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A Study of Sensitivity of Nonlinear Oscillations of a CLD Parallel Circuit to Parametrization of Tunnel Diode

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Abstract

Tunnel diode aka Esaki diode [1] is a peculiar nonlinear electronic element possessing negative ohmic resistance. We consider a multi-mesh circuit composed of three elements: a charged capacitor, C, a self-inductor, L, and an Esaki diode, D, (CLD) all three in parallel. We parametrize the I-V characteristics of the diode and derive the circuit equation; this is a nonlinear differential equation. Applying a Computer Algebra System (CAS) specifically *Mathematica* [2] we solve the circuit equation numerically conducive diode dependent parametric solution; in general the solution has a damped oscillatory character. In this note we investigate the sensitivity of the oscillations as a function of diode's parameters. We establish the fact that for a set of parameters the tunnel diode behaves almost as an ohmic resistor and that the circuit equation tends toward classic CLR-parallel circuit with linear damped oscillations. *Mathematica* simulation assists visualizing the transition.

Keywords. Tunnel Diode, Electrical Nonlinear Oscillations, Computer Algebra System, *Mathematica*

1. Introduction

Negative electric resistance is an unfamiliar concept to the most physicists. Classic physics and electric engineering circuits aside embodying a variety of elements of interest most likely do include ohmic resistors as well. Ohmic resistors are quantified as positive, opposing the flow of electric currents. A curious question is "what is the impact of an element with a negative resistance in a typical classic circuit?" A tunnel diode, aka Esaki diode is an element possessing negative resistance. Detailed information about the characteristics and manufacturing of this element is detailed in [1] and references within. Suffices to note the inventor Leo Esaki awarded the Physics Nobel Prize, 1957 [1]. In short, the characteristics of the tunnel diode is different from a regular diode. A regular diode acts as an one-way open gate in the forward direction and is closed in the reverse direction. A tunnel diode acts as an open gate in both directions. However, in the forward direction it acts peculiarly possessing positive and negative resistances, in the reverse direction it acts like an ohmic resistor. To get a feel about the negative resistance we jump ahead looking over the right panel of Figure 4. It depicts I-V characteristics of a tunnel diode. This is the plot of the current through the diode vs. the applied voltage across it. For the forward positive voltage the current has a "sinusoidal" shape possessing three different slopes. For the low and high voltages the slopes are positive signifying regular, ohmic resistor. These slopes may be compared to the similar unique positive slope of the ohmic resistor depicted on the right panel of the Figure 1. For the intermediate voltage segment the slope is negative corresponding to a negative ohmic resistance. These peculiarities make the intuitive prediction of its impact on the characteristics of a typical electric circuit challenging.

To quantify the impact of a tunnel diode we consider a classic electric circuit with a well-known characters embodying an ohmic resistor. We then replace the resistor with an Esaki diode analyzing the circuit characteristics. Comparing the characteristics of the signals of these two circuits quantitatively reveals the impact of the diode.

To address these issues we craft this investigating note. It composed of three sections. In addition to Introduction, in Sect. 2 we lay the fundamentals of the physics of the problem. We formulate the problem detailing solution. Section 3 is the Conclusions and remarks with suggestions to extend the investigation.

2. Physics of the problem and its solution

Among various potential candidates of classic electric circuits with well-known characters we consider a CLR-parallel circuit. The circuit composed of a charged capacitor, C, a self-inductor, L, and an ohmic resistor, R all parallel. Figure 1 shows the circuit and the I-V characteristics of the ohmic resistor.

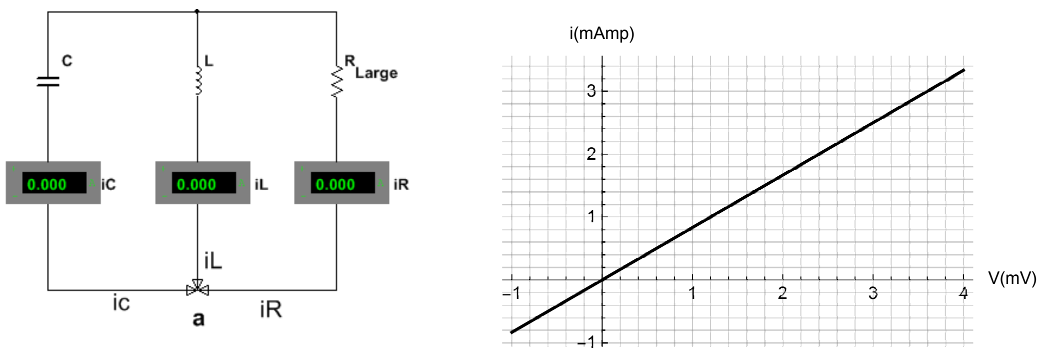

Figure 1. An CLR-parallel circuit. A charged capacitor, C, is connected parallel to the other two elements, L, and R, (left panel). I-V characteristics of the ohmic resistor is shown on the right

