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Comments on Some Transformation Methods of PV Solar Cells' I-V Characteristics from their Implicit Forms to Explicit Ones

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Abstract: Most of the times, an implicit form of a PV solar cell's I-V characteristic is used in the analysis of the electrical behavior of any circuit enclosing that electronic component. That analysis aims notably to determine the conductance at different points of the characteristic and then to extract component's model parameters such as the reverse saturation current (I_s), the ideality factor (n), the series and shunt resistances (R_s and R_{sh}, respectively). When numerical simulations are performed using the above mentioned form of the I-V characteristic, the process is quite slow. Explicit forms of that characteristic are in great demand, since simulations using them are about several tens fold faster than the implicit ones. References on explicit forms of PV solar cells' I-V characteristics are rather scarce in the literature. The main objective of this work is an analysis of four selected techniques transforming those I-V characteristics from their implicit forms to explicit ones. Those techniques are namely: (i) the area's, (ii) the generalized area's, (iii) the trial function's, and (iv) the Lambert W-function's methods, respectively. The analysis is conducted notably in terms of the device operation conditions, kind of solar cell's model and assumptions, related implicit form of the I-V characteristic, derived explicit form(s), outcomes' expressions, method's applications and further comments.

Key words: Area's methods; Extraction of solar cells' parameters; Generalized area's method; Lambert W-function's method; Trial function's method.

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1. Introduction

A PV solar cell usually operates under illumination. Its electrical behavior is described by an equivalent circuit which is composed by one or several (generally non-ideal) diodes in series with one or more resistances (R_{s_i}) and in parallel with the following components: a current generator, one or more shunt resistances (R_{sh_i}) and a load (R_L). From that general non-ideal dissipative many-diodes solar cell's model, various sub-models can be derived depending on the considered model's assumptions, i.e. device operating under darkness or illumination; one or more diodes, series and shunt resistances; zero series resistances and finite shunt resistances; finite series resistances and zero conductance; etc. In most of those sub-models, the solar cell's current-voltage (I-V) characteristic is generally presented in an implicit form. Photo-detectors and diodes' I-V characteristics are generally expressed in implicit forms. With these forms, one always has to resort to iterative or other numerical routines and thus to unpleasant computational delays when analyzing any circuit inclosing the above-mentioned electronic components. In order to avoid that kind of drawback, attempts have been made in different works to set up methods for the transformation of some solar cells' I-V characteristics from their implicit forms to explicit ones. A flight over the literature on that subject shows that the list of such methods is rather short. Four methods have been selected for an analysis in this paper. The presentation of each method will be performed in terms of the: (i) considered solar cell's model, operation conditions, assumptions and I-V characteristic's implicit form; (ii) transformation procedure; (iii) resulting I-V characteristic's explicit form; (iv) outcomes, applications and/or results and comments.

2. Principles of the Selected Methods

2.1. Area's Method

In the first quadrant, the I-V curve of a PV solar cell under illumination in given by the following expression developed by earlier workers [1]-[3] in terms of the dissipative non-ideal single-exponential diode model, with zero conductance ($R_{sh}\to\infty$) and finite series resistance (R_s):

$$I = I_L - I_s\left\{exp\left[\frac{q(V+R_sI)}{nkT}\right] - 1\right\} \quad (1)$$

From that I-V characteristic's implicit form (eq. (1)), simple calculations lead to the following V-I explicit function:

$$V = \frac{nKT}{q}\ln\left(\frac{I_L+I_s-I}{I_s}\right) - R_sI \quad (2)$$

where I_L, I_s, n, q, K, T stand for, respectively, the light generated current, diode dark saturation current, diode ideality factor, electronic charge (absolute value), Boltzmann constant, and solar cell's absolute temperature. A straightforward integration of eq. (2) between $I = 0$ (thus $V = V_{oc}$, the open circuit voltage) and $I = I_{sc}$, the short circuit current (thus V=0), provides an analytical expression for the area (A) between the V-I curve and the two axes. Then, using the approximation

$$I_{sc} \simeq I_L + I_s \tag{3}$$

one obtains the area's expression in the next form:

$$A = V_{oc} I_{sc} - \frac{R_s I_{sc}^2}{2} - \frac{nKT}{q} I_{sc} \tag{4}$$

Eq. (4) is referred to as the formula of the area's method for the determination of the solar cell's series resistance [4].

