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Fuzzy m-β-Irresolute Function

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Abstract

In [3], fuzzy m-β-open set is introduced. Using this concept as a basic tool, in this paper we introduce β-irresolute function in fuzzy m-space, termed as fuzzy m-β-irresolute function. Afterwards, it is shown that fuzzy m-β-irresolute function is fuzzy m-e^*-continuous function [3] as well as fuzzy almost e^*-continuous function [3], but not conversely. Lastly some applications of this newly defined function are given.

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1. Introduction

Fuzzy minimal structure (m-structure, for short) is introduced by Alimohammady and Roohi in [1] as follows : A family $\mathcal{M}$ of fuzzy sets in a non-empty set X is said to be fuzzy minimal structure on X if $\alpha 1_X \in \mathcal{M}$ for every $\alpha \in [0, 1]$. A more general version of fuzzy minimal structure (in the sense of Chang) are introduced in [5, 8] as follows : A family $\mathcal{F}$ of fuzzy sets in a non-empty set X is a fuzzy minimal structure on X if $0_X \in \mathcal{F}$ and $1_X \in \mathcal{F}$. Throughout this paper, we use the notion of fuzzy minimal structure in the sense of Chang. Using this concept in [2] fuzzy m-space is introduced and studied. Fuzzy m-open set [2], fuzzy m-β-open set [3], fuzzy m-e^*-open set [3] are introduced and found their interrelations in [3]. In [3], fuzzy m-compact, fuzzy m-e^*-compact, fuzzy m-P-compact, fuzzy m-P-closed spaces are introduced. Here we introduce fuzzy m-β-compact, fuzzy m-semicompact, fuzzy m-S-closed, fuzzy m-s-closed spaces. Introducing fuzzy m-β-irresolute function, we have shown that fuzzy m-β-compact space remains invariant under fuzzy m-β-irresolute function. Again it is shown that the image of a fuzzy m-β-compact space under fuzzy m-β-irresolute function is fuzzy m-semicompact as well as fuzzy m-S-closed space.

2. Preliminaries

A fuzzy set [10] A is a mapping from a non-empty set X into the closed interval $I = [0, 1]$, i.e., $A \in I^X$. The support [10] of a fuzzy set A, denoted by $suppA$ and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement [10] of a fuzzy set A in X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ [10] while AqB means A is quasi-coincident (q-coincident, for short) [9] with B, i.e., there exists $x \in X$ such that $A(x) + B(x) > 1$. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. For a fuzzy point x_α and a fuzzy set A in X, $x_\alpha \in A$ means $x_\alpha \leq A$, i.e., $A(x) \geq \alpha$.

3. Some Well-Known Definitions, Proposition, Lemma and Theorem in Fuzzy m-Space

Let X be a non-empty set and m be a fuzzy minimal structure on X. Then the members of m are called fuzzy m-open sets and (X, m) is called a fuzzy m-space [2]. The complement of a fuzzy m-open set in a fuzzy m-space is called a fuzzy m-closed set.

Definition 3.1 [2]. Let (X, m) be a fuzzy m-space. For $A \in I^X$, the fuzzy m-closure and fuzzy m-interior of A, denoted by $mclA$ and $mintA$ respectively, are defined as follows :

$$mclA = \bigwedge\{F : A \leq F, 1_X \setminus F \in m\}$$

$$mintA = \bigvee\{D : D \leq A, D \in m\}$$

.

It is to be noted that given a fuzzy minimal structure m on X, if $A \in I^X$, then $mintA$ may not be an element of m. But if m satisfies M-condition (i.e., m is closed under arbitrary union) [2], then $mintA$ is an element of m.

Proposition 3.2 [2]. Let (X, m) be a fuzzy m-space. Then for any $A \in I^X$, a fuzzy point $x_\alpha \in mclA$ iff for any $U \in m$ with $x_\alpha qU$, UqA.