To show the impact of the resistor, R, first we consider the two ends of the resistor is shunted; this drops the resistor off the circuit. The circuit equation for the remaining tank circuit, CL-parallel is,

$$\frac{d^2}{\mathrm{dt}^2}q(t)+0\frac{d}{\mathrm{dt}}q(t)+\frac{1}{LC}q(t)=0 \tag{1}$$

In Equation (1) the zero coefficient of $\frac{d}{dt}q(t)$ indicates the resistance, R is large, indicating a shunned circuit.

Solution of the circuit equation, Equation (1) is shown in Figure 2.

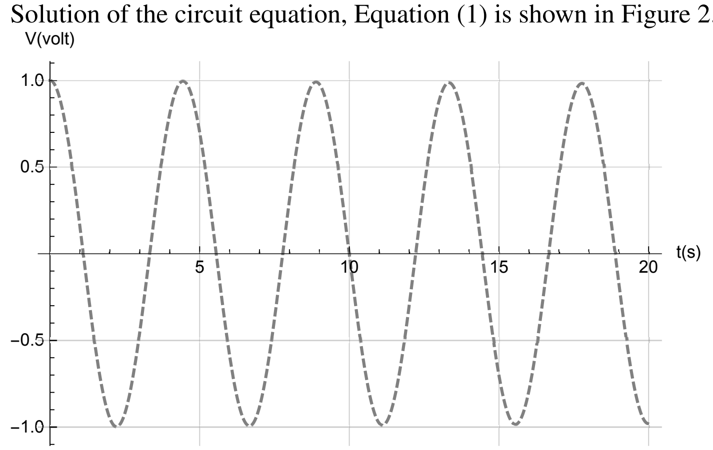

Figure 2. Voltage vs. time for a CL tank circuit.

As expected the charge oscillates, so does the voltage across the elements.
The circuit with un-shunted resistor is depicted in Figure 1 as well. The circuit equation is,

$$\frac{d^2}{\mathrm{dt}^2}q(t)+\left(\frac{1}{RC}\right)\frac{d}{\mathrm{dt}}q(t)+\frac{1}{LC}q(t)=0 \tag{2}$$

The quantity $\frac{1}{RC}$ is the damping factor. Solution of Equation (2) is depicted in Figure 3. The voltage across the elements is damped signifying the impact of the positive resistor,R.

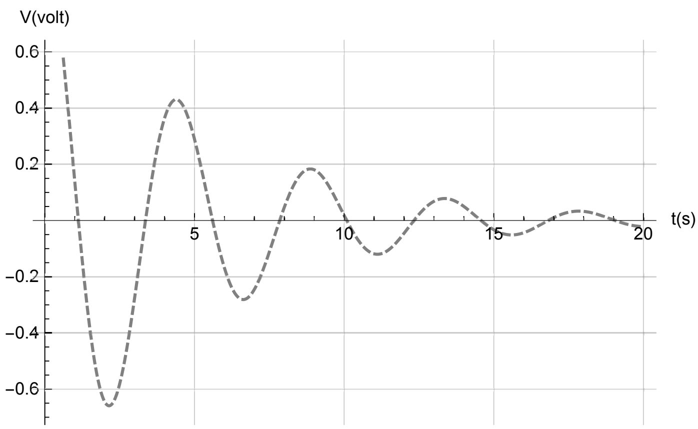

Figure 3. Voltage vs. time for the CLR-parallel circuit shown in Figure 1.
With this brief introduction we lay the foundation for replacing the ohmic resistor with a tunnel diode. The circuit is shown in Figure 4.

