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The Graeco-Latin Square and Hyper Graeco-Latin Square Designs

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Abstract

As an experimental design model the Graeco-Latin square is an extension of a Latin square and can simultaneously control three sources of nuisance variability. The following aspects of this model will be discussed: a brief history, estimation and ANOVA, use for the analysis of experimental data (example with R code given), model generation and a test for non-additivity. An R example of the Hyper Graeco-Latin square model, which extends the Graeco-Latin square to controlling four sources of nuisance variability, will also be discussed.

Keywords: Graeco-Latin, Euler, mutually orthogonal, non-additivity, design generation, Youden square, Hyper Graeco-Latin, Replications.

1. Definition

A Latin square is an arrangement of n Latin letters (A, B, C etc.) in an $n \times n$ square so that no letter appears more than once in the same row or column. In experimental design such a design is used to simultaneously control two sources of nuisance variability. Such a design needs relatively few observations when compared to a full factorial design e.g. a three factor factorial design with r replicates where each of the factors has n levels will need rn^3 observations.

A Graeco-Latin (Euler) square is an $n \times n$ square where one Latin square (made up of Greek letters) is superimposed on another Latin square (made up of Latin letters) such that each pair of Latin and Greek letters occur only once (are orthogonal).

Table 1: Example of a 3×3 Graeco-Latin square

Aα	Bβ	Cγ
Bγ	Cα	Aβ
Cβ	Aγ	Bα

Such a design is used to simultaneously control three sources of nuisance variability.

2. Short History

The first appearance of Graeco-Latin squares in mathematical literature was in a publication by Euler in 1782. In this paper Euler conjectured that there can be no Graeco-Latin square of size $4k+2, k = 0,1,2,\cdots$.

The case $k=1$, i.e. a 6×6 Graeco-Latin square, received considerable attention in the literature. The problem is concerned with an investigation on arranging 6 officers, who have different ranks, from each of 6 regiments in a 6×6 grid such that each rank and regiment occurs exactly once in each row and column (36 officers problem). Much of Euler's paper is dedicated to outlining his argument of the non-existence of a 6×6 Graeco-Latin square and the plausibility of the non-existence of one of size $n=4k+2, k=0,1,2,\cdots$.

Table 2: Example of a near 6×6 Graeco-Latin square

Aα	Bγ	Cε	Dβ	Eδ	Fς
Bβ	Cδ	Dς	Eγ	Fε	Aα
Cγ	Dε	Eα	Fδ	Aς	Bβ
Dδ	Eς	Fβ	Aε	Bα	Cγ
Eε	Fα	Aγ	Bς	Cβ	Dδ
Fς	Aβ	Bδ	Cα	Dγ	Eε

The above table shows two 6×6 Latin squares superimposed. Since Aα and Bβ appear twice and Aδ and Bε do not appear at all, these squares are not orthogonal. Gaston Tarry (1900) was the first to confirm the nonexistence of a Graeco-Latin square of order 6. In 1959 counterexamples of Euler's conjecture were provided by Bose and Shrikhande (Graeco-Latin square of order 22) and Parker (order 10). In 1960 these three authors joined forces to prove the existence of all Graeco-Latin squares of order $n=4k+2, k=2,3,\cdots$.

3. Model Estimation and ANOVA

Ronald Fisher (1926) applied Latin squares to the design of field experiments in agriculture. The statistical analysis of the Graeco-Latin square is a straightforward extension of that of the Latin square. The statistical model for this design is

$$y_{ijkl}=\mu+\alpha_i+\beta_j+\gamma_k+\delta_l+\varepsilon_{ijkl}\,,\; i,j,k,l=1,2,\cdots,n$$

where $\alpha_i,\beta_j,\gamma_k$ and δ_l are the main effects associated with rows, columns, Latin letters and Greek letters respectively. It is assumed that $\varepsilon_{ijkl}\sim IN(0,\sigma^2)$ and $\sum_i\alpha_i=\sum_j\beta_j=\sum_k\gamma_k=\sum_l\delta_l=0$.