Lemma 3.3 [2]. Let (X, m) be a fuzzy m-space. For $A, B \in I^X$, the following statements are true:

(i) $A \leq B \Rightarrow mintA \leq mintB$, $mclA \leq mclB$,

(ii) $mint0_X = 0_X$, $mint1_X = 1_X$, $mcl0_X = 0_X$, $mcl1_X = 1_X$,

(iii) $mintA \leq A \leq mclA$,

(iv) $mclA = A$ if $1_X \setminus A \in m$, $mintA = A$, if $A \in m$,

(v) $mcl(1_X \setminus A) = 1_X \setminus mintA$, $mint(1_X \setminus A) = 1_X \setminus mclA$,

(vi) $mcl(mclA) = mclA$ and $mint(mintA) = mintA$.

Definition 3.4 [3]. Let (X, m) be a fuzzy m-space and $A \in I^X$. Then A is said to be

(i) fuzzy m-regular open if $A = mint(mclA)$,

(ii) fuzzy m-semiopen if $A \leq mcl(mintA)$,

(iii) fuzzy m-preopen if $A \leq mint(mclA)$,

(iv) fuzzy m-β-open if $A \leq mcl(mint(mclA))$.

The complement of the above mentioned sets are called their respective closed sets.

The infimum of all fuzzy m-semiclosed (resp., fuzzy m-preclosed, fuzzy m-β-closed) sets containing a fuzzy set A in a fuzzy m-space (X, m) is called fuzzy m-semiclosure (resp., fuzzy m-preclosure, fuzzy m-β-closure) of A, denoted by $msclA$ (resp., $mpclA$, $m\beta clA$).

The family of all fuzzy m-regular open (resp., fuzzy m-semiopen, fuzzy m-preopen, fuzzy m-β-open) sets is denoted by $mRO(X)$ (resp., $mSO(X)$, $mPO(X)$, $m\beta O(X)$). The family of all fuzzy m-β-closed sets in a fuzzy m-space (X, m) is denoted by $m\beta C(X)$.

Definition 3.5 [3]. Let (X, m) be a fuzzy m-space and $A \in I^X$. The fuzzy m-δ-closure and fuzzy m-δ-interior of A, denoted by $m\delta clA$ and $m\delta intA$ respectively, are defined by :

$$m\delta clA = \{x_\alpha \in X : Aqmint(mclU), \mathit{for\ all}\, U \in m,\, x_\alpha qU\}$$

$$m\delta intA = \bigvee\{W : W \leq A, W \in mRO(X)\}$$

.

Definition 3.6 [3]. Let (X, m) be a fuzzy m-space and $A \in I^X$. Then A is called fuzzy m-e^*-open if $A \leq mcl(mint(m\delta clA))$.
The complement of a fuzzy m-e^*-open set is called fuzzy m-e^*-closed.

The family of all fuzzy m-e^*-open sets in a fuzzy m -space (X, m) is denoted by $me^*O(X)$.

Remark 3.7. It is shown in [3] that $m\beta O(X) \subseteq me^*O(X)$. But not conversely. And arbitrary union of fuzzy m-β-open sets is fuzzy m-β-open.

Definition 3.8 [3]. Let (X, m) and $(Y, m^{'})$ be two fuzzy m-spaces and $f : (X, m) \to (Y, m^{'})$ be a function. Then f is called fuzzy
(i) $(m, m^{'})$-e^*-continuous if $f^{-1}(A) \in me^*O(X)$ for all $A \in m^{'}$,
(ii) $(m, m^{'})$-almost-e^*-continuous if $f^{-1}(A) \in me^*O(X)$ for all $A \in m^{'}RO(Y)$.

4. Fuzzy $(m, m^{'})$-β-Irresolute Function : Some Characterizations

In this section we first introduce fuzzy m-β-nbd (resp., fuzzy m-β-q-nbd) of a fuzzy point and then introduce fuzzy $(m, m^{'})$-β-irresolute function and characterize it in several ways.

Definition 4.1. A fuzzy set A in a fuzzy m-space (X, m) is called a fuzzy m-β-nbd of a fuzzy point x_α if there exists a fuzzy m-β-open set U in X such that $x_\alpha \leq U \leq A$.

