplest population model of single species [13] is the Malthusian Model. The model is named after Thomas Robert Malthus, who wrote an essay on the principle of population in 1798 which was earliest and one of the most influential books on population. This model is also known as the exponential growth model.

"The rate of change of the population is proportional to the current population."

Then Malthusian model has the following form:

$y(t) = y_o e^{rt}$

Where, y o = y (0) is the initial population size

y (t) is the current population size.

r = is ‘the population growth rate, sometimes called Multhusian parameter and,

t =time.

Logistic Growth Model

When population is too large, Malthusian model fails to determine the population. Verhulst proposed that, a self-limiting process should operate when population become too large. For, this reason this model is also known as [13] Verhulst model.

According to the statement of this model

We have,

$$\frac{dy}{dt} = ry - by^2$$

Or, $$\frac{dy}{dt} = ry\left(1 - \frac{yb}{r}\right)$$

Or, $$\frac{dy}{dt} = ry\left(1 - \frac{y}{\frac{r}{b}}\right)$$

Hence, $\frac{dy}{dt} = ry\left(1 - \frac{y}{k}\right)$………………. (1)

Where, r and k are positive constants. This is called logistic growth model. Here, $\frac{dy}{dt} = ry\left(1 - \frac{y}{k}\right)$ is the per capita birth rate and K is the carrying capacity of the environment which is determined by the available sustaining resources.

From (1) we can write

$$\frac{dy}{y(1-\frac{y}{k})} = rdt$$

Or, $\frac{dy}{y(k-y)} = rdt$

Or, $\int (\frac{1}{y} + \frac{1}{k-y})\, dy = r\int dt$

Or, lny – ln(k-y) - lnA = rt

Or, ln$\frac{Ay}{(k-y)} = rt$

Or, $\frac{Ay}{(k-y)} = e^{rt}$

Or, Ay = (k-y)e^{rt}……………………. (2)

When, y (0) =y0

Or, Ay0 = (k-y0)e^{0}

Hence

A = $\frac{(k-y0)}{y0}$

Put this value in (4) we have

$$\frac{(k-y0)}{y0} y = (k-y)e^{rt}$$

Or, ky –yy0 = ky0e^{rt}-yy0e^{rt}

Or, y (k-y0+y0e^{rt}) =ky0e^{rt}

Or, y = y (t) =$\frac{ky0}{ke^{-rt}+y0(1-e^{-rt})}$

Hence,

$$y(t) = \frac{ky0}{ke^{-rt}+y0(1-e^{-rt})}$$

This is the complete solution of logistic growth model.

1. From the solution we have,

$$y\ (t) = \frac{ky0}{ke^{-rt}+y0(1-e^{-rt})}$$

When $t \to \infty, we\ have$

$$y\ (t)\ \to \frac{ky0}{y.0+y0(1-0)}$$

Or, $y(t)\ \to \frac{ky0}{y0}$

Hence,

$y\ (t)\ \to\ k$

This is the limiting behavior of the model as $t \to \infty$.

The Logistic Growth Model

$$\frac{dy}{dt} = ry\left(1-\frac{y}{k}\right)\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

is a single species model and r, k are constants.

For equilibrium point

$$\frac{dy}{dt} = 0$$

$$ry\left(1-\frac{y}{k}\right) = 0$$

Or, $y\left(1-\frac{y}{k}\right) = 0$

Or, $y=0$ or, $\left(1-\frac{y}{k}\right) = 0$

Hence

$$y=k$$

So, y = 0, k are the equilibrium points.

a) $y = 0\ is\ unstable\ .Since\ linearization\ it\ gives\ \frac{dy}{dt} \approx ry$ and so y grows exponentially from any small initial value.

b) $For\ linerization\ about\ \ N = k, we\ put\ N = k + \varepsilon,$ With ε small so that we have.

$$\frac{d(k+\varepsilon)}{dt} = r(k+\varepsilon)\left(1-\frac{k+\varepsilon}{k}\right)$$

Or, $\frac{d\varepsilon}{dt} = r(k+\varepsilon)\left(1-1-\frac{\varepsilon}{k}\right)$

Or, $\frac{d\varepsilon}{dt} = r(k+\varepsilon)\left(-\frac{\varepsilon}{k}\right)$

Or, $\frac{d\varepsilon}{dt} = -r\varepsilon$, to the first order

Or, $\frac{d(y-k)}{dt} \approx -r(y- \ k)$

Which gives y$\rightarrow$ k as t $\rightarrow \infty$

Hence, y=k is stable.