(i) Parameter estimation by minimizing error sum of squares:

The parameters $\mu,\alpha_i,\beta_j,\gamma_k$ and δ_l are estimated by minimizing

$S=\sum_{ijkl}\varepsilon_{ijkl}^2=\sum_{ijkl}[y_{ijkl}-(\mu+\alpha_i+\beta_j+\gamma_k+\delta_l)]^2$ with respect to $\mu,\alpha_i,\beta_j,\gamma_k$ and δ_l i.e. equating

$\frac{\partial S}{\partial \mu}, \frac{\partial S}{\partial \alpha_i}, \frac{\partial S}{\partial \beta_j}, \frac{\partial S}{\partial \gamma_k}$ and $\frac{\partial S}{\partial \delta_l}$ to 0 and solving for the parameters. This leads to $\hat{\mu} = \bar{y}_{....}$,

$\hat{\alpha}_i = \bar{y}_{i...} - \bar{y}_{....}, \hat{\beta}_j = \bar{y}_{.j..} - \bar{y}_{....}, \hat{\gamma}_k = \bar{y}_{..k.} - \bar{y}_{....}, \hat{\delta}_l = \bar{y}_{...l} - \bar{y}_{....}, i, j, k, l = 1,2,\cdots,n$.

(ii) Parameter estimation by re-parameterization:

Consider the identity

$$\varepsilon_{ijkl} = \bar{\varepsilon}_{....} + (\bar{\varepsilon}_{i...} - \bar{\varepsilon}_{....}) + (\bar{\varepsilon}_{.j..} - \bar{\varepsilon}_{....}) + (\bar{\varepsilon}_{..k.} - \bar{\varepsilon}_{....}) + (\bar{\varepsilon}_{...l} - \bar{\varepsilon}_{....})$$

$$+ (\varepsilon_{ijkl} - \bar{\varepsilon}_{i...} - \bar{\varepsilon}_{.j..} - \bar{\varepsilon}_{..k.} - \varepsilon_{...l} + 3\bar{\varepsilon}_{....}).$$

From the model

$\varepsilon_{ijkl} = y_{ijkl} - \mu - \alpha_i - \beta_j - \gamma_k - \delta_l$ and

using $\sum_i \alpha_i = \sum_j \beta_j = \sum_k \gamma_k = \sum_l \delta_l = 0$ you have that

$\bar{\varepsilon}_{....} = \bar{y}_{....} - \mu$, $\bar{\varepsilon}_{i...} = \bar{y}_{i...} - \mu - \alpha_i$, $\bar{\varepsilon}_{.j..} = \bar{y}_{.j..} - \mu - \beta_j$, $\bar{\varepsilon}_{..k.} = \bar{y}_{..k.} - \mu - \gamma_k$,
$\bar{\varepsilon}_{...l} = \bar{y}_{...l} - \mu - \delta_l$

$$E_i = \bar{\varepsilon}_{i...} - \bar{\varepsilon}_{....} = (\bar{y}_{i...} - \mu - \alpha_i) - (\bar{y}_{....} - \mu) = \bar{y}_{i...} - \bar{y}_{....} - \alpha_i$$

$$E_j = \bar{\varepsilon}_{.j..} - \bar{\varepsilon}_{....} = (\bar{y}_{.j..} - \mu - \beta_j) - (\bar{y}_{....} - \mu) = \bar{y}_{.j..} - \bar{y}_{....} - \beta_j$$

$$E_k = \bar{\varepsilon}_{..k.} - \bar{\varepsilon}_{....} = (\bar{y}_{..k.} - \mu - \gamma_k) - (\bar{y}_{....} - \bar{y}_{....}) = \bar{y}_{..k.} - \bar{y}_{....} - \gamma_k$$

$$E_l = \bar{\varepsilon}_{...l} - \bar{\varepsilon}_{....} = (\bar{y}_{...l} - \mu - \delta_k) - (\bar{y}_{....} - \mu) = \bar{y}_{...l} - \bar{y}_{....} - \delta_l$$

