"Science Stays True Here"
Journal of Mathematics and Statistical Science (ISSN 2411-2518, USA), Vol.10, Issue 4, 71-87 | Science Signpost Publishing

Parametric Equations Giving Solution Sets Memorial Conjecture

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Abstract

It was proposed by Paul Erdös in 1948 and known as the Erdos-Strauss Delusion guess that for a positive integer P ≥2, the sum of three positive unit fractions is $\frac{4}{P}$ and the solution sets are always. So far the proof for every positive integer P could not be done. So far,with the help of computer programs, it has been found that this second is satisfied up to $P \geq 10^{17}$. This study, it is aimed to find the parametric equations that give the solution sets for positive integers P. Quantitative research methods and experimental designs in this method were used in this study. The topics of rational equations, identities and first-order equations with two unknowns were used in the study. The positive integers P were divided into three groups and these groups were analyzed separately as even numbers P, prime numbers P and odd numbers P, respectively. From the sets of non-prime odd numbers that give a remainder when divided by 3 for the positive integers P, it was found that the solution sets for the numbers that give a remainder when divided by , parametric equations that give solution sets for all P numbers except for some P numbers whose factors all give a remainder when divided by 3, was found.All parametric equations found are equations that do not exist in the literature. Based on this study, new studies can be conducted for the set of numbers for which parametric equations cannot be found.İf parametric equations satisfying this set of numbers can be found, the conjecture will be completely proved.

Keywords: Erdös Strauss Delusion, Parametric Equations, Number theory , Algebra

1. Objective

Erdos –Strauss Conjecture; For a positive integer P ≥2, the sum of the three positive unit fractions $\frac{4}{p}$ and that's the solutions sets are always. So far, it has not been proved for every positive integer P. So far, with the help of computer programs, it has been found that this assumption is satisfied up to $P \geq 10^{17}$. In this study, it is aimed to find the parametric equations of the solution sets that satisfy the assumption according to the numbers P in this assumption. When the studies on this subject are examined, solutions were tried to be found according to the remainder of the set of P numbers 4. P numbers q being a positive integer; 4q, 4q+1, 4q+2 and 4q+3 as sets of numbers and tried to find their solutions. İn the studies carried out so far, no consensus has been reached on the solution sets of prime numbers, and the prime numbers that can be written in the form of 4q+3 have been proved ,but the prime number that can be written in the form of 4q+1 have not been proved.İn this study while finding the parametric equations of the solution sets that satisfy the conjecture, operations were not performed according to the remainders divided by 4 as in the literature . İnstead, studies were carried out on even P numbers, odd P numbers and prime P numbers. The majority of the parametric equations found in the study, which give the solution sets, are also equations that are not in the literature. If all factors of odd P numbers that give remainder 1 when divided by 3 also give remainder 1 when divided by 3, parametric equations of certain number sets were found.

1.1. Scope Of The Study

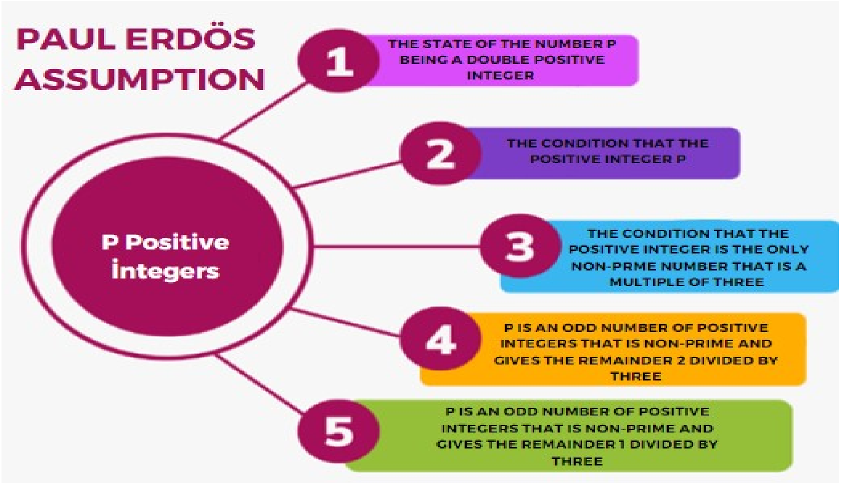

***Figure-1*:** Decomposition of P numbers in Paul Erdös' Conjecture for solution sets.

As seen in Figure-1, the scoop of this research is determined by the number P in the equations in the Erdös-Strauss Assumption.The number P was examined by dividing it into 5 number sets and parametric equations giving solution sets in each number set were tried to be found.