Definition 4.2. A fuzzy set A in a fuzzy m-space (X, m) is called a fuzzy m-β-q-nbd of a fuzzy point x_α if there exists a fuzzy m-β-open set U in X such that $x_\alpha qU \leq A$. If, in addition, A is fuzzy m-β-open, then A is called fuzzy m-β-open-q-nbd of x_α.

Definition 4.3. Let (X, m) and $Y, m^{'})$ are two fuzzy m-spaces. A function $f : (X, m) \to (Y, m^{'})$ is said to be fuzzy $(m, m^{'})$-β-irresolute if $f^{-1}(A) \in m\beta O(X)$ for each $A \in m^{'}\beta O(Y)$.

Theorem 4.4. Let (X, m) and $(Y, m^{'})$ are two fuzzy m-spaces and $f : (X, m) \to (Y, m^{'})$ be a function where m and $m^{'}$ satisfy M-condition. Then the following statements are equivalent :

(a) f is fuzzy $(m, m^{'})$-β-irresolute,

(b) for each fuzzy point x_α in X and each $A \in m^{'}\beta O(Y)$ with $f(x_\alpha) \leq A$, there exists $B \in m\beta O(X)$ such that $x_\alpha \leq B$ and $f(B) \leq A$,

(c) $f^{-1}(B) \in m\beta C(X)$ for each $B \in m^{'}\beta C(Y)$,

(d) for each fuzzy point x_α in X, the inverse of each fuzzy $m^{'}$-β-nbd B of $f(x_\alpha)$ in Y is a fuzzy m-β-nbd of x_α in X,

(e) for each fuzzy point x_α in X and each fuzzy $m^{'}$-β-nbd B of $f(x_\alpha)$, there exists a fuzzy m-β-nbd C of x_α in X such that $f(C) \leq B$,

(f) for each fuzzy set D in X, $f(m\beta clD) \leq m^{'}\beta cl(f(D))$,

(g) for each fuzzy set B in Y, $m\beta cl(f^{-1}(B)) \leq f^{-1}(m^{'}\beta clB)$.

Proof. (b) $\Rightarrow$ (a) Let A be a fuzzy $m^{'}$-β-open set in Y and x_α, a fuzzy point in $f^{-1}(A)$. Then $f(x_\alpha) \leq A$. By (b), there exists a fuzzy m-β-open set B in X such that $x_\alpha \leq B$ and $f(B) \leq A$. Thus $B \leq f^{-1}(A)$. We have to show that $f^{-1}(A) \leq mcl(mint(mcl(f^{-1}(A))))$. As $B \in m\beta O(X)$, $x_\alpha \leq B \leq mcl(mint(mclB)) \leq mcl(mint(mcl(f^{-1}(A)))) \Rightarrow f^{-1}(A) \leq mcl(mint(mcl(f^{-1}(A))))$.

(a) $\Rightarrow$ (c). Let $B \in m^{'}\beta C(Y)$. Then $1_Y \backslash B \in m^{'}\beta O(Y)$. By (a), $f^{-1}(1_Y \backslash B) = 1_X \backslash f^{-1}(B) \in m\beta O(X) \Rightarrow f^{-1}(B) \in m\beta C(X)$.

(c) $\Rightarrow$ (a) Straightforward.

(a) $\Rightarrow$ (d) Let x_α be a fuzzy point in X and B, a fuzzy $m^{'}$-β-nbd of $f(x_\alpha)$ in Y. Then there exists $U \in m^{'}\beta O(Y)$ such that $f(x_\alpha) \leq U \leq B$. Then $x_\alpha \in f^{-1}(U) \leq f^{-1}(B)$. By (a), $f^{-1}(U) \in m\beta O(X)$. Hence the proof.

(d) $\Rightarrow$ (e) Since $f(f^{-1}(B)) \leq B$, the result follows by taking $C = f^{-1}(B)$.