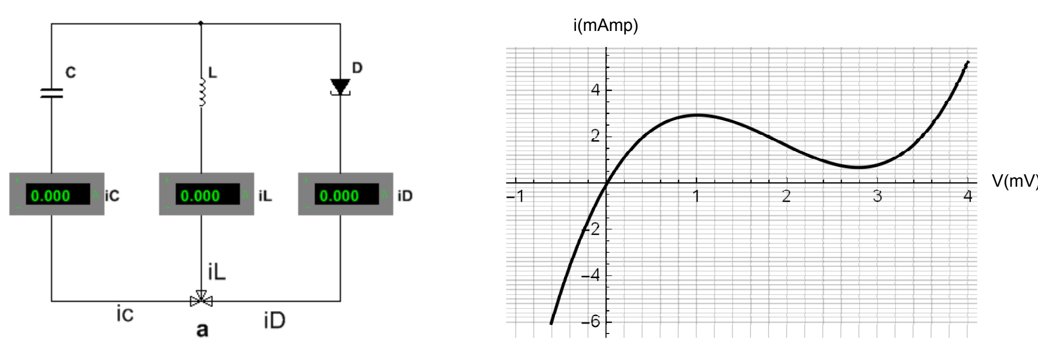

Figure 4. A CLD-parallel circuit. A charged capacitor, C, is connected parallel to the other two elements, L, and D, (left panel). I-V characteristics of the tunnel diode is shown on the right.

The I-V character of the tunnel diode shown on the right panel of Figure 4 indicates for the forward voltage it acts quite different from the one shown on the right panel of Figure 1. For the latter as shown, for the low and high voltage range it acts almost as an ohmic resistor, for the intermediate range between the adjacent maximum and minimum the negative slope signifies a negative resistor. The I-V character of the diode mathematically is parametrized [3],

$$i(V)=\alpha-\beta(V-\delta)+\gamma(V-\delta)^3 \tag{3}$$

Equation (3) is the dependence of the current through the diode as a function of the voltage across its ends. Quantities $\alpha,\beta,\gamma$and δ are specifics to the diode on hand. Parameterizing the curve with four parameters, $\{\alpha,\beta,\gamma,\delta\}$, gives the freedom adjusting its general character. For instance by adjusting these parameters the shown values of current extrema is controlled. By the same token their associated voltage separation is controlled as well. The curve has an infection point,(δ,α). And hence in Eq3, α, with dimension of current signifies a special current, it is the ordinate of the inflection point. Furthermore, by adjusting the combination of these parameters namely, $\left(\alpha+\beta\delta-\gamma\delta^{3}\right)$, the zero value of the current at V = 0 is enforced. Figure 4 is the global I-V character of a tunnel diode. Accordingly, it makes its impact on the analysis of the circuits challenging. To be as general as possible we consider a wide range of family of parameters. Each set of parameters alters the I-V character impacting the signal accordingly. Among the atlas of studied cases we present three representatives. Values of the set of parameters are shown in the corresponding figure caption.

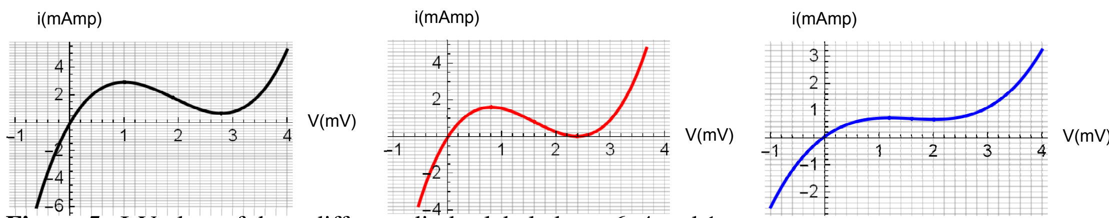

Figure 5. I-V plots of three different diodes labeled case6, 4 and 1.

Parameters associated with the graphs from left to right of Figure 5 are: case6=$\{\alpha \to 1.8, \beta \to 1.9, \gamma \to 0.8, \delta \to 1.9\}$, case4=$\{\alpha \to 0.8, \beta \to 1.5, \gamma \to 0.8, \delta \to 1.6\}$, case1=$\{\alpha \to 0.7, \beta \to 0.1, \gamma \to 0.2, \delta \to 1.6\}$, respectively. As shown, the selected parameters do alter the I-V characters compatible with the objectives of our investigation. For instance, the left most plot shows no zero current. The second set, case4, indicates at a certain voltage the current diminishes. The last plot corresponds to case1 shows for an intermediate voltage range the negative resistance drops off entirely. As a useful search tool we craft a *Mathematica* simulation program, its front-end screen is shown in Figure 6. Clicking on any of the icons of the first row displays a magnified image of the corresponding I-V pairs; the snapshot shown compares case6 (Black) vs. case4 (Red).