2. Introduction

This study started with a research question.

Research Question: Are there parametric equations of Paul Erdos that give sets of solutions satisfying the famous Erdos Strauss Conjecture? Can we find these equations? In the literature review based on the research question; for the positive integer $P \geq 2$, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{4}{P}$ there is a parametric equation that satisfies the equation $x = P, y = \frac{P+1}{3}, z = \frac{P.(P+1)}{3}$. This equation is known as 'Egyptian Fractions'. However, for the assumption to be satisfied in this equation,the number P must be a number that is a multiple of 3 and gives the remainder of 2 when divided by 3 (Bleicher, MN.1998). In general, solution sets could not be constructed for all P numbers. In the Diophantine equation $4xyz = x.y + y.z + x.z$ obtained by adjusting the assumption for the positive integer $P \geq 2$, P= 4q+1 was assumed and solutions were tried to be found using the Chinese Remainder Theory. However, these solutions do not cover all of them (Bright, M.& Lougran, D.2020). In this study, there are no parametric solution sets that give all solution sets in general. For a positive integer $P \geq 2$, the so-called polylogarithmic solution sets are; $a_k . (\log P)^k + a_{k-1} . (\log P)^{k-1} + \ldots\ldots + a_0$ for such polynomials, it is arranged as Diophantine equation and its lower and upper limits, that is , the solution range, were tried to be found (Elsholtz, C. & Terence, T. 2013). There are some studies on the number of solution sets that satisfy the assumption of a positive integer $P \geq 2$. However this study is not a study for all P numbers (Sander, JW. 1994).Studies have been reached.When the studies in the literature are examined,it is seen that some of the studies focus on positive integers $P \geq 2$. For example,the primes of positive integers $P \geq 2$ or the ones that can be transformed into Diophantine Equations. However, there is no study for all positive integers $P \geq 2$ as a whole.In this study, the positive integers $P \geq 2$ are systematically divided into sets of numbers in general and parametric equations that give solutions are found. Many of the parametric equations found are not available in the literature. As such, it will fill the aforementioned gaps in the literature.

3. Method

The quantitative research method was used in this study. The experimental design was used in the quantitative research method.

3.1 Hypothesis Based On The Research Question

***Hypothesis-1*:**Among the positive integers P > 2 in the Erdos Conjecture,there is at least one set of solutions satisfying the equation when the integers P are even.

***Hypothesis-2*:**In the Erdos Conjecture, solutions are satisfying the equation for the primes P with $P \geq 2$.

***Sub-Hypothesis-1*:**Among the primes P mentioned in Hypothesis-2, there is a solution set of the equation for the double prime number P $= 2$.

***Sub-Hypothesis-2*:**The solutions that satisfy the equation for the primes P given in Hypothesis-2 that give the remainder 3 when divided by 4(these primes can be written as $P = 4q + 3$ with q being a positive integer) have a parametric equation.

Sub-Hypothesis-3: Hypothesis-2 gives the remainder of the given prime numbers P over 4 For prime numbers (which can be written as P=4q+1 and q is a positive integer), there is at least one solution set that satisfies the equation.

***Hypothesis-3*:**For non-prime odd numbers P, there are solution sets that satisfy the equation for numbers that are multiples of 3 and give the remainder 2 when divided by 3.

***Sub-Hypothesis-1*:** Parametric equations exist for the solution sets that provide the equation for odd numbers that are multiples of three and not pride among the P numbers in hypothesis -3.

***Sub-Hypothesis-2*:**There are sets of solutions that satisfy the equation for odd numbers that give 2 when divided by 3 in the number P specified in hypothesis-3.

***Hypothesis-4*:** Single non-prime numbers have solution sets that satisfy the equation for numbers that give a remainder of 1 when divided by 3.

***Sub-Hypothesis-1*:** For the numbers P specified in Hypothesis-4, at least one of the factors is the solution sets that satisfy the equation in numbers that give a reminder of 2 when divided by 3.