The Proposed Growth Model

The derivative of new function is defined as

$$y(t)=k\left(\frac{2}{e^{-2ct}\frac{k-y0}{k+y0}+1} - 1\right)$$

The Proposed growth function is very similar to the Gompertz[2] and Logistic model[1] but the inflection point for the logistic model is at the 50% of the curve and Gompertz model is at the 37% of the Long-term saturation levels but the proposed growth model shows the 10% of the long-term saturation a symmetric model.

4.1 Solution of Proposed Growth Model

Another model for a growth function for a limited population is given by the proposed function, which is the solution for the differential equation

$$\frac{dy}{dt} = c(y+k)(1- \ \frac{y}{k}\ ) \ \ \ldots\ldots\ldots (1)$$

Where c is the Growth rate and k is the carrying capacity, y is a population Number. By using variable separable method in equation (1)

$\frac{dy}{dt} = \frac{c}{k}$ (y+k) (k-y)

or $\frac{dy}{dt} = \frac{c}{k}(k^2 - y^2)$

Or $\frac{dy}{(k^2-y^2)} = \frac{c}{k}$ dt

Or $\int \frac{dy}{k^2-y^2} = \frac{c}{k}\int dt$

Or $\frac{1}{2k}$ ln$\left|\frac{k+y}{k-y}\right|$+d1 = $\frac{c}{k}$t +d2

Or $\frac{1}{2k}$ ln| $\frac{k+y}{k-y}$| =$\frac{c}{k}$t +d2-d1

Or $\frac{1}{2k}$ ln| $\frac{k+y}{k-y}$| = $\frac{c}{k}$t+ M

Or $\frac{k+y}{k-y}$ =e^{2ct}. e^{2kM}……………………… (2)

when t=0 and y (0) =y0

e^{2kM}= $\frac{k+y0}{k-y0}$

From (2) we can write

$\frac{k+y}{k-y}$ =e^{2ct}. $\frac{k+y0}{k-y0}$

Or $\frac{k-y}{k+y}$ = e^{-2ct} . $\frac{k-y0}{k+y0}$

Or 1+ $\frac{k-y}{k+y}$ = e^{-2ct} . $\frac{k-y0}{k+y0}$+1

Or $\frac{2k}{k+y}$ = e^{-2ct} . $\frac{k-y0}{k+y0}$ +1

Or 2k = (k+y) (e^{-2ct} . $\frac{k-y0}{k+y0}$ +1)

Or y = $\frac{2k-k(e^{-2ct}.\frac{k-y0}{k+y0}+1}{e^{-2ct}.\frac{k-y0}{k+y0}+1}$

Or y = k ($\frac{2}{e^{-2ct}.\frac{k-y0}{k+y0}+1}$ -1)

When t→∞ we have

y(t) = k ($\frac{2}{e^{\infty}+1} - 1$)

= k

∴ y(t) → k

This is the limiting behavior of the Model as t→∞

4.2 Comparison among Logistic model, Gompertz growth model, the Proposed Model

1. By using both (Gompertz model & Logistic model), we can plot graphical representation of growth level for any given data. Similarly, by using methods, we can graphically represent the manpower sector & their related data.

2. The New growth function is very similar to the gompertz and Logistic model but the inflection point

for the logistic model is at the 50% of the curve and Gompertz model is at the 37% of the Long-term structure levels but the proposed growth model shows the 10% of the long-term saturation a symmetric model.

