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A New Characterization of Hall Effects on Flow Through Porous Medium in a Rotating Parallel Plate Channel

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Abstract

This paper presents the study of unsteady flow of an incompressible electrically conducting viscous fluid in a rotating porous media, with a variable pressure gradient and in the presence of hall current. Here we consider three different cases, like impulsive change, cosine and sine oscillations of pressure gradient. Here, it is proved in this paper that, the rotational and Lorenz forces are having significant effect on velocity profile in the presence of pressure gradient and hall current.

Keywords: Unsteady flows, rotating channels, Hall current effects, pressure gradient, impulsive change, Cosine Oscillations and Sine Oscillations.

Introduction

We assess and calculate the positions and velocities with respect to a fixed frame of reference, applying its magnetic field. The dynamics of geophysics as a field of study has become a vital branch of fluid dynamics owing to the enormous work being carried to explore the atmosphere. Hall effects have extended its influence even on the studies launched in the area of astrophysics, where it is used to study the celestial occurrences like solar storms or even the dynamics working on the stellar, solar structures and the matter present between one planets and the other and between one star and the other. Hide and Roberts [1], gave an explanation for the observed continuation and secular variation of the geomagnetic field. Also Dieke [2] discussed an important in the solar physics mixed up in the sunspot development. A phenomenon (It was discovered by Edwin Hall in 1879) that occurs when an electric current moving through a conductor is exposed to an external magnetic field applied at a right angle, in which an electric potential develops in the conductor at a right angle to both the direction of current and the magnetic field. The Hall effect was a direct result of Lorentz forces acting on the charges in the current, and was named after American physicist Edwin Herbert Hall (1855- 1938). Hall current effect is also indispensable when the fluid is an ionized gas with low density or we are applying the high range of magnetic field. Because the electrical conductivity of the fluid will then be a tensor and a Hall current is provoked. Which is likely to be central in many engineering situations has been discussed by Sutton and Sherman [3]. The Hall effects on the flow of ionized gas between parallel plates under uniform transverse magnetic field have been premeditated by Sato [4]. Nanda and Mohanty [5] considered the hydromagnetic rotating channel flows. Datta and Jana [6] presented the Hall effects on unsteady Couette flow.

Hall effects on hydromagnetic convective flow through a channel with conducting walls is given by Datta and Jana [7], they discussed the flow nature with non-dimensional parameters. Mandal et al. [8] have studied the combined effects of rotation and Hall current on steady flow. Mandal et al. [9] discussed the effects of Hall current on flow between thick arbitrarily conducting plates. Ghosh [10] has analysed the unsteady hydromagnetic flow in a rotating channel with oscillating pressure gradient. Nagy et al. [11] discussed the effects of Hall currents and rotational force on Hartmann flow under general wall conditions. Kanch et al. [12] discussed the Hall Effect on unsteady Couette flow under boundary layer approximations.

Chauhan and Agrawal [13] studied the Hall effects on flow in a channel partially filled with a porousmedium into a rotating system. Sarkar et al. [14] have examined the combined effects of Hall currents and rotation on steady hydromagnetic flow. Nadeem et al. [15] discussed the numerical solutions of peristaltic flow of a Newtonian fluid under the effects of magnetic field and heat transfer in a porous concentric tubes. Nadeem and Akbar [16] discussed the influence of heat transfer and variable viscosity in vertical porous annulus with peristalsis. Nadeem et al. [17] have investigated the influence of heat and mass transfer on Newtonian bio-magnetic fluid of blood flow throughout a tapered porous artery with a stenosis.

The heat generation/absorption and thermo-diffusion on an unsteady free convective flow of radiating and chemically reactive second grade fluid near an infinite vertical plate through a porous medium and taking the Hall current into account have been studied by Veera Krishna and Chamkha [19].

Motivated from the above studies, in this paper, we have considered the unsteady flow of an incompressible electrically conducting viscous fluid in the course of porous medium in a rotating system with pressure gradient as a variable and taking hall current into account.

Formulation and Solution of the Problem

We have consider the unsteady flow of an incompressible electrically conducting viscous fluid in the course of porous medium in a rotating system between two infinitely long horizontal parallel walls separated by a distance h with pressure gradient as a variable and taking hall current into account. We choose a Cartesian frame of reference with the x-axis along the channel wall at y = 0. The configuration of the problem given in Fig.2. A uniform transverse magnetic field H_0 is applied perpendicular to the channel walls. Since the channel walls are infinite in extent and the flow is unsteady, the physical variables are the function of y and t only.