Sub-Hypothesis-2: For the numbers P stated in Hypothesis-4 ,there were sets of solutions that satisfy the equation for those numbers if it is the number that gives the remainder of 1 when all its factors are divided by 3.

3.2 Mathematical Topics and Concepts Used in the Research Process

***Definition*:** For x,y,z positive integers and integer P≥ 2; $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{4}{P}$ There are always sets of solutions that satisfy the equation. Since the solutions of this equation cannot be proved for every P number it arrives at,it remained as an assumption. This assumption is called the Erdös-Strauss assumption.

3.2.1 Mathematical Topics Used in the Research Process

In this study,rational equations and their properties, factorization rules and their identities ,first-order equations with two unknowns and finding solution sets are used.

4. Results

4.1. Proof Of Hypothesis-1

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{4}{p}$;

P = 2.k, k $\in z^{+}$

$$\frac{1}{X}=\frac{2}{2K}=\frac{1}{K}\rightarrow x=k$$

$$\frac{1}{y}+\frac{1}{z}+\frac{2}{2k}=\frac{y+z}{y.z}=\frac{1}{k}$$

$$\rightarrow ky+kz=yz$$

$$\rightarrow ky=yz-kz$$

$$\rightarrow ky=z(y-k)$$

$$\rightarrow \frac{ky}{y-k} = z$$

$$\rightarrow \frac{ky-k^2}{y-k} + \frac{k^2}{y-k} = z$$

$$\rightarrow k + \frac{k^2}{y-k} = z$$

$$\rightarrow y - k = 1 \ \ and\ z = k + k^2$$

$$\rightarrow y = k+1, z = k^2 + k$$

According to this ;

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{4}{p}$;

$P = 2k, (x,y,z) = (k, k+1, k^2 + k$ ……………………………………………………………………..(1)

***Example-1*:** x, y, z are positive integers,if P=12 ;
$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{4}{12}$ find a solution to the equation.

***Solution-1*:** For the solution set given in (1),if P=2k=12, then k=6. According to this ;
If x =k =6, y = k+1=7, z=k^2+k =42 then (x, y, z)=(6, 7, 42) is a solution to the equation.

4.2. Proof Of Hypothesis-2

When the prime numbers are examined, it is the only prime number that is 2 pairs. When prime numbers greater than 3 are examined these numbers are written as P = 4q + 1 or P = 4q + 3.

4.2.1. Proof Of Sub-Hypothesis-1

For P = 2k, (x, y, z) = $(k, k+1, k^2)$ given in (1), P = 2k = 2;
x = k = 1, y = k + 1 = 2 and z = $k^2 + k$ satisfies the equation (x, y, z)=(1, 2, 2)…………………(2)

4.2.2. Proof Of Sub-Hypothesis-2

$$\frac{4}{p}=\frac{4\left(\frac{p+1}{2}\right)}{p.\left(\frac{p+1}{2}\right)}=\frac{1}{\left(\frac{p+1}{2}\right)}.\frac{2.(p+1)}{p}$$

$$\frac{4}{p}=\frac{1}{\left(\frac{p+1}{2}\right)}.\frac{2.(p+1)}{p}=\frac{1}{\left(\frac{p+1}{2}\right)}.(2+\frac{2}{p})$$

$$\frac{4}{p}=\frac{1}{\left(\frac{p+1}{2}\right)}.\left(2+\frac{2}{p}\right)=\frac{1}{\left(\frac{p+1}{2}\right)}.(2+\frac{1}{p}+\frac{1}{p})$$

$$\frac{4}{p}=\frac{1}{\left(\frac{p+1}{2}\right)}.\left(2+\frac{1}{p}+\frac{1}{p}\right)=\frac{1}{\left(\frac{p+1}{4}\right)}+\frac{1}{p.(\frac{p+1}{2})}+\frac{1}{p.(\frac{p+1}{2})}$$

According to this ; (x, y, z) =$\left(\left(\frac{p+1}{4}\right), p.\left(\frac{p+1}{2}\right), p.(\frac{p+1}{2})\right)$ it becomes.