2. Graphical comparison Gompetz, Logistic Model, the proposed model:

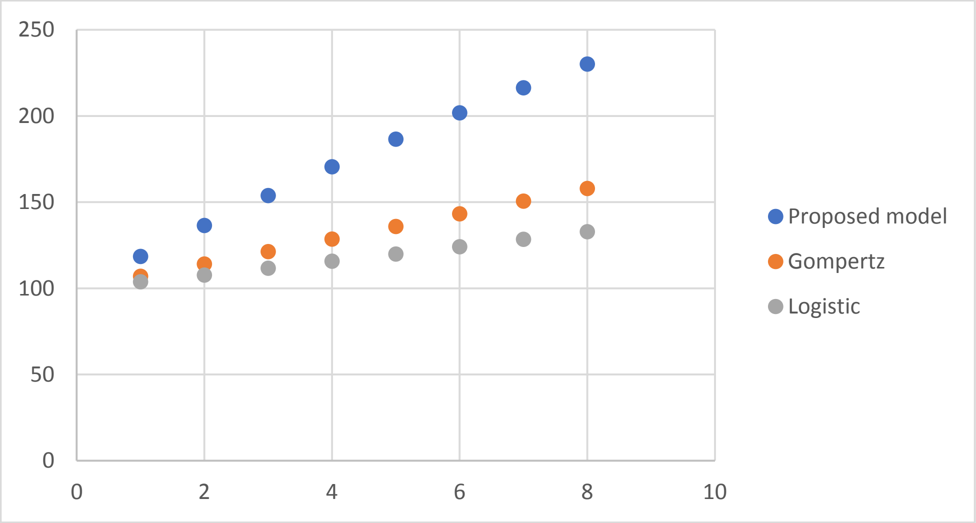

Fig 1: Comparision Logistic,Gompertz,and The proposed Model

Experiments

Data Collection

The secondary data had been collected from Bangladesh Bureau of Statistics, Ministry of Finance, Ministry of Labor and Employment, Ministry of Expatriate Welfare and Overseas Employment, Bureau of Manpower, Employment and Training, IOM, BIDS and Bangladesh Bank.

Model on Remittances Data

For Remittances transaction, we used the data from 2009 to 2019. Which are given in the chart below:

Year	Remittances (In Billion Taka)
2009-2010	760.11
2010-2011	829.91
2011-2012	1018.82
2012-2013	1156.47
2013-2014	1105.84
2014-2015	1189.93
2015-2016	1168.57

2016-2017	1010.99
2017-2018	1231.56
2018-2019	1380.07

Table-1: The comparison among all the selective model

Time	**Remittances (In Billion Taka)**	**Proposed Model (**y = k $\left(\frac{2}{e^{-2ct}\cdot\frac{k-y0}{k+y0}+1}-1\right)$	**Error in Percentage**	**Gompertz Model** y_fit=k*exp(Ln(Y0/k)*exp(-c*t))	**Error in Percentage**	**Logistic Model** y_fit=((k*y0)/((k*exp(-c*t))+(y0*(1-exp(-c*t)))))	**Error in Percentage**
1	760.11	850.9463	-11%	804.4997	-6%	794.0553	-4%
2	829.91	931.2643	-11%	846.8952	-2%	827.4931	0%
3	1018.82	1001.46	2%	887.1783	15%	860.272	18%
4	1156.47	1062.186	9%	925.2768	25%	892.2526	30%
5	1170.15	1114.259	5%	961.1578	22%	923.3104	27%
6	1189.93	1158.576	3%	994.822	20%	953.3365	25%
7	1210.35	1196.049	1%	1026.298	18%	982.2392	23%
8	1231.56	1227.563	0%	1055.635	17%	1009.944	22%
9	1380.07	1253.945	10%	1082.903	27%	1036.395	33%
	Total		**1%**		**15%**		**19%**
	Fitting Data						
Y0	k	c					
760.11	1380.07	0.1					