The unsteady boundary layer equations for the flow through a loosely porous medium along x and z-directions in a rotating frame of reference using Brinkman model are

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} + 2\Omega w = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\frac{\partial^2 u}{\partial y^2} - \frac{\mu_e J_z H_o}{\rho} - \frac{v}{k}u \qquad \text{-----(1)}$$

$$\frac{\partial w}{\partial t} - v_0 \frac{\partial w}{\partial y} - 2\Omega w = v\frac{\partial^2 w}{\partial y^2} - \frac{\mu_e J_z H_o}{\rho} - \frac{v}{k}u \qquad \text{------ (2)}$$

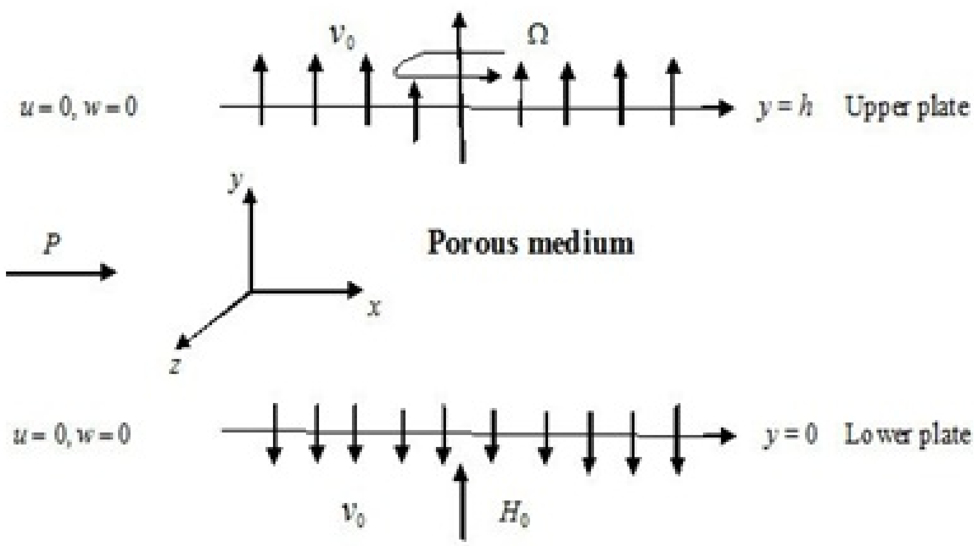

Figure 1: Physical Geometry of the problem

The initial and boundary conditions are

$u = 0, w = 0,$ $\quad t \leq 0,$ $\quad 0 \leq y \leq h$ ---------- (3)

$u = 0, w = 0,$ $\quad v = v_0,$ $\quad t > 0, \quad y = 0$ *and* $y = h$ ------------*(4)*

The generalized Ohm's law comes essentially from the momentum equation of motion for the electron fluid. Its derivation can be found in some plasma physics books. It can be written, on taking Hall currents into account and neglecting ion-slip and thermo-electric effect, as (Cowling [18])

$$J + \frac{\omega_e \tau_e}{H_0}(J \times H) = \sigma(E + \mu_e q \times H) \qquad \text{--------------- (5)}$$

The right hand side is the electric field in the moving frame, The first term on the left hand side comes from the electron drag on the ions. The second term is the Hall term and has to do with the idea that electrons and ions can decouple and move separately, The magnetic Reynolds number assumed small, so that the induced magnetic field effect is negligible in comparison with applied magnetic field. The electron atom collision frequency is relatively high as compared to the ion collision frequency, due to this the electron pressure gradient is neglected but, Hall Effect remains present. The relation $\Delta\,.\,H = 0$ for magnetic field implies $H_y = H_0 = constant$, everywhere in the fluid. Further, the equation of the conservation of the current density is $\nabla.\,J = 0, gives\, J_y = constant.$ This constant is zero since $J_y = 0$ at the places which are electrically non-conducting. Thus $J_y = 0$ everywhere in the flow, Since the induced magnetic field is neglected, Maxwell's equation becomes $\nabla \times E = 0$ which implies $\frac{\partial E_x}{\partial y} = 0$ and $\frac{\partial E_z}{\partial y} = 0$. That is E_x = constant everywhere in the flow. In view of the above assumption, the equation (5) gives

$$J_x - m\,J_z = -\,\sigma\mu_e H_0\, w \qquad \text{------------(6)}$$

$$J_z + m\,J_x = -\,\sigma\mu_e H_0\, u \qquad \text{------------(7)}$$

We solve the equations (6) and (7), we get

$$J_x = \frac{\sigma\mu_e H_0}{1+m^2}\,(mu - w) \qquad \text{------------ (8)}$$

$$J_z = \frac{\sigma\mu_e H_0}{1+m^2}\,(u + mw) \qquad \text{-------------(9)}$$

On making use of (8) and (9), the momentum equation (1) and (2) along x- and z- directions become

$$\frac{\partial u}{\partial t} - v_0\frac{\partial u}{\partial y} + 2\Omega w = -\frac{1}{\rho}\frac{\partial \rho}{\partial x} + v\frac{\partial^2 w}{\partial y^2} + \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(u + mw) - \frac{v}{k}u \text{ --- } (10)$$

$$\frac{\partial w}{\partial t} - v_0\frac{\partial w}{\partial y} - 2\Omega \mathrm{u} = v\frac{\partial^2 w}{\partial y^2} - \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(w - mu) - \frac{v}{k}w \qquad \text{---(11)}$$