Since there are prime numbers that give the remainder of 3 when the number P is divided by 4;
P=4q+3, q $\in z^{+}$

According to this ;

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{4}{p}$ for;
(x, y, z)=(q+1, (4q+3).(2q+2), (4q+3).(2q+2)) …………………………(3)

***<u>Example-2</u>*:** x, y, z are positive integers,if P=11 ;
$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{4}{11}$ find a solution to the equation.

<u>Solution-2:</u> When analyzed according to the solution set given in (3);
If P=4q+3=11 , q=2
(x,y,z)=(q+1, (4q+3).(2q+2), (4q+3).(2q+2));
x=3 , y=66 , z=66.

4.2.3. Proof Of Sub-Hypothesis-3

$\boldsymbol{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{4}{p};}$ P= 4q+1 prime number

$\boldsymbol{\frac{1}{x}+\frac{1}{y}} < \boldsymbol{\frac{4}{p}}$, x, y, p positive integer

$P.(x+y)<4x.y$

P. x + P. Y < 4xy fort his equality to be equality;

Let the number $\frac{1}{z}$ be written as P.m, m $\in R^{+}$.

$P.x + P.y + P.m = 4xy$
$P.y + P.m = x(4y.P)$

$x = \frac{P(y+m)}{4y-P}$
becomes .....……………………………………………………………………………………..(4)

$P.x + P.y + P.m = 4xy$
$P.x + P.m = 4xy - P.y$
$P.x + P.m = y(4x - P)$

$y = \frac{Px+Pm}{4x-P} =$
$\frac{P(x+m)}{4x-P}$ …………………………………………………………………………………………(5)

When (4) and (5) are examined, it is seen that these two numbers are symmetrical when x and y are swapped.

$y = \frac{Px+Pm}{4x-P} \rightarrow t = \frac{x+m}{4x-P}$ let it
be ...…………………………………………………………………………...(6)

$y = P.t$
becomes………………………………………………………………………………………...…….(7)

$P = 4q +$
1………....…………………………………………………………………………….……………..
(8)

When the necessary operations are carried out between (4), (5), (6), (7), (8);
$t = a.b.c$ *and* a, b, c are positive integers.
Let $m = a.b^2$ be the same as m given in (4) and following.
Equality $q = t.(4n - 1) - n - m$ follows. After finding the appropriate positive integer n;

$x = q + n$

$y = p.t$

$z = \frac{x.y}{m}$ becomes.

$(P, m) = 1$
……………………………………………………………………………………………….……(9)

Example-3: Find the solution satisfying the Erdös Conjecture for the prime $P = 4q + 1 = 2137$.

Solution-3: For the values $a = 2$, $b = 3$,$c = 2$;
For $t = a.b.c = 2.3.2 = 12$ and $m = 2.3^2 = 18$;
$q = t.(4n - 1) - n - m$
$q = 12.(4n - 1) - n - 18$ equation with $n = 12$. Accordingly, $q = 534$ and $P = 4q + 1 = 2137$.

$x = q + n = 534 + 12 = 546$

$y = P.t = 2137.12 = 25644$

$z = \frac{x.y}{m} = \frac{546.25644}{18} = 777868$

$\frac{4}{P} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ the solution of the equation has been found.$\frac{4}{2137} = \frac{1}{546} + \frac{1}{25644} + \frac{1}{777868}$

4.3. Proof of Hypothesis-3

4.3.1.Proof of Sub-Hypothesis-1

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{4}{P}$, *P*=*3k*, $k \in z^+$ let it be

$\frac{1}{x} = \frac{1}{3k} \rightarrow x = 3k$

$\frac{1}{y} + \frac{1}{z} = \frac{3}{3k} = \frac{1}{k}$

$\frac{1}{y} + \frac{1}{z} = \frac{1}{k} \rightarrow ky + kz = yz$

$\rightarrow ky = yz - kz$

$\rightarrow ky = z(y - k)$

$\rightarrow z = \frac{ky}{y-k} = \frac{ky-k^2}{y-k}$

$\rightarrow z = k + \frac{k^2}{y-k}$

$\rightarrow$ In the case of $y - k = 1, z = k + k^2$ and $y = k + 1$

According to this;

For the $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{4}{P}$;

In the case $P = 3k$, there are $(x, y, z) = (3k, k + 1, k^2 + k)$ solution set………..…………….(10)

Example-4: x, y, z are positive integers,if P=15 ;
$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{4}{15}$ find a solution to the equation.