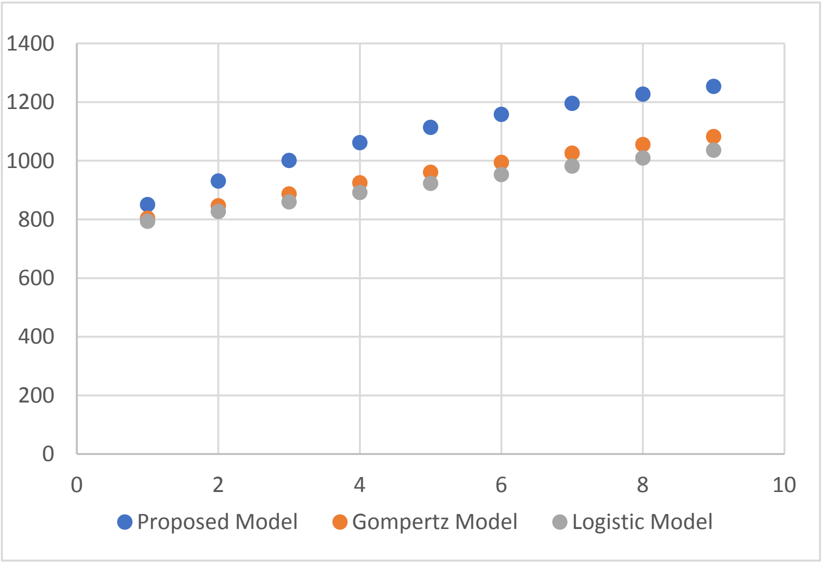

Fig2: Comparision Logistic,Gompertz,and The proposed Model using Remittance Data

For GDP, we used the data from 2013 to 2029. Which are given in the chart below:

Year	**GDP (In Million Taka)**
2013-2014	13.436744
2014-2015	15.158022
2015-2016	17.328637
2016-2017	19.758154
2017-2018	22.504793
2018-2019	25.361770

Table-2: The comparison among all the selective model

Time	GDP (In Million Taka	Proposed Model (y = k $(\frac{2}{e^{-2ct}\cdot\frac{k-y0}{k+y0}+1}$ -1)	Error in Percentage	Gompertz Model y_fit=k*exp(Ln(Y0/k)*exp (-c*t))	Error in Percentage	Logistic Model y_fit=((k*y0)/((k*exp(-c*t))+(y 0*(1-exp(-c*t)))))	Error in Percentage
1	13.436744	15.32577	-12%	14.35595	-6%	14.12875	-5%
2	15.158022	16.98019	-11%	15.23268	0%	14.81212	2%
3	17.328637	18.4085	-6%	16.06345	8%	15.483	12%
4	19.758154	19.62638	1%	16.84609	17%	16.13777	22%
5	22.504793	20.65388	9%	17.57954	28%	16.77322	34%
6	25.36177	21.51306	18%	18.26366	39%	17.38653	46%
Total			0%		14%		19%
Fitting Data							
Y0	k	c					
13.43674	25.36177	0.11					

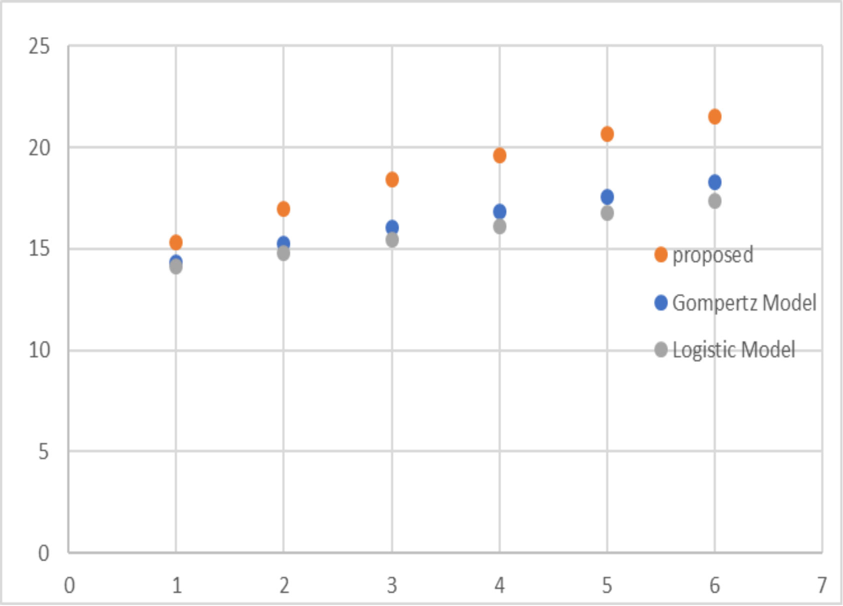

Fig3: Comparision Logistic,Gompertz,and The proposed Model using GDP Data

For Overseas Employment (Female Workers), we used the data from 1991 to 2020. Which are given in the chart below:

Year	**Overseas Employment (Female Workers)**
1991-1995	9496
1996-2000	14125
2001-2005	29057
2006-2010	107911
2011-2015	331714
2016-2020	439138

Table-3: The comparison among all the selective model

Time	Overseas employment (Female Worker)	Proposed Model (y = k $\left(\frac{2}{e^{-2ct}\cdot\frac{k-y0}{k+y0}+1}-1\right)$	Error in Percentage	Gompertz Model y_fit=k*exp(Ln(Y0/k)*exp (-c*t))	Error in Percentage	Logistic Model y_fit=((k*y0)/((k*exp(-c*t))+(y0*(1-exp(-c*t)))))	Error in Percentage
1	9496	18173.81	-91%	10143.45	-7%	9585.748	-1%
2	14125	26933.11	-91%	10929.21	23%	9775.082	31%
3	29057	35670.95	-23%	11758.46	60%	9968.069	66%
4	107911	44380.45	59%	12632.32	88%	10164.78	91%
5	331714	53054.81	84%	13551.87	96%	10365.27	97%
6	439138	61687.37	86%	14518.15	97%	10569.62	98%
Total			4%		59%		63%
Fitting Data							
Y0	k	c					
9400	439138	0.02					

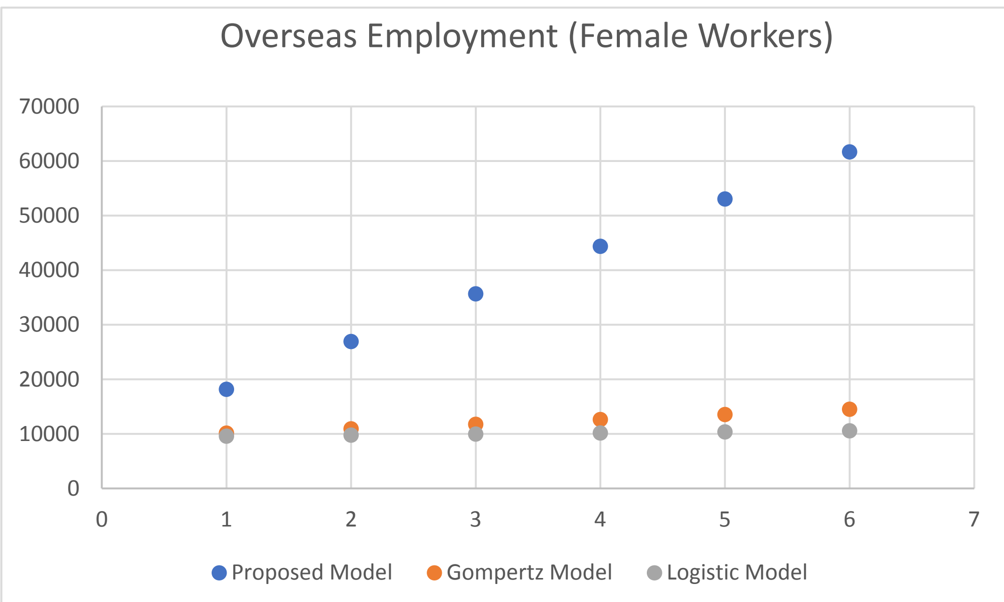

Fig 4: Overseas Employment

Conclusion

The main destination in this work has been successfully achieve. We compare different kinds of growth model with our Proposed Model and prediction of the Growth of Manpower in Bangladesh. We found that our proposed model shows the best result from other model result, and error are so closed the obtaining result. The inflection point is so high than Gompertz and Logistic Model. The Proposed growth function is very similar to the Gompertz and Logistic model but the inflection point for the logistic model is at the 50% of the curve and Gompertz model is at the 37% of the Long-term saturation levels but the proposed growth model shows the 10% of the long-term saturation a symmetric model. We can be predicted what's number of people are going aboard in future from Bangladesh by our proposed Model. In future, we have plan to compare the porposed model with the regression model [16].

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