(e) $\Rightarrow$ (b) Let x_α be a fuzzy point in X and A, any fuzzy $m^{'}$-β-open set in Y with $f(x_\alpha) \leq A$. Then A is a fuzzy $m^{'}$-β-nbd of $f(x_\alpha)$ in Y. By (e), there exists a fuzzy m-β-nbd C of x_α in X such that $f(C) \leq A$. Therefore, there exists $U \in m\beta O(X)$ such that $x_\alpha \leq U \leq C$ and so $f(U) \leq f(C) \leq A \Rightarrow f(U) \leq A$.

(c) $\Rightarrow$ (f) Let D be any fuzzy set in X. Then $m^{'}\beta cl(f(D))$ is fuzzy $m^{'}$-β-closed set in Y as $m^{'}$ satisfies M-condition. By (c), $f^{-1}(m^{'}\beta cl(f(D))) \in m\beta C(X)$. Now $D \leq f^{-1}(f(D)) \leq f^{-1}(m^{'}\beta cl(f(D)))$, i.e., $m\beta clD \leq m\beta cl(f^{-1}(m^{'}\beta cl(f(D)))) = f^{-1}(m^{'}\beta cl(f(D)))$ as m satis-

fies M-condition. Therefore, $f(m\beta clD) \leq m'\beta cl(f(D))$.
(f) $\Rightarrow$ (c) Let $B \in m'\beta C(Y)$. Put $D = f^{-1}(B)$. By (f), $f(m\beta clD) \leq m'\beta cl(f(D)) = m'\beta cl(f(f^{-1}(B))) \leq m'\beta clB = B$. Thus $m\beta clD \leq f^{-1}(f(m\beta clD)) \leq f^{-1}(B) = D$. Hence $D = f^{-1}(B) \in m\beta C(X)$.
(f) $\Rightarrow$ (g) Let $B \in I^Y$. Let $D = f^{-1}(B)$. By (f), $f(m\beta clD) \leq m'\beta cl(f(D))$, i.e., $m\beta clD \leq f^{-1}(m'\beta cl(f(D)))$, i.e., $m\beta cl(f^{-1}(B)) \leq f^{-1}(m'\beta cl(f(f^{-1}(B)))) \leq f^{-1}(m'\beta clB)$.
(g) $\Rightarrow$ (f) Let $D \in I^X$. By (g), $m\beta cl(f^{-1}(f(D))) \leq f^{-1}(m'\beta cl(f(D))) \Rightarrow m\beta clD \leq f^{-1}(m'\beta cl(f(D))) \Rightarrow f(m\beta clD) \leq m'\beta cl(f(D))$.

Theorem 4.5. A function $f : (X, m) \to (Y, m')$ is fuzzy (m, m')-β-irresolute function iff for each fuzzy point x_α in X and any fuzzy m'-β-open-q-nbd V of $f(x_\alpha)$ in Y, there exists a fuzzy m-β-open-q-nbd U of x_α in X such that $f(U) \leq V$.

Proof. Let $f : (X, m) \to (Y, m')$ be fuzzy (m, m')-β-irresolute function and x_α be a fuzzy point in X. Let V be a fuzzy m'-β-open-q-nbd of $f(x_\alpha)$ in Y. Then $f^{-1}(V)$ ($= U$, say) is a fuzzy m-β-open-q-nbd of x_α in X such that $f(U) = f(f^{-1}(V)) \leq V$.