We introduce the non-dimensional variables

$x^* = \frac{x}{h}$, $y^* = \frac{y}{h}$, $u^* = \frac{uh}{v}$, $w^* = \frac{wh}{v}$, $q^* = \frac{qh}{v}$,

$t^* = \frac{tv}{h^2}$, $\omega^* = \frac{\omega h^2}{v}$, $p^* = \frac{ph^2}{\rho v^2}$

making use of non-dimensional variables, the equations (10) and (11) becomes to (dropping asterisks)

$$\frac{\partial u}{\partial t} - Re\frac{\partial u}{\partial y} + 2K^2 w = f(t) + \frac{\partial^2 u}{\partial y^2} - \frac{M^2}{1+m^2}(u + mw) - Du \quad \text{-------(12)}$$

$$\frac{\partial w}{\partial t} - Re\frac{\partial w}{\partial y} + 2K^2 u = \frac{\partial^2 w}{\partial y^2} - \frac{M^2}{1+m^2}(w - mu) - Dw \quad \text{-------(13)}$$

Where, $M^2 = \frac{\sigma\mu_e^2 H_0^2 h^2}{\rho v}$ is the Hartmann number, $m = \tau_e\omega_e$ is the hall parameter, $D = \frac{k}{h^2}$ is the Darcy parameter (Permeability parameter), $K^2 = \frac{\Omega^2 h^2}{v}$ the rotation parameter, Re = $\frac{v_0 h}{v}$ the Reynolds number and $f(t) = -\frac{1}{\rho}\frac{\partial p}{\partial x}$ is the non-dimensional pressure gradient.

Corresponding non-dimensional initial and boundary conditions are

$u = 0,\quad w = 0,\quad t \le 0,\quad 0 \le y \le 1$ --------- (14)

$u = 0,\ w = 0,\quad t > 0,\quad y = 0$ and $y = 1$ --------- (15)

Combining equations (12) and (13), Let q = u + iw and i = $\sqrt{-1}$, we get the momentum equation in terms of complex velocity q where, u is the velocity along the x-direction and w is the velocity along the z-direction, is given as –

$$\frac{\partial q}{\partial t} - Re\frac{\partial q}{\partial y} = f(t) + \frac{\partial^2 q}{\partial y} - \left(\frac{M^2}{1+im}\ 2iK^2 + D\right)q \quad \text{------- (16)}$$

The initial and boundary conditions are -

$q = 0,\quad t \le 0,\quad 0 \le y \le 1$ ----------- (17)

$q = 0,\quad t > 0,\quad y = 0$ and $y = 1$ ----------- (18)

Taking the Laplace transform of the equation (16), we have

$$\frac{\partial^2 \bar{q}}{\partial y^2} - Re\frac{\partial \bar{q}}{\partial y} - \left(\frac{M^2}{1+im} - 2iK^2 + D\right)\bar{q} = \bar{f}(s) \quad \text{---------- (19)}$$

The transformed boundary conditions are

$\bar{q}(0,s) = 0$, and $\bar{q}(1,s) = 0$ --------(20)

The solution of the equation (19) subjected to the boundary condition (20) are given by -

$$\bar{q}(\mathrm{y, s}) = \frac{\bar{f}(s)}{\lambda_1+s}\left[1 - e^{-\frac{1}{2}Rey}\,\frac{sinh\sqrt{\lambda_2+s(1-y)}}{sinh\sqrt{\lambda_2+s}} - e^{-\frac{1}{2}Re(1-y)}\,\frac{sinh\sqrt{\lambda_2+s(y)}}{sinh\sqrt{\lambda_2+s}}\right] \text{------- (21)}$$

Where $\lambda_1 = \frac{M^2}{1+im} - 2iK^2 + D$ and $\lambda_2 = \frac{Re^2}{4} + \frac{M^2}{1+im} - 2iK^2 + D$ and we assume

$$f(t) = P_0 + P_1 e^{i\omega t} + P_2 e^{-i\omega t} \qquad \text{-------------} \qquad (22)$$

Where ω is the frequency of oscillation ; $P_0, P_1, and\ P_2$ are real constants. Taking the inverse Laplace transforms to the equation (21), and we obtain the solution for the complex velocity q as,

$$q(y,t) = \frac{P_0}{\lambda_1}\left[1 - e^{-\frac{1}{2}Re\,y}\ \frac{\sinh(a-ib)(1-y)}{\sinh\,(a-ib)} - e^{\frac{1}{2}Re(1-y)}\ \frac{\sinh(a-ib)(y)}{\sinh\,(a-ib)}\right] +$$

$$\frac{P_1}{\lambda_1+i\omega}\left[1 - e^{-\frac{1}{2}Re\,y}\ \frac{\sinh(a_1 \pm ib_1)(1-y)}{\sinh(a_1 \pm ib_1)} - e^{\frac{1}{2}Re(1-y)}\ \frac{\sinh(a_1 \pm ib_1)(y)}{\sinh(a_1 \pm ib_1)}\right] e^{i\omega t} +$$