2.2. Generalized Area's Method

The I-V characteristic of the PV solar cell under illumination is considered in relation to the dissipative non-ideal single-exponential diode model, with finite series and shunt resistances. Plotted in the first quadrant, the expression of that I-V curve is:

$$I = I_L - I_s \left[exp\left(\frac{V + \mathrm{r}I}{n\gamma}\right) - 1\right] - g(V + \mathrm{r}I) \tag{5}$$

where $r = R_s$, $\gamma = {KT}/{q}$ and $\gamma = 1/R_{sh}$. From eq. (5), implicit equations can be derived for I_{sc} and $V_{oc,}$ assuming successively that the voltage and the current are equal to zero. The first step in the method [5] consists to transform V and I of eq. (5) into the following dimensionless variables:

$$x = V/V_{oc};\ y = I/I_{sc} \tag{6}$$

That yields an implicit form $y(x)$ of the solar cell's I-V curve in the (x, y) plane. The expression of $y(x)$ incloses five dimensionless quantities, i.e.: j_L, j_s, p, q and u, which are expressed in terms of the following parameters: I_L, I_s, V_{oc}, I_{sc}, n, r, g and γ. The curve cuts the (x, y) axes in two points of unit coordinates and that provides two implicit relations for j_L and j_s in terms of the quantities p, q and u. In the second step, a new coordinates system (X, Y) is introduced. It arises from a counterclockwise rotation of the (x, y) axes by an angle α, such that:

$$\cos\alpha = p/h; \sin\alpha = q/h;\ h = (p^2 + q^2)^{1/2} \tag{7}$$

The transformation equations between the two coordinates systems are:

$$x = (pX - qY)/h;\; y = (qX - pY)/h \tag{8}$$

With such a transformation, one gets the following explicit form of the solar cell's I-V characteristic in the (X, Y) plane:

$$Y = \frac{h}{p}\left[j_L + j_s - \left(uh + \frac{q}{h}\right)X - j_s\exp(hX)\right] \tag{9}$$

The next step in the method is to determine the area A of the region bounded by the I-V curve and the (V, I) axes. This is performed by applying the coordinates transformations of eqs. (6) and (7)-(9), successively. The following expression is obtained for that area:

$$A = (I_L + I_s)(V_{oc} - \mathrm{r}I_{sc}) + I_{sc}(1 + g\mathrm{r})\left(\frac{\mathrm{r}I_{sc}}{2} - \gamma n\right) + V_{oc}\,g\left(\gamma n - \frac{V_{oc}}{2}\right) \tag{10}$$

Eq. (10) is the formula of the generalized area's method for the determination of the next three solar cell's parameters: r, g and n. The method can be extended to the many-diodes model of the solar cell's equivalent circuit, for which the expression of the I-V curve in the first quadrant is the following [5]:

$$I = I_L - \sum_{i=1}^{N} I_{si}\left\{\exp\left[\frac{(V+\mathrm{rI})}{n_i\gamma}\right] - 1\right\} - g(V + \mathrm{rI}) \tag{11}$$

2.3. Trial Function's Method

The I-V characteristic of a PV solar cell under darkness has the following implicit form in the framework of the ideal single-exponential diode model in series with a resistance R_s ($n = 1$ and $R_{sh} \rightarrow \infty$):

$$I = I_s\left[exp\left(\frac{V - R_s I}{V_{th}}\right) - 1\right] \tag{12}$$

where $V_{th} = KT/q$ is the thermal voltage. The first stage in the method [6] is to rewrite eq. (12) as following in terms of normalized variables i and u:

$$i = \exp(u - i) \tag{13.1}$$

$$i = R_s(I + I_s)/V_{th} \tag{13.2}$$

$$u = \frac{(V + R_s I_s)}{V_{th}} + ln\left(\frac{R_s I_s}{V_{th}}\right) \tag{13.3}$$