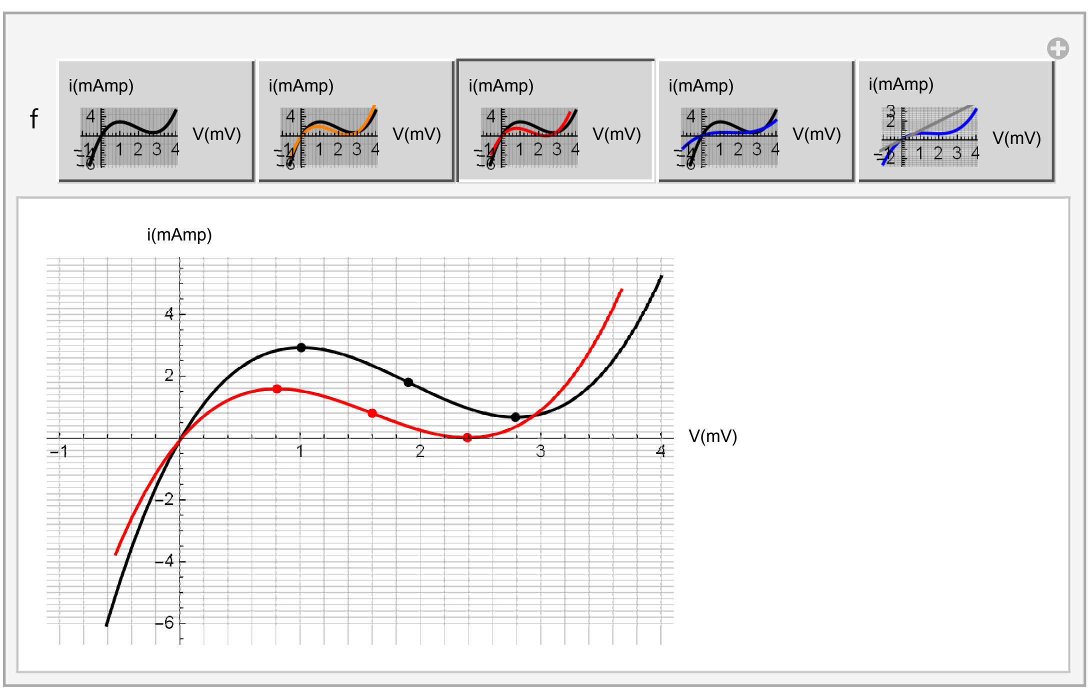

Figure 6. Snapshot of *Mathematica* simulation. I-Vs of case6 (Black) vs. case4 (Red) are compared.

For the circuit shown in Figure 4 applying the conservation of current at node a yields ,

$$i_C + i_L + i_D = 0 \tag{4}$$

where, i_C, i_L, and i_D are the currents in the capacitor, self-inductor and diode, respectively.

Substituting for $i_C = C\frac{d}{\mathrm{dt}}V(t)$, $i_L = \frac{1}{L}\int V(t)\mathrm{dt}$ and $i_D(t) = \alpha - \beta[V(t) - \delta] + \gamma[V(t) - \delta]^3$ results the circuit equation,

$$\frac{d^2}{\mathrm{dt}^2}V(t) + \frac{1}{C}\left\{-\beta + 3\gamma[V(t) - \delta]^2\right\}\frac{d}{\mathrm{dt}}V(t) + \frac{1}{LC}V(t) = 0 \tag{5}$$

We solve Equation (5) for three cases of interest. Parameters of these three cases are given in the above text. Suitably we set the values $\{C, L\} = \{5., 0.1\}$, units are implicit. To investigate the impact of the diode vs. the circuit shown in Figure 1 we solve its corresponding well-known circuit equation. Results are shown in Figure 7.