Conversely, let x_α be a fuzzy point in X and V be any fuzzy m'-β-open set containing $f(x_\alpha)$. Let K_α be a positive integer such that $1/K_\alpha < \alpha$. Then $0 < 1 - \alpha + 1/n = t_n$ (say) < 1, for all $n \geq K_\alpha$. Now $y_{t_n} qV$ for each $n \geq K_\alpha$, where $y = f(x)$. Then by hypothesis, there exists a fuzzy m-β-open set U_n in X such that $x_{t_n} qU_n$ and $f(U_n) \leq V$, for all $n \geq K_\alpha$. Put $U = \bigcup_{n \geq K_\alpha} U_n$. Then $U \in m\beta O(X)$ (by Note 3.7) such that $f(U) \leq V$. Also $t_n + U_n(x) > 1$ for all $n \geq K_\alpha \Rightarrow 1 - \alpha + 1/n + U_n(x) > 1$ for all $n \geq K_\alpha \Rightarrow \alpha < U_n(x) + 1/n$ for all $n \geq K_\alpha \Rightarrow \alpha \leq sup_{n \geq K_\alpha} U_n(x) = U(x) \Rightarrow x_\alpha \leq U$. Hence by Theorem 4.4, F is fuzzy (m, m')-β-irresolute function.

5. Mutual Relationship

In [4], fuzzy (m, m')-irresolute function is defined and studied. In this section we first show that fuzzy (m, m')-irresolute function and fuzzy (m, m')-β-irresolute function are independent concepts. In [3], we have introduced fuzzy (m, m')-e^*-continuous function and fuzzy (m, m')-almost-e^*-continuous function. It is obvious that fuzzy (m, m')-β-irresolute function is fuzzy (m, m')-e^*-continuous function as well as fuzzy (m, m')-almost-e^*-continuous function. But the converses are not true, in general.

Definition 5.1 [4]. Let (X, m) and $(Y, m^{'})$ be two fuzzy m-spaces. Then a function $f : (X, m) \to (Y, m^{'})$ is said to be fuzzy $(m, m^{'})$-irresolute if $f^{-1}(A) \in mSO(X)$ for each $A \in m^{'}SO(Y)$.

Remark 5.2. The next two examples show that fuzzy $(m, m^{'})$-irresolute function and fuzzy $(m, m^{'})$-β-irresolute function are independent concepts.

Example 5.3. Fuzzy $(m, m^{'})$-β-irresolute function $\not\Rightarrow$ fuzzy $(m, m^{'})$-irresolute function
Let $X = \{a, b\}$, $m = \{0_X, 1_X, A\}$, $m^{'} = \{0_X, 1_X, C\}$ where $A(a) = 0.5, A(b) = 0.4$ and $C(a) = 0.6, C(b) = 0.5$. Then (X, m) and $(X, m^{'})$ are two fuzzy m-spaces. Consider the function $f : (X, m) \to (X, m^{'})$ defined by $f(a) = b, f(b) = a$. We claim that f is fuzzy $(m, m^{'})$-β-irresolute function, but not fuzzy $(m, m^{'})$-irresolute function. Now $mSO(X) = \{0_X, 1_X, U\}$ where $A \leq U \leq 1_X \setminus A$ and $m^{'}SO(X) = \{0_X, 1_X, V\}$ where $V \geq C$. Again any fuzzy set in (X, m) is fuzzy m-β-open in (X, m) and $m^{'}\beta O(X) = \{0_X, 1_X, W\}$ where $W \not\leq 1_X \setminus C$. Let $B \in m^{'}SO(X)$ be defined by $B(a) = B(b) = 0.6$. Now $[f^{-1}(B)](a) = B(f(a)) = B(b) = 0.6$, $[f^{-1}(B)](b) = B(f(b)) = B(a) = 0.6$ and so $f^{-1}(B) \notin mSO(X)$. Therefore, f is not fuzzy $(m, m^{'})$-irresolute function. Since any fuzzy set in (X, m) is fuzzy m-β-open in (X, m), f is clearly fuzzy $(m, m^{'})$-β-irresolute function.