In order to get an approximate solution of i as an explicit function of u, a trial function $i_t(u)$, which produces roughly the correct solution for all values of u, is introduced. Once a proper trial function has been implemented, the following explicit form of u_t as a function of i_t is derived from eq. (13.1):

$$u_t = i_t + \ln i_t \tag{14}$$

Then a solution for $i(u)$ is found by performing a Taylor series expansion of the eq. (13.1) about $u = u_t$:

$$i(u) = i_t + (u - u_t)\left.\frac{di}{du}\right|_{u_t} + \frac{1}{2}(u - u_t)^2 \left.\frac{d^2 i}{du^2}\right|_{u_t} + \cdots \tag{15}$$

Provided that the quality of the trial function is high, a very precise approximation to the correct solution can be achieved by bounding the expansion to the first few terms. To second-order expansion, the approximation solution is:

$$i(u) \approx i_t(u)\left\{1 + \frac{u - u_t(u)}{1 + i_t(u)} + \frac{1}{2}\frac{[u - u_t(u)]^2}{[1 + i_t(u)]^3}\right\} \tag{16}$$

The main problem in the procedure is to find a proper trial function. This one must have the correct asymptotic behavior for large positive and negative values of u. A detailed determination of such a function leads to the following form [6]:

$$i_t(u) = \begin{cases} exp(u)[1 - exp(u)], & \text{for } u \le u_0 \\ u + a\, exp\left(\frac{u_0 - u}{b}\right) - \ln\left\{\frac{u - u_0}{2} + \left[\left(\frac{u - u_0}{2}\right)^2 + \left(\frac{c}{2}\right)^2\right]^{1/2}\right\}, & \text{for } u \ge u_0 \end{cases} \tag{17}$$

where:

$$u_0 = -2.303; \quad a = 2.221; \quad b = 6.804; \quad c = 1.685 \tag{18}$$

2.4. Lambert W-Function's Method

As considered in relation to the dissipative non-ideal single-exponential diode model containing a series resistance (R_s) and two shunt resistances (R_{sh1} and R_{sh2}), the I-V characteristic of a PV solar cell under darkness has the following implicit form:

$$I = I_s\left\{exp\left\{\frac{1}{nV_{th}}\left[V\left(\frac{R_s}{R_{sh2}}\right) - R_s I\right]\right\} - 1\right\} + \frac{V - R_s I}{R_{sh1}} + \frac{V}{R_{sh2}} + \frac{VR_s}{R_{sh1}R_{sh2}} \tag{19}$$

The special omega function (also referred to as the Lambert W-function) is used here to transform eq. (19) into explicit forms I (V) and V (I). For any real (or in general complex) variable x, the function W (x) is the solution of the following equation [7]:

$$W(x)exp[W(x)] = x \tag{20}$$

The values of this special function can be calculated through available efficient and accurate algorithms. From eq. (19), our computations come up to the following expressions for the above-said explicit forms, in good accordance with results from [7]:

$$I = \frac{nV_{th}}{R_s} W\left\{\frac{I_s R_{sh1} R_s}{(R_s+R_{sh1})nV_{th}} exp\left[\frac{(V+R_s I_s)R_{sh1}}{nV_{th}(R_{sh1}+R_s)}\right]\right\} + \frac{V-I_s R_{sh1}}{R_{sh1}+R_s} + \frac{V}{R_{sh2}} \tag{21}$$

$$V = -\frac{nV_{th}R_{sh2}}{R_{sh2}+R_s} W\left\{\frac{I_s R_{sh1}(R_s+R_{sh2})}{nV_{th}(R_{sh1}+R_{sh2}+R_s)} exp\left[\frac{IR_{sh1}R_{sh2}+I_s R_{sh1}(R_{sh2}+R_s)}{nV_{th}(R_{sh1}+R_{sh2}+R_s)}\right]\right\} + \frac{IR_{sh2}}{R_{sh2}+R_s}\left(R_s + \frac{R_{sh1}R_{sh2}}{R_{sh1}+R_{sh2}+R_s}\right) + \frac{I_s R_{sh1} R_{sh2}}{R_{sh1}+R_{sh2}+R_s} \tag{22}$$

Expressions for the explicit forms I (V) and V (I) are also proposed in the following particular cases: (i) infinite R_{sh2}; (ii) infinite R_{sh1}; (iii) infinite R_{sh1} and R_{sh2}; and (iv) zero R_s.