Solution-4: If the operations are performed according to the solution sets in (10); If $P = 3k = 15$, $k = 5$ happens. According to this;

$x = 3k = 15$
$y = k + 1 = 6$
$z = k^2 + k = 30$ results are revealed.

5.3.2. Proof of Sub-Hypothesis-2

for $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{4}{P}$;

Let $P = 3k + 2. k \epsilon z^+$

$\frac{1}{x} = \frac{1}{3k+2} \rightarrow x = 3k + 2$

$\frac{1}{y} + \frac{1}{z} = \frac{3}{3k+2} \rightarrow (y + z).(3k + z) = 3yz$

$\rightarrow 2y + 3yk + 3kz + 2z = 3yz$

$\rightarrow 3ky + 2y = 3y - 3kz - 2z$

$\rightarrow y(3k + 2) = z(3y - 3k - z)$

$$\rightarrow z = \frac{(3k+2)y}{3y-3k-2}$$

$$\rightarrow z = \frac{(3k+2).y - \left(\frac{3k+2}{3}\right)^2}{3y-3k-2} + \frac{\frac{(3k+2)^2}{3}}{3y-3k-2}$$

$$\rightarrow z = \frac{3k+2}{3} + \frac{(3k+2)^2}{3(3y-3k-2)}$$

$\rightarrow \text{let } 3y - 3k - 2 = 1.3y = 3k + 3 \rightarrow y = k + 1$

$\rightarrow \text{If } y = k + 1 \text{ then } z = \frac{3k+2}{3} + \frac{(3k+2)^2}{3.1}$

$\rightarrow z = \frac{3k+2}{3}(1 + 3k + 2) = \frac{3k+2}{3}.3(k + 1)$

And z becomes $z = (3k + 2)(k + 1)$

According to this;

For $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{4}{P}$;

In case of $P = 3k + 2$ $(x, y, z) = (3k + 2, k + 1, (3k + 2).(k + 1))$ ……………………………(11)

Example-5: x, y, z are positive integers,if P=35 ;
$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{4}{35}$ find a solution to the equation.

Solution-5: If operations are carried out according to the set of solutions in (11); If $P = 3k + 2 = 35$, then k=11.
According to this ;
$x = 3k + 2 = 35$
$y = k + 1 = 12$
$z = (3k + 2).(k + 1) = 420$ are found.

4.4. Proof Of Hypothesis-4
4.4.1. Proof Of Sub-Hypothesis-1

Let $P = 6.t + 1$, t be a positive integer.

If $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{4}{6t+1} \rightarrow x = \frac{1}{6t+1}$ used;

$\frac{1}{y}+\frac{1}{z}=\frac{3}{6t+1} \rightarrow 6yt+6zt+y+z=$
$3yz$ ……………………………………………………………….(12)

m, n, p, r are positive integers;

if we take $y=mt+n$ and $z=p.t+n$ and substitute them in (12);

$6mt^2+6p=3mp \rightarrow 2m+2p=m.p$

$\rightarrow 2m=m.p-2p$

$\rightarrow 2m=p.(m-2)$

$\rightarrow p=\frac{2m}{m-2}$
becomes……………………………………………………..…………(13)

$3nr=n+r \rightarrow n=3nr-r$

$\rightarrow n=r(3n-1)$

$\rightarrow r=\frac{n}{3n-1}$
becomes…………………………………………………………………..……(14)

If the equations in (13) and (14) are written in the equation 6n + 6r + m + p =3mr + 3np

$6n+m+\frac{2m}{m-2}=3m.\frac{n}{3n-1}+3n.\frac{2m}{m-2}$ when the necessary operations are applied to equality;

$36n^2-m^2\frac{6n}{3n-1}=-12mn \rightarrow 36n^2-12mn+m^2=0$

$\rightarrow (6n-m)^2=0$

$\rightarrow m=6n$ can be
found…………………………………………………………(15)

If the equality in (15) is written in (13) and (14);

$p=\frac{6n}{3n-1}$ and $r=\frac{n}{n-1}$ can be
found……………………………………………………………………………(16)

According to this;

