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Finding the Number at Any Digger After the Comma In Decimals

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Abstract

In this study, it is aimed to develop a method that enables to find the digit in any digit after the comma in the decimal form of the number $\frac{1}{n}$ for a natural number n whose prime factors are not 2 and 5, and based on this method, the fraction $\frac{a}{n}$, a being a natural number greater than one, is followed by the decimal point. It is aimed to generalize the method to give the number in any of its digits. It started with a question "What is the digit in the 2021th digit after the comma in the decimal form of the number $\frac{1}{2021}$?" As a method, modular arithmetic rules, Euler Function, Euler Theorem, Chinese Remainder Theorem were used. As a result of the studies, since finding the digit in k. digits from the beginning after the comma of the number $\frac{1}{n}$ actually means finding the digit in the first digit of the number 10k-1. $\frac{1}{n}$ after the comma, the form of the number 10^{k-1} as n.p +q, 0 < q < n has been found. When the number of n.p +q found is multiplied by the number $\frac{1}{n}$, the number p $+\frac{q}{n}$ will be formed, and the first digit after the comma in the decimal form of the number $\frac{q}{n}$ will be the result we are looking for. The coding of this study was done, and the program that gives the number in any digit after the comma was made. Thanks to this method, it was possible to generalize within the number $\frac{a}{n}$ (a natural number greater than one).

Keywords: Decimals, Euler Function, Modular Arithmetic, Number Theory

1. Introduction

Research question: What is the digit in the 2021th digit after the comma in the decimal formof the number $\frac{1}{2021}$?"

In the studies I started with the research question, by multiplying the number formed by 10^{2020} to find the 2021th digit after the comma, the first digit of the number after the comma will be the digit I am looking for. Then, since the remainder from the division of 10^{2020} with 2021 is important, I determined which topics I would use to find the remainder. When I examine the studies on this subject,

Let the prime number of n be n = pa. qb. rc……….tv. The value of φ (n) = $(p^a - p^{a-1}). (q^b - q^{b-1}). (r^c - r^{c-1})$………… $(t^v - t^{v-1})$ is called the Euler Function of the number n. a, b being prime numbers among them; It is the Euler Function value of φ (b) of the number x, which provides the equivalence of $a^x \equiv 1(\bmod b)$. However, there is a possibility that the positive divisors of this value will also provide (Taşçı, D. 2007).

Let p1, p2, p3, …………, p_n be prime numbers in pairs.

$x \equiv k_1 \pmod{p_1}$

$x \equiv k_2 \pmod{p_2}$

$x \equiv k_3 \pmod{p_3}$

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$x \equiv k_n \pmod{p_n}$

The number of x that provides the equivalence is one (Balcı, S. 1993). I found the topics. After finding the answer to my research question, I found that it can be generalized. First, I was able to find the digit in any k. digits after the comma in the decimal form of $\frac{1}{n}$, but not 2 and 5 in the prime factors of n. Then, in the decimal form of an $\frac{a}{n}$ shaped number, where a is a natural number greater than one, I was able to find the digit in any k. digit after the comma.

2. Method

2.1. Euler Function

Let $n = p^a . q^b . r^c$ be the prime number of a positive integer n.

So it will be $\varphi(n) = (p^a - p^{a-1}).(q^b - q^{b-1}).(r^c - r^{c-1})$. This value is called the Euler function of the number n.

2.2. Euler's Theorem

Let a,b be two prime integers between them.

The x value that provides the equivalence of $a^x \equiv 1 \pmod{b}$ is the value φ(b). The x value can also be positive divisors of φ(b). However, the equivalence is precise in φ(b).

2.3. Chinese Remainder Theorem

Where the numbers $p_1, p_2, p_3, \ldots, p_n$ are prime numbers two by two,

$x \equiv k_1 \pmod{p_1}$

$x \equiv k_2 \pmod{p_2}$

$x \equiv k_3 \pmod{p_3}$

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$x \equiv k_n \pmod{p_n}$

It has only one solution according to the equivalence system ($p_1, p_2, p_3,\dots, p_n$).

3. Conclusion

3.1. Answer to the Research Question

Let it be $\frac{1}{2021} = 0, a_1\ a_2\ a_3\ a_4\dots\ a_{2020}\ a_{2021}\dots$ the decimal writing of $\frac{1}{2021}$. So the reult of $10^{2020} . \frac{1}{2021}$ will be $10^{2020} . \frac{1}{2021} = a_1\ a_2\ a_3\ a_4\dots\ a_{2020}, a_{2021}$

In this case, the first digit of this number after the comma is a_{2021}.

So, 2021 = 43.47 .

Now let's find the value of $10^{2020} \equiv x \pmod{43}$ and $10^{2020} \equiv y \pmod{47}$.

If $10^{2020} \equiv x \pmod{43}$ is (10, 43) = 1, then φ(43) = 42. $10^{42} \equiv 1 \pmod{43}$ exits.