3. Results and Discussion

3.1. On the Area's Method

This method is the simplest amongst the selected techniques to transform a PV solar cell's I-V characteristic from its implicit form to an explicit one. As indicated in eq. (4), the method allows the determination of one of the two following solar cell's parameters when the other is known: (i) the series resistance (R_s) and the ideality factor (n). Through that method, the values of one of those parameters are therefore affected by the other parameter as demonstrated in an earlier work [8]. Indoor I-V measurements have been performed using the round blue commercial single-crystal silicon solar cell of figure 1 (total area S_t = 78.5 cm^2; active area S_a =75.1 cm^2).

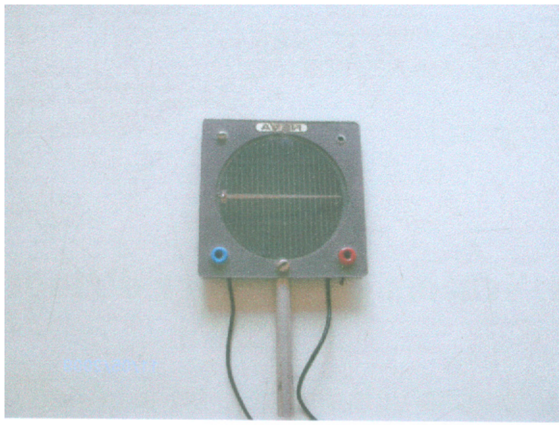

Fig. 1. View of the PV solar cell used for the test of the area's and the generalized area's methods.

With a light intensity of 300 Wm^{-2} and T = (302 ± 2) K, the test of the method has led to R_s = (0.030 ± 0.016) Ω, i.e. $R_S' = \frac{R_S}{S_t}$ =0.4 mΩ cm^{-2}. In the method, the value n = 1.40 has been used for the ideality factor as derived from the Singal's method [9]. The previous value of R_S' is quite in agreement with results from [8]. As a matter of fact, I-V measurements under different illumination levels on a single-crystal silicon N/P junction solar cell of total area S_t = 23 cm^2 and at T = 295 K, have led those workers to the following range of R_S' values: [-0.2; 0.46] mΩ cm^{-2}. The negative values obtained for R_s (and thus R_S') indicate that the area's method introduces a significant uncertainty in the results. At its turn, the ideality factor of the PV solar cell of figure 1 has been determined using the same method where R_s has been taken equal to (0.10 ± 0.01) Ω as derived from the Singal's method. With a light intensity of 300 Wm^{-2} and T = 321 K, the value obtained has been: n = 1.45. A comparison of that value with results from many other methods, together with comments of their overall results, is provided in an earlier work [10].

3.2. On the Generalized Area's Method

From eq. (10), this method enables one to determine the PV solar cell's parameters $r = R_s$, n and $g = 1/R_{sh}$ as follows. Firstly, the cell's I-V characteristics under three different light intensity levels (E_i, i = 1, 2, 3) at the same temperature (T) are constructed. Secondly, the area (A_i) of the region limited by each I-V curve and the (I, V) axes is calculated. Finally, the next system of three linear equations has to be solved for the unknown quantities r, n and g:

$$\rho_i = \left(\frac{I_{sc}}{2V_{oc}}\right)_i \mathrm{r} + \left(\frac{1}{V_{oc}}\right)_i \gamma n + \left(\frac{V_{oc}}{2I_{sc}}\right)_i g - \left(\frac{1}{I_{sc}}\right)_i \gamma g n \quad (23)$$

where

$$\rho_i = \left(\frac{I_{sc}V_{oc} - A}{I_{sc}V_{oc}}\right)_i \tag{24}$$

An experimental test of the present method has been performed on the device of figure 1 under the following operation conditions:

$E_1 = 240$ Wm^{-2}; $I_{sc1} = 0.324$A; $V_{oc1} = 0.410$V; $A_1 = 0.1221$W; T = 321 K;

$E_2 = 260$ Wm^{-2}; $I_{sc2} = 0.336$A; $V_{oc2} = 0.402$V; $A_2 = 0.1234$W;

$E_3 = 340$ Wm^{-2}; $I_{sc3} = 0.279$A; $V_{oc3} = 0.403$V; $A_3 = 0.1083$W.