$y=m.t+n=6n.t+n \rightarrow y=6nt+n$

$z = p.t + r = \frac{6n}{3n-1}.t + \frac{n}{n-1} \to z = \frac{6nt}{3n-1} + \frac{n}{n-1}$ becomes
$x = 6t + 1$ becomes

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{4}{6t+1}$ if it is written in its place and the necesary actions are taken;

$x = 6t + 1$
$y = n.(6t + 1)$

$z = \frac{n}{3n-1}.(6t + 1)$ is found in the form……………………………………………………………………(15)

Accordingly, if the number $P = 6.t + 1$ has at least one factor that gives the remainder of 2 when divided by 3 as stated in sub-hypothesis-6;
$P = (3.q + 2).w$ is a number in the form of. According to this;
x=(3.q+2).w
y=n.((3.q+2).w)

$z = \frac{n}{3n-1}.((3.q + 2).)$ the solution set becomes………………………………………………………(16)

Since the numbers 3n-1 are divided by 3, there will be numbers that give the remainder of 2; $3n - 1$ and $3.q + 2$ there are always n and w numbers that make their numbers equal.

Example-6: x,y,z are positive integers, if $P = 119$;
Find a solution of the $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{4}{119}$ equation.

Solution-6; (16) if the operations are performed according to the solution set, $P = 119 = 17.7$ is found There is at least one multiplier in the number P, that is 3 divided by the remaining 2, and this multiplier is 17. According to this;

If $P = (3.q + 2).w = 17.7, q = 5$ and $w = 7$ becomes.
$x = (3.q + 2).w = 17.7 = 119$
$y = n.\big((3.q + 2).w\big) = n.17.7$
$z = \frac{n}{3n-1}.\big((3.q + 2).w\big) = \frac{n}{3n-1}.17.7$ becomes.

Since the number $3n - 1$ also means the number that gives the remainder of 2 when divided by 3; for $3n - 1 = 17;\ n = 6$ comes out.

$x = (3.q + 2).w = 17.7 = 119$
$y = n.\big((3.q + 2).w\big) = n.17.7 = 6.17.7 = 714$
$z = \frac{n}{3n-1}.\big((3.q + 2).w\big) = \frac{n}{3n-1}.17.7 = 6.7 = 42$
The solution set satisfying the equation is (x, y, z)=(119, 714, 42) becomes.

4.4.2. Proof Of Sub-Hypothesis-2

When the numbers given in sub-hypothesis-2 are examined, it is seen that the multipliers of these numbers are all written as $6k + 1$, k will be a positive integer. For example, in the number 91=7.13, this is the case. According to this ;,

In the $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{4}{p}$ equation P numbers under the desired conditions $P = (6k + 1).(6m + 1)$ let it be. Let one multiplier in the number P be defined as (6k + 1), while the multiplication of all other multipliers is $(6m + 1)$. Where a, b and c are positive integers;

$\frac{1}{x} = \frac{1}{a.(6m+1)}$ and $\frac{1}{y}+\frac{1}{z} = \frac{1}{b.(6m+1)}$ be accepted in the form……………………………………………….(17)

$\frac{1}{a.(6m+1)} + \frac{1}{b.(6m+1)} = \frac{1}{c.(6m+1)}$ let the number P in the equation, c=6k+1 have a multiplier under the desired conditions. $(a + b).c = 4ab$ becomes. When necessary arrangements are made so that the number a remains alone when it is done;

a becomes $a = \frac{bc}{4b-c}$ The values of a and b can be found for $c = 6k + 1$ under the desired conditions.

For example; If $c = 6k + 1 = 7$ then $b = 2$ and $a = 14$……………………………………………(18)

Let the values of a and b written in thr parametric equation in (17). According to that;
If $\frac{1}{x} = \frac{1}{a(6m+1)} = \frac{1}{2(6m+1)}$ then $x = 2(6m + 1)$

$\frac{1}{y}+\frac{1}{z} = \frac{1}{14(6m+1)}$ If the equation is made according to the general solution given in hypothesis-1;
$y = 84m + 15$ *and* $z = (84m + 14).(84m + 15)$ and the final version of the parametric equation is;

$x = 2(6m + 1)$
$y = 84m + 15$
$z = (84m + 14).(84m + 15)$ ……………………………………………………..………………………..(19)

Example-7; $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{4}{7.13}$ find the roots of the equation.