$$\frac{P_1}{\lambda_1-i\omega}\left[1 - e^{-\frac{1}{2}Re\,y}\ \frac{\sinh(a_2-ib_2)(1-y)}{\sinh(a_2-ib_2)} - e^{-\frac{1}{2}Re(1-y)}\ \frac{\sinh(a_2-ib_2)(y)}{\sinh(a_2-ib_2)}\right] e^{i\omega t} -$$

$$\left(\frac{P_0}{\lambda_1} + \frac{P_1}{\lambda_1+i\omega} + \frac{P_1}{\lambda_1-i\omega}\right)*\left[1 - e^{-\frac{1}{2}Re\,y}\ \frac{\sinh(1/2)(1-y)}{\sinh(1/2)Re} - e^{\frac{1}{2}Re(1-y)}\ \frac{\sinh(1/2)Re(y)}{\sinh(1/2)Re}\right] e^{-\lambda_1 t}$$

$$+\, 2\sum_{n=1}^{\infty} n\pi\left[(-1)^n e^{\frac{1}{2}Re(1-y)} - e^{-\frac{1}{2}Re\,y}\right]\left(\frac{P_0}{S_1} + \frac{P_1}{S_1+i\omega} + \frac{P_2}{S_1-i\omega}\right)\frac{\sin n\pi y}{\lambda_1+S_1}\, e^{S_1 t} \qquad \text{-----(23)}$$

In the equation (23), the lower sign is valid for $2K^2 + \frac{mM^2}{1+m^2} + mD > \omega$ and the upper sign is valid for $2K^2 + \frac{mM^2}{1+m^2} + mD < \omega$. The equation (23) represents the velocity of the fluid in the general case. Now we shall consider the following special cases.

Case – 1. Velocity distribution for impulsive pressure gradient :

In this case $P_1 = P_2 = 0$, then the equation (23) reduces to

$$q(y,t) = \frac{P_0}{\lambda_1}\left[1 - e^{-\frac{1}{2}Re\,y}\ \frac{\sinh(a-ib)(1-y)}{\sinh\,(a-ib)} - e^{\frac{1}{2}Re(1-y)}\ \frac{\sinh(a-ib)(y)}{\sinh\,(a-ib)}\right] +$$

$$-\ \frac{P_0}{\lambda_1}\left[1 - e^{-\frac{1}{2}Re\,y}\ \frac{\sinh(1/2)(1-y)}{\sinh(1/2)Re} - e^{\frac{1}{2}Re(1-y)}\ \frac{\sinh(1/2)Re(y)}{\sinh(1/2)Re}\right] e^{-\lambda_1 t} +$$

$$+ 2\sum_{n=1}^{\infty} n\pi \left[(-1)^n e^{\frac{1}{2}Re(1-y)} - e^{-\frac{1}{2}Re\, y}\right] \left(\frac{P_0}{S_1}\right) \frac{\sin n\pi y}{\lambda_1+S_1} e^{S_1 t} \qquad \text{------------ (24)}$$

Case – 2. Velocity distribution for cosine oscillations of pressure gradient :

In this case $P_0 = 0$ *and* $P_1 = P_2 = \frac{P}{2}$, then the equation (23) reduce to

$$q(y,t) = \frac{P}{2} \left\{ \left[1 - e^{-\frac{1}{2}Re\, y} \frac{\sinh(a_1 \pm ib_1)(1-y)}{\sinh(a_1 \pm ib_1)} - e^{\frac{1}{2}Re(1-y)} \frac{\sinh(a_1 \pm ib_1)(y)}{\sinh(a_1 \pm ib_1)}\right] \frac{e^{i\omega t}}{\lambda_1 + i\omega} + \left[1 - e^{-\frac{1}{2}Re\, y} \frac{\sinh(a_2 - ib_2)(1-y)}{\sinh(a_2 - ib_2)} - e^{-\frac{1}{2}Re(1-y)} \frac{\sinh(a_2 - ib_2)(y)}{\sinh(a_2 - ib_2)}\right] \frac{e^{i\omega t}}{\lambda_1 - i\omega} \right\}$$

-

$$\frac{P}{2}\left(\frac{1}{\lambda_1+i\omega} + \frac{1}{\lambda_1-i\omega}\right) * \left[1 - e^{-\frac{1}{2}Re\, y} \frac{\sinh(1/2)(1-y)}{\sinh(1/2)Re} - e^{\frac{1}{2}Re(1-y)} \frac{\sinh(1/2)Re(y)}{\sinh(1/2)Re}\right] e^{-\lambda_1 t}$$

$$+ 2\sum_{n=1}^{\infty} n\pi P \left[(-1)^n e^{\frac{1}{2}Re(1-y)} - e^{-\frac{1}{2}Re(y)}\right] \left(\frac{1}{S_1+i\omega} + \frac{1}{S_1-i\omega}\right) \frac{\sin n\pi y}{\lambda_1+S_1} e^{S_1 t} \quad \text{---(25)}$$