Example 5.4. Fuzzy $(m, m^{'})$-irresolute function $\not\Rightarrow$ fuzzy $(m, m^{'})$-β-irresolute function
Let $X = \{a, b\}$, $m = \{0_X, 1_X, A\}$, $m^{'} = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.7$ and $B(a) = 0.6, B(b) = 0.7$. Then (X, m) and $(X, m^{'})$ are two fuzzy m-spaces. Consider the identity function $i : (X, m) \to (X, m^{'})$. Now $mSO(X) = \{0_X, 1_X, V\}$ where $V \geq A$ and $m\beta O(X) = \{0_X, 1_X, A, U\}$ where $U \not\leq 1_X \setminus A$. Again, $m^{'}SO(X) = \{0_X, 1_X, C\}$ where $B \leq C$ and $m^{'}\beta O(X) = \{0_X, 1_X, W\}$ where $W \not\leq 1_X \setminus B$. We claim that i is fuzzy $(m, m^{'})$-irresolute function, but not fuzzy $(m, m^{'})$-β-irresolute function. Now $[i^{-1}(C)](a) = C(i(a)) = C(a) \geq B(a)$, $[i^{-1}(C)](b) = C(i(b)) = C(b) \geq B(b)$ and $B \geq A \Rightarrow i^{-1}(C) \geq A \Rightarrow i^{-1}(C) \in mSO(X)$ which shows that i is fuzzy $(m, m^{'})$-irresolute function. But $W(a) = 0.6, W(b) = 0.3$ being a fuzzy $m^{'}$-β-open set in $(X, m^{'})$ and $i^{-1}(W) = W \notin m\beta O(X) \Rightarrow i$ is not fuzzy $(m, m^{'})$-β-irresolute function.

Example 5.5. Fuzzy (m, m')-e^*-continuous, fuzzy (m, m')-almost-e^*-continuous function $\not\Rightarrow$ fuzzy (m, m')-β-irresolute function

Let $X = \{a, b\}$, $m = \{0_X, 1_X, A\}$, $m' = \{0_X, 1_X\}$ where $A(a) = 0.5, A(b) = 0.6$. Then (X, m) and (X, m') are two fuzzy m-spaces. Consider the identity function $i : (X, m) \to (X, m')$. Clearly i is fuzzy (m, m')-e^*-continuous as well as fuzzy (m, m')-almost-e^*-continuous function. Now any fuzzy set in (X, m') is fuzzy m'-β-open. Consider the fuzzy set B defined by $B(a) = 0.5, B(b) = 0.3$. Then $B \in m'\beta O(X)$. Now $i^{-1}(B) = B \not\leq mcl(mint(mclB)) = 0_X \Rightarrow B \notin m\beta O(X) \Rightarrow i$ is not fuzzy (m, m')-β-irresolute function.

Result 5.6. In a fuzzy m-space (X, m), $m\delta clA = mclA$, for all $A \in mSO(X)$.

Proof. It is clear from definition that $mclA \leq m\delta clA$. So we have to show that $m\delta clA \leq mclA$, for all $A \in mSO(X)$.

Let x_α be a fuzzy point in X such that $x_\alpha \in m\delta clA$, but $x_\alpha \notin mclA$. Then by Proposition 3.2, there is a fuzzy m-open set U in X with $x_\alpha qU$, but $U \not{q} A$. Then $U \leq 1_X \setminus A \Rightarrow mint(mclU) \leq mint(mcl(1_X \setminus A)) = 1_X \setminus mcl(mintA) \leq 1_X \setminus A$ (as $A \in mSO(X)$, $A \leq mcl(mintA) \Rightarrow 1_X \setminus A \geq 1_X \setminus mcl(mintA)) \Rightarrow mint(mclU) \not{q} A \Rightarrow x_\alpha \notin m\delta clA$, a contradiction.

Definition 5.7. A fuzzy m-space (X, m) is called fuzzy

(i) mT_β-space if every fuzzy m-β-open set in X is fuzzy m-open,

(ii) mT_{e^*}-space if every fuzzy m-e^*-open set in X is fuzzy m-β-open.

Remark 5.8. In a fuzzy m-space (X, m), if a fuzzy set A is fuzzy m-semiopen, then A is fuzzy m-e^*-open iff it is fuzzy m-β-open and so a function $f : (X, m) \to (Y, m')$ where X is fuzzy mT_{e^*}-space and Y is fuzzy mT_β-space is fuzzy (m, m')-β-irresolute iff it is fuzzy (m, m')-e^*-continuous.