The following results have been obtained as the most realistic values of the parameters:

$R_s = (0.83 \pm 0.39)$ Ω; $R_s' = 10.6$ mΩ cm^{-2}; $n = 1.50$; $R_{sh} = (1923 \pm 122)$ Ω.

Nevertheless, as noticed during the computation process, the present method introduces a more significant uncertainty than the area's method in the values of $r = R_s$, n and $g = 1/R_{sh}$. This major drawback likely explains why it is almost impossible to find out in the relevant literature any application of eq. (11) to extend the present method to solar cell's dissipative non-ideal many-diodes models.

3.3. On the Trial Function's Method

Computations of the authors [6] lead to the following main results of this method: (i) in comparison with the exact numerical solution, the relative error in the trial function of eqs. (17) - (18) exhibits a maximum of 12% near $u = 0$. (ii) However, that error is much less in other regions and becomes even negligibly small at large positive and negative values of u. (iii) When combining that trial function with the above-mentioned Taylor series expansion, the error is made arbitrary small for all values of u. (iv) Especially, to second-order expansion, the solution for i(u) is in an excellent agreement with the exact numerical solution. (v) The next other trial function:

$$i_t(u) = \ln[1 + \exp(u)] \tag{25}$$

is much simpler, but far less accurate than $i_t(u)$ of eqs. (17) – (18). It however has the advantage of being a continuous function for all values of u.

3.4. On the Lambert W-Function's Method

The effectiveness of the present method has been tested by the authors [7] by making a comparison between the use of W-function type solutions versus iteration and approximate solutions methods. The particular case of infinite R_{sh2} has been considered in their model, together with the following data: I_s =

10^{-12}A; $n = 1$; $R_s = 1k\Omega$; $R_{sh1} = 1M\Omega$. The current (I) was calculated for forward voltages (V) up to one volt, using eq. (21) with infinite R_{sh2} (exact solution (i)). Eq. (19) (with infinite R_{sh2}) was employed in the case of iterative solutions (ii), whereas approximate solutions (iii) were computed using the expression presented in ref. [11]. The results obtained in the tests exhibit notably the following features. Computation with W-function type solutions (i) is typically 40 times faster than iteration method (ii) under the above-quoted conditions. The approximate method (iii) is about twice faster than the W-function type method. Nevertheless, the approximate solutions present the major drawback of producing a noticeable error (≈3% maximum) [11].

4. Conclusion

Four methods which transform PV solar cells' I-V characteristics from their implicit forms to explicit ones have been selected for analysis in this work. For any of them, the transformation procedure has been briefly presented. From one method to the other, the involved solar cell's model, assumptions and operation conditions, together with the resulting explicit analytical function(s) I(V) or V(I) and the practical interest (or applications), have been also stated. The first two methods, i.e. the area's and the generalized area's methods, apply to PV solar cells operating under illumination. The area's method, which is the simplest amongst the four, allows the determination of either R_s (provided n is known) or n (if R_s is known). With the generalized area's method, the solar cell's parameters R_s, n and R_{sh} can be extracted from I-V characteristics of the solar cell under three illumination levels and a given temperature. A discussion of (and comments on) the results of our experimental test of the two methods on a single-crystal silicon solar cell are reported in this work. The other two methods, i.e. the trial function's and the Lambert W-function's ones, apply to solar cells operating in darkness conditions. For both of them, it is shown that the results are in very good agreement with the exact numerical solutions, with the important advantage to be many tens faster than the related iteration methods. All the four methods are good alternatives to be considered for diodes (photodetectors or solar cells) models to be used for electronic circuit simulation purposes.

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