Case – 3. Velocity distribution for sine oscillations of pressure gradient : In this case $P_0 = 0$ *and* $P_1 = P_2 = \frac{P}{2i}$, then the equation (23) reduce to

$$q(y,t) = \frac{P}{2i} \left\{ \left[1 - e^{-\frac{1}{2}Re\, y} \frac{\sinh(a_1 \pm ib_1)(1-y)}{\sinh(a_1 \pm ib_1)} - e^{\frac{1}{2}Re(1-y)} \frac{\sinh(a_1 \pm ib_1)(y)}{\sinh(a_1 \pm ib_1)}\right] \frac{e^{i\omega t}}{\lambda_1 + i\omega} + \left[1 - e^{-\frac{1}{2}Re\, y} \frac{\sinh(a_2 - ib_2)(1-y)}{\sinh(a_2 - ib_2)} - e^{-\frac{1}{2}Re(1-y)} \frac{\sinh(a_2 - ib_2)(y)}{\sinh(a_2 - ib_2)}\right] \frac{e^{i\omega t}}{\lambda_1 - i\omega} \right\}$$

-

$$\frac{P}{2i}\left(\frac{1}{\lambda_1+i\omega} + \frac{1}{\lambda_1-i\omega}\right) * \left[1 - e^{-\frac{1}{2}Re\, y} \frac{\sinh(1/2)(1-y)}{\sinh(1/2)Re} - e^{\frac{1}{2}Re(1-y)} \frac{\sinh(1/2)Re(y)}{\sinh(1/2)Re}\right] e^{-\lambda_1 t}$$

$$+ 2\sum_{n=1}^{\infty} n\pi P_i \left[(-1)^n e^{\frac{1}{2}Re(1-y)} - e^{-\frac{1}{2}Re(y)}\right] \left(\frac{1}{S_1+i\omega} + \frac{1}{S_1-i\omega}\right) \frac{\sin n\pi y}{\lambda_1+S_1} e^{S_1 t} \text{---(26)}$$

for the impulsive change of pressure gradient, the non-dimensional shear stresses at the wall y = 0 are given by

$$\tau_x + i\tau_z = \left(\frac{\partial q}{\partial y}\right)_{y=0} = \frac{P_0}{\lambda_1}\left[\left\{\left(\frac{1}{2}\right)Re + (a - ib)\right\}\coth(a-ib) + \frac{\left(\frac{1}{2}\right)Re + (a-ib)}{\sinh(a-ib)} e^{\frac{1}{2}Re}\right] -$$

$$\frac{P_0 Re}{\lambda_1}\coth\frac{Re}{2} e^{-\lambda_1 t} + 2\pi^2\ P_0 \sum_{n=1}^{\infty} \left[(-1)^n e^{\frac{1}{2}Re} - 1\right] \left(\frac{n^2}{s_1}\right) \frac{1}{\lambda_1+s_1} e^{s_1 t} \quad \text{----(27)}$$

for the cosine oscillations of pressure gradient, the non-dimensional shear stresses at the wall y = 0 are given by

$$\tau_x + i\tau_z = \left(\frac{\partial q}{\partial y}\right)_{y=0} = \frac{P}{2}\left[\left\{\left(\frac{1}{2}\right)Re + (a_1 \pm ib_1)\right\}\coth(a_1 \pm ib_1) + \frac{\left(\frac{1}{2}\right)Re + (a_1 \pm ib_1)}{\sinh(a_1 \pm ib_1)} e^{\frac{1}{2}Re}\right]$$

$$\frac{e^{i\omega t}}{\lambda_1+i\omega} + \left[\left\{\left(\frac{1}{2}\right)Re + (a_2 - ib_2)\right\}\coth(a_2 - ib_2) + \frac{\left(\frac{1}{2}\right)Re + (a_2 - ib_2)}{\sinh(a_2 - ib_2)} e^{\frac{1}{2}Re}\right] \frac{e^{-i\omega t}}{\lambda_1 - i\omega}$$

$$- \quad \frac{PRe}{2}\left(\frac{1}{\lambda_1+i\omega} + \frac{1}{\lambda_1-i\omega}\right)\coth\left[\frac{1}{2}Re\right] e^{\lambda_1 t} + \pi^2 P \sum_{n=1}^{\infty} \left[(-1)^n e^{\frac{1}{2}Re} - 1\right]$$

$$\left(\frac{1}{s_1+i\omega}+\frac{1}{s_1-i\omega}\right)\frac{1}{\lambda_1+s_1}e^{s_1 t} \quad \text{----------(28)}$$

for the sine oscillation of pressure gradient, the non-dimensional shear stresses at the wall y = 0 are given by

$$\tau_x + i\tau_z = \left(\frac{\partial q}{\partial y}\right)_{y=0} = \frac{P}{2i}\left\{\left[\left\{\left(\frac{1}{2}\right)Re + (a_1 \pm ib_1)\right\}\coth(a_1 \pm ib_1) + \frac{\left(\frac{1}{2}\right)Re - (a_1 \pm ib_1)}{\sinh\,(a_1 \pm ib_1)}e^{\frac{1}{2}Re}\right]\right.$$