Solution-7; $\frac{1}{a.(6m+1)}+\frac{1}{b.(6m+1)}=\frac{4}{c.(6m+1)}$ parametric equation for $c=7$ in the equation if it is written instead of $m=2$ since it is created;
$x=26, y=183\ and\ z=183.182$ becomes.

When the $\frac{1}{26}+\frac{1}{183}+\frac{1}{183.182}=\frac{4}{p}$ equation is solved; $\mathrm{P}=7.13$ revenue. Accordingly, (18) is found for $c=7$, i.e. the number P in the desired conditions with a factor of 7 in the number P.

Now take $c=6k+1=19$ given in (18) and in these conditions, $a=95$ and $b=5$ satisfying the equation $a=\frac{bc}{4b-c}$ accordingly;

Let the values a and b found in (17) be written in the parametric equations. According to this;
$\frac{1}{x}=\frac{1}{95.(6m+1)}=\frac{1}{95.(6m+1)}$ if $x=95.(6m+1)$

$\frac{1}{y}+\frac{1}{z}=\frac{1}{5.(6m+1)}$ if the equation is made according to the general solution given hypothesis-1;
$y=30m+6\ and\ z=(30m+5)(30m+6)$, and the parametric equation of this equation is the final form;
$x=95(6m+1)$
$y=30m+6$
$z=(30m+5).(30m+6)$ it is found in the form………………………………………………………..(20)
The parametric equation found is actually the equation formed when one of the multipliers of the number P is 19.

Example-8: $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{19.13}$ find the roots of the equation.

Solution-8: $\frac{1}{a.(6m+1)}+\frac{1}{b.(6m+1)}=\frac{4}{c.(6m+1)}$ parametric equation for $c=19$ in the equation if it is written instead of $m=2$ since it is created;
$x=95.13, y=66\ and\ z=65.66$ becomes.

When the $\frac{1}{95.13}+\frac{1}{66}+\frac{1}{65.66}=\frac{4}{p}$ equation is solved; $P=19.13$ revenue. Accordingly, for $c=19$, that is , the number P is found in the desired conditions with a factor of 19 in the number P.

4.4.2.1. P numbers with a Multiplier of 4q + 3

In the general, the P numbers under the desired conditions are $4q + 1$ and $4q + 3$. If $b = q + 1$ is taken when a multiple in P numbers is $c = 4q + 3$ then the positive natural numbers a and b are found from the equation $a = \frac{bc}{4b-c}$.

When editing the equation $\frac{1}{a.(6m+1)} + \frac{1}{b.(6m+1)} = \frac{4}{c.(6m+1)}$;

Taking $b = q + 1$ in the expression $a = \frac{bc}{ab-c}$ gives $a = (q + 1).(4q + 3)$ and $c = (4q + 3)$. If the found values are written instead in the equation;

$x = (q + 1).(4q + 3).(6m + 1)$

$y = (q + 1).(6m + 1) + 1$

$z = (q + 1).(6m + 1).[(q + 1).(6m + 1) + 1]$ it is found in the form.

If the given in example-7 is taken as $c = 7 = 4q + 1$, then $q = 1$ becomes. For $6m + 1 = 13$, $x = 5.19.13 = 182$, $y = 5.13 + 1 = 66$, $z = 65.66 = 4290$ becomes.

In sub-hypothesis-2, a general parametric equation was found for the solution sets of numbers with a multiplier of $4q + 3$ from the P numbers under the desired conditions. However, no general parametric has been found for P numbers with a multiplier of $4q + 1$.

5. Conclusion and Discussion

5.1. General Evaluation

In this study, based on the research question, the purpose and scope of the research were determined and the necessary hypothesis and sub-hypothesis were established. As a result of the research, the answer to the research question was found except for some of the numbers in the sup-hypothesis-2 in hypothesis-4. So for P numbers; Gives a remainder when divided by 3, and gives a remainder when all of its multipliers are divided by 3 parametric equations have been found that give the solution sets for all P numbers in the assumption except for some of these types of non-prime P numbers.

The parametric equations found do not give all the sets of solutions, of course. Because many of the parametric equations found are equations that are not in the literature. Since there is a set of solutions in the equations in the literature; The solution sets found in this study do not represent all. However, since the purpose is to prove that the assumption is always met, the solution sets found mean that the assumption is correct, except for some of the numbers determined in sub-hypothesis-2 in hypothesis-4.