$10^{2020} = (10^{42})^{48}. 10^4 \equiv x \pmod{43}$

$10^4 \equiv x \pmod{43}$

x = 24 ……………………………………………………………………………………(1)

For $10^{2020} \equiv y \pmod{47}$, (47, 10) = 1, so φ(47) = 46

becomes $10^{46} \equiv 1 \pmod{47}$

If $10^{2020} = (10^{46})^{43}. 10^{42} \equiv y \pmod{47}$

$10^{42} \equiv y \pmod{47}$

$(10^2)^{21} \equiv y \pmod{47}$

$2^{21} \equiv y \pmod{47}$

$(2^6)^3. 2^3 \equiv y \pmod{47}$

$17^3 . 8 \equiv y \pmod{47}$

y = 38 …………………………………………………………………………………………(2)

$10^{2020} \equiv 24 \pmod{43}$

If $10^{2020} \equiv 38 \pmod{47}$, then $k=10^{2020}$

$k \equiv 24 \pmod{43}$

Let's find the value of k so that k is 38 (mod 47) .

If $k \equiv 24 \pmod{43}$ then k = 43.m + 24 ve m ∈ Z …………………………………………………………(3)

$43.m + 24 \equiv 38 \pmod{47}$

$43.m \equiv 14 \pmod{47}$

$- 4.m \equiv 14 \pmod{47}$

$m \equiv -\frac{7}{2} \pmod{47}$

$m \equiv 20 \pmod{47}$

$m = 47.t + 20,\ t \in Z$……………………………………………………………………………………………(4)

$k = 43.m + 24$

$k = 43.(47.t + 20) + 24$

$k = 2021.t + 884$

Accordingly, the remainder of the division of 10^{2020} by 2021 will be 884,

So $10^{2020} = 2021.p + 884,\ p \in Z$.

For $10^{2020} . \frac{1}{2021} = a_1\ a_2\ a_3 \ldots a_{2020}, a_{2021}$

$(2021.p + 884). \frac{1}{2021} = p + \frac{884}{2021} = p + 0{,}4374\ldots = p{,}43\ldots$ the 2021th digit of the number $\frac{1}{2021}$ after the comma is 4.

3.2. If n is a Prime Number

If n is a prime number, the k. digit after the decimal point of $\frac{1}{n}$ will actually be the first digit after the decimal of $10^{k-1}.\frac{1}{n}$,

In the equivalence of $10^{k-1} \equiv x \pmod{n}$, x will be found. For the number x found, $\frac{x}{n}$ will be the first digit after the comma. $\varphi(n) = n-1$ Euler Function will be used when finding the x value.

3.3. The Case of n Being Two Different Prime Factors

If n has two prime factors that are not 2 and 5, it is written as $n = p.q$. When the number $\frac{1}{n}$, which is after the comma and in the begining of the line k., in the digimal form is found ,

$10^{k-1} \equiv x \pmod{p}$

By applying the Chinese Remainder Theorem for the number x, y, which provides the $10^{k-1} \equiv y \pmod{q}$ equivalence,

In the equation of $10^{k-1} \equiv z \pmod{n}$,the z number is found. The first digit after the decimal point of $\frac{z}{n}$ is the result.

3.4. The Case of n Being Three Different Prime Factors

Where n is a positive integer,

Let $n = p.\ q.\ r$ be the prime factors excluding 2 and 5 . When the number in the k. digit after the comma is asked in the form of the number, In the form of the number $10^{k-1}.\frac{1}{n}$, the first digit after the comma is the desired value. According to this,

$10^{k-1} \equiv x \pmod{p}$, $p \neq 2$, $p \neq 5$, $(p, 10) = 1$

$10^{k-1} \equiv y \pmod{q}$, $q \neq 2$, $q \neq 5$, $(q, 10) = 1$

$10^{k-1} \equiv z \pmod{r}$, $r \neq 2$, $r \neq 5$, $(r, 10) = 1$

These equations are solved by Euler's Theorem and Modular Arithmetic rules and x, y, z are found.

where $w = 10^{k-1}$ an integer,

$w \equiv x \pmod{p}$

$w \equiv y \pmod{q}$

$w \equiv z \pmod{r}$ According to the Chinese Remainders Theorem, w = m. n + z when the number w is found.

So $10^{k-1} = m.n + z$.

The value $10^{k-1}.\frac{1}{n} = (m.n + z)\ .\frac{1}{n} = m + \frac{z}{n}$ is found. First decimal digit of $\frac{z}{n}$ value, in decimal form, the number $\frac{1}{n}$ becomes the k. digit after the comma.