$$\frac{e^{i\omega t}}{\lambda_1+i\omega} + \left.\left[\left\{\left(\frac{1}{2}\right)Re + (a_2 - ib_2)\right\}\coth(a_2 - ib_2) + \frac{\left(\frac{1}{2}\right)Re - (a_2 - ib_2)}{\sinh\,(a_2 - ib_2)}e^{\frac{1}{2}Re}\right]\frac{e^{-i\omega t}}{\lambda_1 - i\omega}\right\}$$

$$- \quad \frac{PRe}{2i}\left(\frac{1}{\lambda_1+i\omega}+\frac{1}{\lambda_1-i\omega}\right)\coth\left[\frac{1}{2}Re\right]e^{-\lambda_1 t} + \pi^2 P\sum_{n=1}^{\infty}\left[(-1)^n e^{\frac{1}{2}Re} - 1\right]$$

$$\left(\frac{1}{s_1+i\omega}+\frac{1}{s_1-i\omega}\right)\frac{1}{\lambda_1+s_1}e^{s_1 t} \quad \text{----------(29)}$$

Results and Discussion

We have considered the unsteady flow of an incompressible electrically conducting viscous fluid in the course of porous medium in a rotating system with pressure gradient as a variable and taking hall current into account. We have computed three different cases based on our study of impulsive change, cosine and sine oscillations of pressure gradient. In this aspect, we have analytically and computationally solved the decisive equations by applying Laplace transform technique. It has been successfully established that the flow behavior is determined owing to the mutual influence of Coriolis force and hydro-magnetic force on each other under the purview or monitoring of pressure gradient and hall current. The flow governed by the non-dimensional parameters for the velocity components u and w with different values of magnetic parameter M, Hall parameter m, rotation parameter K, Reynolds number Re, D the permeability parameter, frequency parameter ω and phage angle ωt in Figures (1,12). Figures (1,4) represent the velocity profiles for impulsive pressure gradient; (5,8) represent the velocity profiles for cosine oscillations of pressure gradient, where as the Figures (9,12) represent the velocity profiles for sine oscillations of pressure gradient. Here we observe that, all the profiles are on negative sides for w. Negative velocity just means velocity in the opposite direction than what would be positive. This will attained only with effect pressure gradient in pertinent directions of the flow field.

We have perceive from Figures (1, 5 & 9) that the velocity component u enhances with add to Hartmann number M with the impulsive change of pressure gradient, and The velocity component w less for the cosine oscillations of while it raises with impulsive change and sine oscillations with an augment in magnetic parameter M, given in Figures (2, 6 & 10).As expected due to the fact that the application of transverse magnetic field results to a resistive type force (called Lorentz force) similar to drag force and upon increasing the values of magnetic parameter, the drag force increases which leads to the deceleration of the flow. It is seen from Figures (3, 7 & 11) that the primary velocity *u* increases with an increase in Hall parameter *m* for sine oscillations of the pressure gradient while it reduces for the impulsive change and cosine oscillations of the pressure gradient. Hence, we conclude that an increase in Hall parameter reduces the Lorentz force in *x*- direction and motion of the fluid particles is reinforced in that direction.

VELOCITY PROFILES WITH IMPULSIVE PRESSURE GRADIENT:

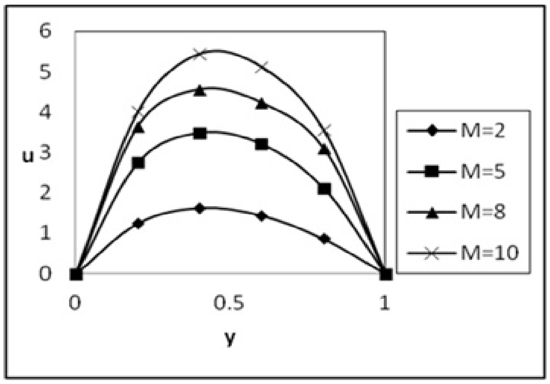

Fig. 1

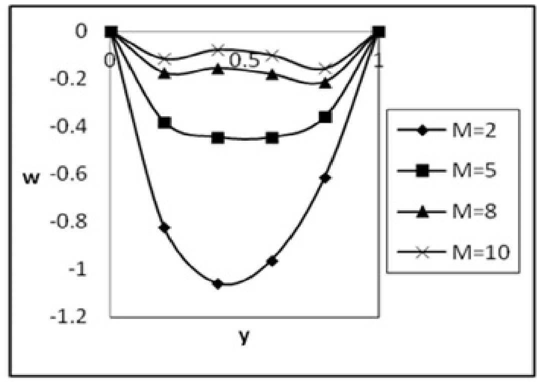

Fig. 2

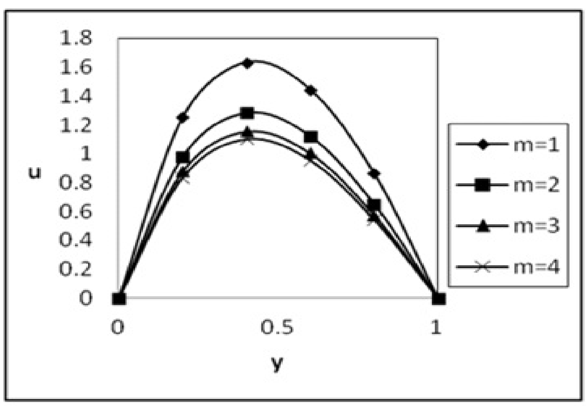

Fig. 3

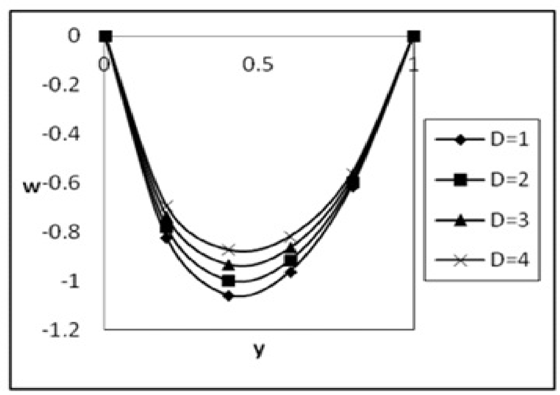

Fig. 4

I. Velocity Profiles with Cosine Oscillations of Pressure Gradient:

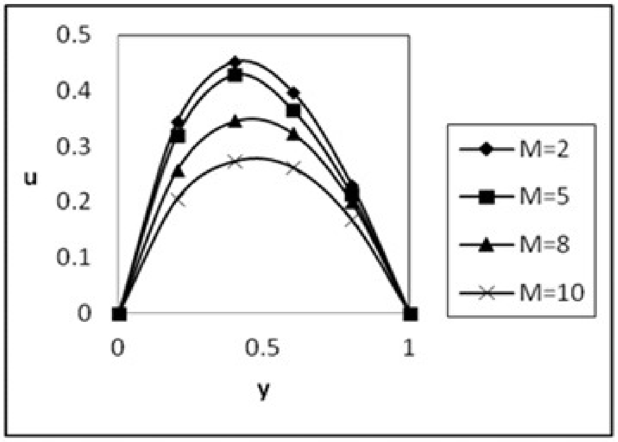

Fig. 5

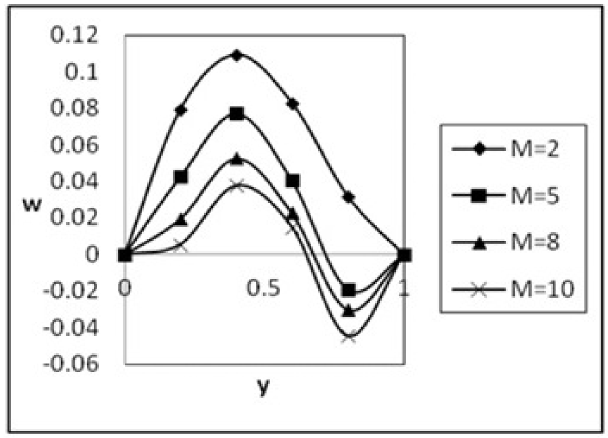

Fig. 6

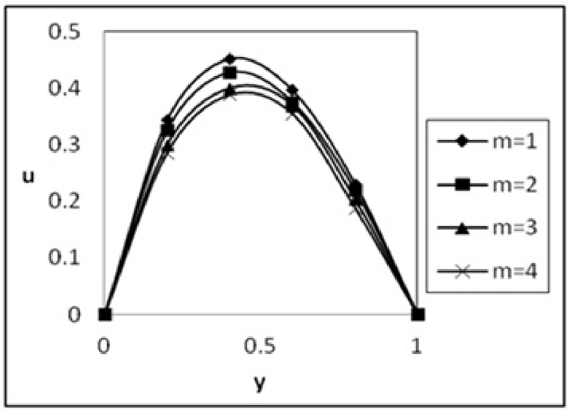

Fig. 7

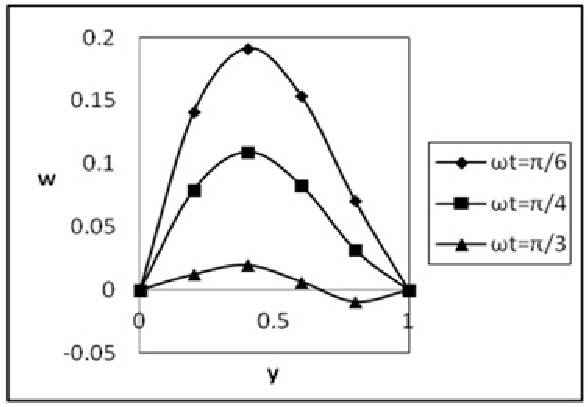

Fig. 8

II. Velocity Profiles with Sine Oscillations of Pressure Gradient:

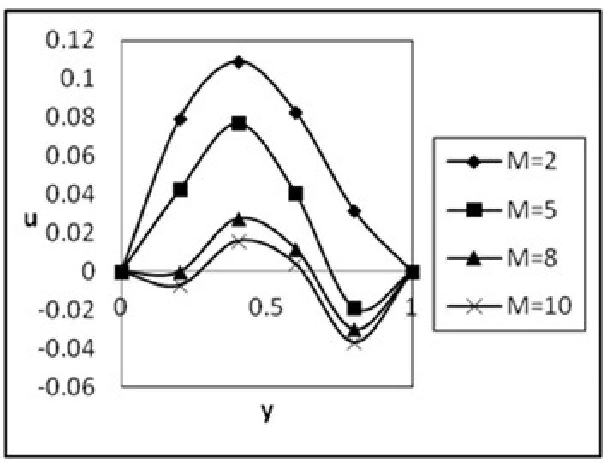

Fig. 9

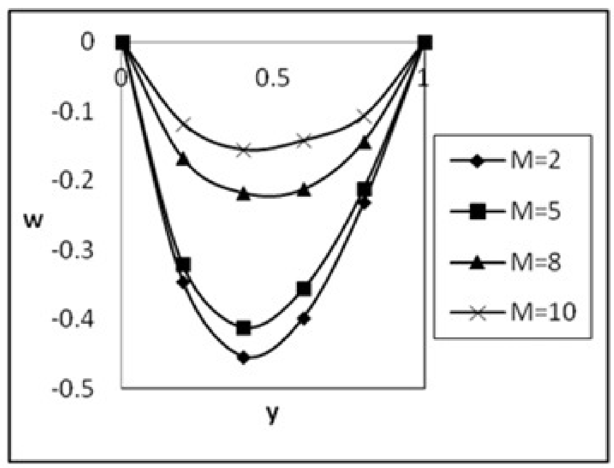

Fig. 10

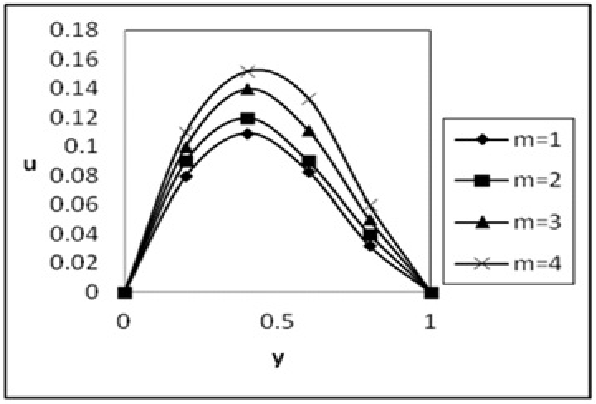

Fig. 11

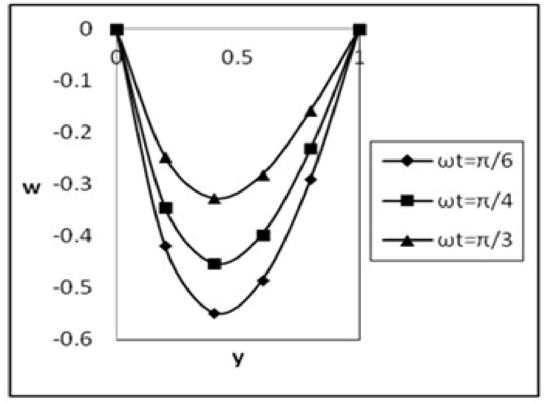

Fig. 12

Conclusions

We have considered the unsteady flow of an incompressible electrically conducting viscous fluid in the course of porous medium in a rotating system with pressure gradient as a variable and taking hall current into account. The conclusions are made as follows.

1. The velocity component for primary flow enhances with increasing M, K and D, and reduces with m, Re for the impulsive change of pressure gradient.

2. The velocity component for secondary flow enhances with increasing M, Re and D, and reduces with m and K for the impulsive change of pressure gradient.

3. The velocity component for primary flow increases with increasing Re, D and ω, and reduces with M, m, K and phase angle ωt for the cosine oscillations of pressure gradient.

4. The velocity for primary flow increases with increasing m and D, and reduces with M, K and phase angle ωt for the sine oscillations of pressure gradient.

5. The magnitude of the velocity for primary flow and for secondary flow enhances initially and then gradually reduces with an increase in Reynolds number Re for sine and cosine oscillations of the pressure gradient respectively.

6. The velocity for secondary flow enhances with increasing M, m, K and phase angle ωt , and reduces with increase in Re, D and frequency of oscillation ω for the impulsive change of pressure gradient.

7. The magnitude of τ_x due to the primary flow decreases for the impulsive change and cosine oscillations with increment in M, m, Re, K and D. For secondary flow it reduces for K and M and increases for m, Re and D.

8. Both the stresses enhance with increase in m, K and D; and reduce with increase in M or Re for sine of the pressure gradient.

9. The shear stress τ_x increases for petite values of M and then it reduces for cosine and sine oscillations of the pressure gradient with an increase in frequency parameter ω

10. The stress τ_z enhances and then it reduces for cosine oscillations. Whereas it initially decreases and then boosts for sine oscillations of the pressure gradient with an increase in ω

11. Finally, the rotational and Lorentz forces are having significant effect on velocity profile in the presence of pressure gradient and hall current.

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