6. Applications

In this section we first recall some definitions for ready references.

Definition 6.1 [6, 7]. Let A be a fuzzy set. A collection $\mathcal{U}$ of fuzzy sets is called a fuzzy cover of A if $sup\{U(x) : U \in \mathcal{U}\} = 1$ for each $x \in suppA$. If, in addition, $A = 1_X$, we get the definition of fuzzy cover of X.

Definition 6.2 [6, 7]. A fuzzy cover $\mathcal{U}$ of a fuzzy set A is said to have a finite subcover $\mathcal{U}_0$, if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigcup \mathcal{U}_0 \geq A$. If, in particular, $A = 1_X$, then the requirement on $\mathcal{U}_0$ is $\bigcup \mathcal{U}_0 = 1_X$.

Definition 6.3. A fuzzy set A in a fuzzy m-space (X, m) is said to be fuzzy m-compact (resp., fuzzy m-e^*-compact) if every fuzzy cover of A by fuzzy m-open (resp., fuzzy $me^*O(X)$) sets in X has a finite subcover $\mathcal{U}_0$ of $\mathcal{U}$. If, in particular, $A = 1_X$, we get the definition of fuzzy m-compact (resp., fuzzy m-e^*-compact) space.

Definition 6.4. A fuzzy set A in a fuzzy m-space (X, m) is said to be fuzzy m-β-compact (resp., fuzzy m-semicompact, fuzzy m-precompact) if for every cover of A by fuzzy m-β-open (resp., fuzzy m-semiopen, fuzzy m-preopen) sets of X has a finite subcover. If, in particular, $A = 1_X$, we get the definition of fuzzy m-β-compact (resp., fuzzy m-semicompact, fuzzy m-precompact) space.

Definition 6.5. A fuzzy m-space (X, m) is said to be fuzzy m-S-closed (resp., fuzzy m-s-closed, fuzzy m-β-closed, fuzzy m-P-closed) if every cover $\mathcal{U}$ of X by fuzzy m-semiopen (resp., fuzzy m-semiopen, fuzzy m-β-open, fuzzy m-preopen) sets of X has a finite subfamily $\mathcal{U}_0$ of $\mathcal{U}$ such that $\bigcup\{mclU : U \in \mathcal{U}_0\} = 1_X$ (resp., $\bigcup\{msclU : U \in \mathcal{U}_0\} = 1_X$, $\bigcup\{m\beta clU : U \in \mathcal{U}_0\} = 1_X$, $\bigcup\{mpclU : U \in \mathcal{U}_0\} = 1_X$).

Remark 6.6. It is clear from definitions that fuzzy m-β-compact space is fuzzy m-compact. The converse is true only in fuzzy mT_β-space. Again, fuzzy m-e^*-compact space is fuzzy m-β-compact. The converse is true only in fuzzy mT_{e^*}-space. Also, fuzzy m-β-compact space is fuzzy m-precompact as well as fuzzy m-P-closed.

Theorem 6.7. Let (X, m) and (Y, m') be two fuzzy m-spaces where X is fuzzy m-β-compact space. Let $f : (X, m) \to (Y, m')$ be fuzzy (m, m')-β-irresolute, surjective function. Then Y is fuzzy m'-semicompact.

Proof. Let $\mathcal{V} = \{V_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of Y by fuzzy m'-semiopen sets of Y. Since fuzzy m-semiopen sets are fuzzy m-β-open, $\mathcal{U} = \{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$ is a fuzzy m-β-open sets of X which covers X as f is fuzzy (m, m')-β-irresolute function. As X is fuzzy m-β-compact space, there is a finite subfamily Λ_0 of Λ such that $\mathcal{U}_0 = \{f^{-1}(V_\alpha) : \alpha \in \Lambda_0\}$ also covers X, i.e., $1_X = \bigcup_{\alpha\in\Lambda_0} f^{-1}(V_\alpha) \Rightarrow 1_Y = f(1_X) = f(\bigcup_{\alpha\in\Lambda_0} f^{-1}(V_\alpha)) = \bigcup_{\alpha\in\Lambda_0} f(f^{-1}(V_\alpha)) \leq \bigcup_{\alpha\in\Lambda_0} V_\alpha \Rightarrow Y$ is fuzzy m'-semicompact space.