3.5. Generalization of the Answer to the Research Question

Where n is a positive integer,

Let n = p. q. r be the prime factors excluding 2 and 5 . When the number in the k. digit after the comma is asked in the form of the number,

In the form of the number $10^{k-1}.\frac{1}{n}$, the first digit after the comma is the desired value. According to this,

$10^{k-1} \equiv x \pmod{p}$, $p \neq 2$, $p \neq 5$, $(p, 10) = 1$

$10^{k-1} \equiv y \pmod{q}$, $q \neq 2$, $q \neq 5$, $(q, 10) = 1$

$10^{k-1} \equiv z \pmod{r}$, $r \neq 2$, $r \neq 5$, $(r, 10) = 1$

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$10^{k-1} \equiv m \pmod{t}$, $t \neq 2$, $t \neq 5$, $(t, 10) = 1$

These equations are solved by Euler's Theorem and Modular Arithmetic rules and x, y, z…..m are found.

where $w = 10^{k-1}$ an integer,

w x (mod p)

w y (mod q)

w z (mod r)

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w m (mod t)

According to the Chinese Remainders Theorem, w = u. n + ü when the number w is found. u, ü are integers.

So 10^{k-1} = u.n + ü

The value $10^{k-1}.\frac{1}{n}$ = (u.n + ü) $.\frac{1}{n}$ = u + $\frac{\text{ü}}{\text{n}}$ is found. First decimal digit of $\frac{\text{ü}}{\text{n}}$ value, in decimal form, the number $\frac{1}{n}$ becomes the k. digit after the comma.

3.6. Finding the Desired Digit of $\frac{\mathbf{a}}{\mathbf{n}}$ Number After Comma

If a is a natural number greater than one and the number in the k. digit after the comma in the decimal form of the number $\frac{\text{a}}{\text{n}}$ is desired to be found,

Let n = p. q. r….t be prime factors excluding 2 and 5. Let n = p. q. r….t be prime factors excluding 2 and 5. For the second digit after the comma in the form of the number $\frac{\text{a}}{\text{n}}$,

In the form of the number $10^{k-1}.\frac{\text{a}}{\text{n}}$, the first digit after the comma is the desired value. According to this,

10^{k-1} ≡ x (mod p), p ≠ 2, p ≠ 5, (p, 10) = 1

10^{k-1} ≡ y (mod q), q ≠ 2, q ≠ 5, (q, 10) = 1

10^{k-1} ≡ z (mod r), r ≠ 2, r ≠ 5, (r, 10) = 1

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10^{k-1} ≡ m (mod t), t ≠ 2, t≠ 5, (t, 10) = 1

These equations are solved by Euler's Theorem and Modular Arithmetic rules and the values of x, y, z, …….., m are found.

where w = 10^{k-1} an integer,

w ≡ x (mod p)

w ≡ y (mod q)

w ≡ z (mod r)

.

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w ≡ m (mod t)

According to the Chinese Remainders Theorem, w = u. n + ü when the number w is found. u, ü are integers.

So 10^{k-1} = u.n + ü

The value ($10^{k-1}.\frac{1}{n}$).a = (u.n + ü) . $\frac{a}{n}$ = u.a + $\frac{ü}{n}$.a is found. First decimal digit of $\frac{ü}{n}$.a value, in decimal form, the number $\frac{a}{n}$ becomes the k. digit after the comma.

4. Conclusion and Discussion

The following results were obtained in this study.

Provided that n is a positive integer, a method has been developed to find the k. digit after the decimal point in the decimal writing of the number. The reason why I multiplied the number by 10^{k-1} in the method found is the first element after the comma in the decimal form of the number $\frac{1}{n}$, it means the first digit of the number $10^{k-1}.\ \frac{1}{n}$ after the comma in the decimal form. In this way, I found the remainder of the prime factors of n divided by 10k-1. Then, with the help of the Chinese Remainder Theorem, I wrote the number 10k-1 as m.p +q. Also the number $10^{k-1}.\ \frac{1}{n}$ is written as (m.p +q). $\frac{1}{n}$. From there I reached the p + $\frac{q}{n}$ value. When I wrote the number $\frac{q}{n}$ as a decimal, the first digit after the comma was the number I was looking for. I was able to generalize the method I found and find the number in any digit after the comma in the decimal form among the numbers $\frac{a}{n}$, where a is a natural number greater than one.The method is the same, and when the p + $\frac{q}{n}$ value, which is the final version of $10^{k-1}.\ \frac{1}{n}$, is found, it is necessary to multiply it by the number a. When it is multiplied by the number a , a.p + $\frac{q.a}{n}$ comes out. When the number $\frac{q.a}{n}$ is written as an integer fraction, the first digit after the comma in the decimal form of the simple fraction part becomes the searched result. Prime factors of n must not have prime factors of 2 and 5. In order to prove Euler's Theorem, if we express the prime factors of n with q, the factors between 10 and q must be prime.

The software of the study was made. For natural numbers with a maximum of three prime factors, the desired digit can be easily found with the help of the program.

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