5.2. Analysis of Hypothesis-1

In the prof for P even numbers, only a parametric equation that satisfies the equation is taken in the solution given in (1). To be requasted in the prof arranged in the form of $\frac{1}{x} = \frac{2}{2k}$ and $\frac{1}{y} + \frac{1}{z} = \frac{2}{2k}$ in the case of ; $\rightarrow k + \frac{k^2}{y-k} = z$

$\rightarrow y - k = 1$ and $z = k + k^2$

$\rightarrow y = k + 1, z = k^2 + k$ in this part, different divisors of the number y-k can also be taken to be sivisors of k^2 and can be found in different parametric equations. However, since hypothesis-1 aims to prove that there is at least one parametric equation that gives the solution, one equation was cansidered sufficent. Also in the expression $P = 2k$ the number $P = 2$ is both since it is both prime and even, it is given in the prime number section.

5.3. Hypothesis-2 i.e. Analysis of P Prime Numbers

In this study, the reason for examining prime numbers separately is examined in a separate section, as odd numbers give a remainder of 1 when divided by 3, a remainder of 2 when divided by 3, and 3 itself is a prime number. All positive integers are a combination of numbers in mod4 that have the remainder 0, 1, 2, 3. If the remainder is 0 or 2, it means that the number is even. Since the prime numbers are odd numbers are odd numbers other than 2, when making proofs of the primes to provide the assumption; the prime numbers were evaluated separately in two separate sections, $4q + 1$ and $4q + 3$ and in 2 and 3.

In the proof of the prime numbers that can be written as sub-hypothesis-3, that is, $4q + 1$, the prof can be completed on the basis that the x and y numbers are symmetrical. While there are n numbers in the $q = t.(4n - 1) - n - m$ value in the prof, the equations $t = a.b.c$ and $m = a.b^2$
Keep the numbers a and c small in positive natural numbers of a, b, c and increase the number b, since the alue of m will qrow rapidly, giving the value of n number can be found in absolute.

5.4. Analysis of Hypothesis-3

For numbers that give the remainder of 1 when divided by 3 from sub-hypothesis-1 given in hypothesis-4, that is, odd numbers that are not prime; Proof can be mad efor those who give the remainder of 2 when at least one of the factors of the numbers is divided b 3. In this proof $P = 6t + 1$ is because the number $z = \frac{n}{3n-1}.(6t + 1)$ was reached as a result of many attempts.
Since the number $3n - 1$ means the number that gives the remainder of 2 when divided by 3, positive integers z can be found.

5.6 Analysis of Hypothesis-4, Sub-Hypothesis-2

In sub-hypothesis-2 given in hypothesis-4, that is for non-prime odd numbers that give the remainder of 1 when the number P is divided by 3, Write the number P as $(6k + 1)(6m + 1)$ and the

$(6k+1)$ multiplier and Asking a and b to come up with positive integers is a result reached after many trials. In this way, the numbers a and b could be found and how many desired multipliers were found in the number P. If there are, the final version of these multipliers is represented as $(6m+1)$. In the method found $(6k+1)$ The parametric equation was found for numbers with a multiplier (6k +1) for P numbers concerning the factor. When the proof of the hypothesis is examined, the numbers a and b are not always. Example-7 and the example-8 could be made for $6k+1=7$ and $6k+1=19$. For a, b and c numbers ;

If there are numbers a and b satisfying the equation $a=\frac{bc}{4b-c}$, then according to these numbers, the parametric equations of the solution set can always be found. But this equation is a multiplier of every $c=6k+1$is not provided for. Therefore, the number c, which is one of the factors in the number P, is divided into two parts. The general parametric equation of those with the form c number $4q+3$ has been found. However, the parametric equation that gives the solution for those with $c=4q+1$ has not been found.

6. Suggestions

Based on this study, studies can be carried out to prove Hypothesis-4 for all factors of sub-hypothesis-2. If this hypothesis can be proven, the Erdös-Strauss Conjecture will be proved. In addition, the methods in this study can be examined and based on these methods, it can be tried to find a prof of the hypothesis that cannot be proven.

7. Resources

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