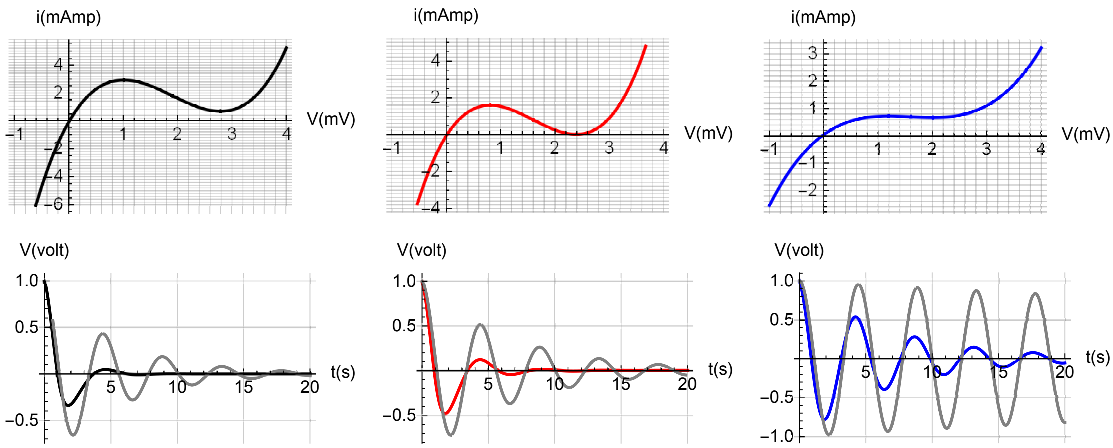

Figure 7. I-V characteristics of the tunnel diodes are on the first row. Frames from left to right correspond to case6, 4 and 1, respectively. The second row are the corresponding voltage vs. time. The solid colored curves are the voltages across the diode, the gray curves are the corresponding CLR circuit.

The frames on the first row are the ones shown in Figure 5. They are plotted for the sake of comparison. The frames shown on the second row are the corresponding voltages vs. time. The solid colored curves are the CLD circuits and the gray signals are the corresponding CLR ones. Comparing the signals reveals the impact of the diode vs. the corresponding ohmic resistor; $\frac{1}{R} = \frac{\mathrm{di}(V)}{\mathrm{dV}}|_{\gamma=0}$. As shown, diodes globally have damping impact. They damp the signals more stronger than the corresponding ohmic resistors. The reason for this enhancement is the damping factor in Equation (5), i.e. the factor in front of $\frac{d}{\mathrm{dt}}V(t)$. For a CLR circuit with a pure ohmic resistor the circuit equation is given by (2), the damping factor is $\frac{1}{\mathrm{RC}}$. For the sake of comparison with a CLD circuit we replace $q(t) = V(t)C$ in (2) resulting a $V(t)$ dependent Equation (5). Here the damping factor is not constant it is parametric and voltage dependent function. The choice of the parameters conduces the shown enhancements. Among the three I-V cases depicted in Figure 7 case1, the far right frame of the first row is the closest to the CLR circuit; exhibiting no negative resistance. As such, their corresponding voltage characters shown in far right frame of the 2$^{\mathrm{nd}}$ row are compatible.

The current through the diode is determined by inserting the solution of Equation (5) in (3). For the cases of interest these are displayed in Figure 8.

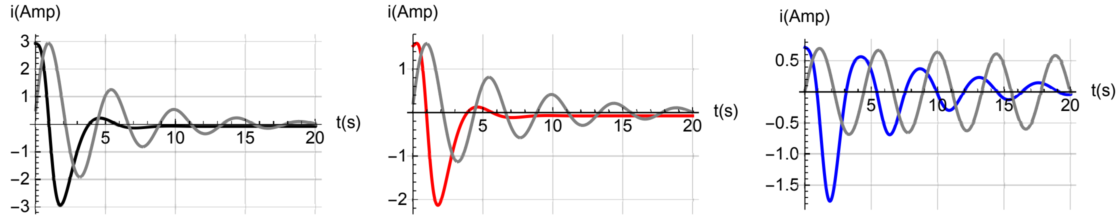

Figure 8. From left to right; the colored signals are associated with the three case6, 4 and 1, respectively. The gray curves are the currents through the resistor in the corresponding CLR circuits.

As observed the tunnel diode irrespective of the detailed parametric specifications damps the current; its dampening impact is stronger than the corresponding ohmic resistance.

3. Conclusions

One of the objectives of our investigation is to analyze the impact of a tunnel diode on the signal character of electric circuits. Among possible choices we choose a well-known circuit that by replacing only one of its elements with a diode converts the circuit to a suitable study case. Briefly we discussed the characteristics of a typical tunnel diode; we display a generic I-V plot of a typical one. To be as general as possible three different types of diodes are introduced. The impact of each diode on the circuit is analyzed and the output are compared. It is shown a tunnel diode is a nonlinear element complicating the circuit equation. As such equations are nonlinear differential equations. Derivation of the circuit equations and their corresponding solutions are a mixed blend of analytic and numeric. Numeric analysis is carried out utilizing a Computer Algebra System (CAS) specifically *Mathematica*. As a general observation a tunnel diode possessing a negative resistance has a strong dampening impact. It damps the voltage and the current in the shown circuits.
As a potential project to extending our investigation one may consider placing a tunnel diode in series with other two elements. An investigator may utilize the footwork of our work analyzing the suggested circuit. Interested readers on *Mathematica* may find [4][5] resourceful.

Acknowledgements

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