Note 6.8. Since every fuzzy m-semicompact space is fuzzy m-S-closed (resp., fuzzy m-s-closed, fuzzy m-precompact, fuzzy m-P-closed) space, then we can state the following theorem.

Theorem 6.9. Let (X, m) and (Y, m') be two fuzzy m-spaces where X is fuzzy m-β-compact space. Let $f : (X, m) \to (Y, m')$ be fuzzy (m, m')-β-irresolute, surjective function. Then Y is fuzzy m'-S-closed (resp., fuzzy m'-β-compact, fuzzy m'-precompact, fuzzy m'-P-closed) space.

Remark 6.10. Since for a fuzzy set A in a fuzzy m-space (X, m), $m\beta clA \leq msclA$, $m\beta clA \leq mpclA$, $m\beta clA \leq mclA$, we can state the following theorem easily.

Theorem 6.11. Let (X, m) and (Y, m') be two fuzzy m-spaces where X is fuzzy m-β-closed space. Let $f : (X, m) \to (Y, m')$ be fuzzy (m, m')-β-irresolute, surjective function. Then Y is fuzzy m'-S-closed (resp., fuzzy m'-s-closed, fuzzy m'-P-closed) space.

Theorem 6.12. Every fuzzy m-β-closed set A in a fuzzy m-β-compact space (X, m) is fuzzy m-β-compact.

Proof. Let A be a fuzzy m-β-closed set in a fuzzy m-β-compact space (X, m). Let $\mathcal{U}$ be a fuzzy cover of A by fuzzy m-β-open sets of X. Then $\mathcal{V} = \mathcal{U} \bigcup (1_X \setminus A)$ is a fuzzy m-β-open cover of X. By hypothesis, there exists a finite subcollection $\mathcal{V}_0$ of $\mathcal{V}$ which also covers X. If $\mathcal{V}_0$ contains $1_X \setminus A$, we omit it and get a finite subcover of A. Consequently, A is fuzzy m-β-compact set.

Theorem 6.13. Let (X, m) and (Y, m') be two fuzzy m-spaces and $f : (X, m) \rightarrow (Y, m')$ be fuzzy (m, m')-β-irresolute function. If A is fuzzy m-β-compact relative to X, then the image $f(A)$ is fuzzy m'-β-compact relative to Y.

Proof. Let $A(\in I^X)$ be fuzzy m-β-compact relative to X and $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of $f(A)$ by fuzzy m'-β-open sets of Y, i.e., $f(A) \leq \bigcup\limits_{\alpha\in\Lambda} U_\alpha \Rightarrow A \leq f^{-1}(\bigcup\limits_{\alpha\in\Lambda} U_\alpha) = \bigcup\limits_{\alpha\in\Lambda} f^{-1}(U_\alpha) \Rightarrow \mathcal{V} = \{f^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a fuzzy cover of A by fuzzy m-β-open sets of X as f is fuzzy (m, m')-β-irresolute function. As A is fuzzy m-β-compact relative to X, there exists a finite subcollection $\mathcal{V}_0 = \{f^{-1}(U_{\alpha_i}) : 1 \leq i \leq n\}$ of $\mathcal{V}$ such that $A \leq \bigcup\limits_{i=1}^{n} f^{-1}(U_{\alpha_i}) \Rightarrow f(A) \leq f(\bigcup\limits_{i=1}^{n} f^{-1}(U_{\alpha_i})) = \bigcup\limits_{i=1}^{n} f(f^{-1}(U_{\alpha_i})) \leq \bigcup\limits_{i=1}^{n} U_{\alpha_i} \Rightarrow \mathcal{U}_0 = \{U_{\alpha_i} : 1 \leq i \leq n\}$ is a finite subcover of $f(A)$. Hence the result.

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