$$E = \varepsilon_{ijkl} - \bar{\varepsilon}_{i...} - \bar{\varepsilon}_{.j..} - \bar{\varepsilon}_{..k.} - \varepsilon_{...l} + 3\bar{\varepsilon}_{....}$$
$$= (y_{ijkl} - \mu - \alpha_i - \beta_j - \gamma_k - \delta_l) - (\bar{y}_{i...} - \mu - \alpha_i) - (\bar{y}_{.j..} - \mu - \beta_j) - (\bar{y}_{..k.} - \mu - \gamma_k)$$

$$- (\bar{y}_{..l} - \mu - \delta_l) + 3(\bar{y}_{....} - \mu)$$

$$= (y_{ijkl} - \bar{y}_{....}) - (\bar{y}_{i...} - \bar{y}_{....}) - (\bar{y}_{.j..} - \bar{y}_{....}) - (\bar{y}_{..k.} - \bar{y}_{....}) - (\bar{y}_{...l} - \bar{y}_{....})$$

Selecting the parameters $\mu, \alpha_i, \beta_j, \gamma_k, \delta_l$ such that $\sum \varepsilon_{ijkl}^2 = \sum(\bar{\varepsilon}_{....}^2 + E_i^2 + E_j^2 + E_k^2 + E_l^2 + E^2)$ is a minimum leads to the same estimates as shown above.

(iii) Parameter estimation by using the normal equations:

Write the model in matrix form as $Y = X\beta + \varepsilon$.

Suppose the order of the Graeco-Latin square is $n \neq 6$.

The first two subscripts of y_{ijkl} are $i, j = 1,2,\cdots,n$. The subscripts of k, l and the form of the X matrix will depend on the Graeco-Latin square layout and

$$X^T X = \begin{pmatrix} n^2 & n\times 1_n^T & n\times 1_n^T & n\times 1_n^T & n\times 1_n^T & n\times 1_n^T \\ n\times 1_n & n\times I_n & J & J & J & J \\ n\times 1_n & J & n\times I_n & J & J & J \\ n\times 1_n & J & J & n\times I_n & J & J \\ n\times 1_n & J & J & J & n\times I_n & J \\ n\times 1_n & J & J & J & J & n\times I_n \end{pmatrix}$$ is a $(4n+1)\times(4n+1)$ matrix,

where 1_n - column vector of n 1's ,

I_n - identity matrix of order n and

J - $n\times n$ matrix of 1's.

The parameters can be estimated from the normal equations $(X^T X)\hat{\beta} = X^T Y$ by taking the restrictions $\sum_i \alpha_i = \sum_j \beta_j = \sum_k \gamma_k = \sum_l \delta_l = 0$ into account.

ANOVA:

When performing an ANOVA the total sum of squares is partitioned by making use of the identity

$$y_{ijkl} - \bar{y}_{....} = (\bar{y}_{i...} - \bar{y}_{....}) + (\bar{y}_{.j..} - \bar{y}_{....}) + (\bar{y}_{...k.} - \bar{y}_{....}) + (y_{...l} - \bar{y}_{....}) + (y_{ijkl} - \bar{y}_{i...} - \bar{y}_{.j..} - \bar{y}_{..k.} - \bar{y}_{...l} + 3\bar{y}_{....})$$

Squaring both sides of the above expression, summing over i, j, k, l, noting that the cross product sums are zero and simplifying the expressions leads to

$SS_{total} = SS_{rows} + SS_{columns} + SS_{Latin} + SS_{Greek} + SS_{error}$, where

$$SS_{total} = \sum_{ijkl} y_{ijkl}^2 - \frac{y_{....}^2}{n^2},$$

$$SS_{rows} = \sum_i \frac{y_{i...}^2}{n} - \frac{y_{....}^2}{n^2} \quad , \quad SS_{Latin} = \sum_j \frac{y_{.j..}^2}{n} - \frac{y_{....}^2}{n^2} \quad , \quad SS_{Greek} = \sum_k \frac{y_{..k.}^2}{n} - \frac{y_{....}^2}{n^2} \quad ,$$

$SS_{columns} = \sum_l \frac{y_{...l}^2}{n} - \frac{y_{....}^2}{n^2}$ and $SS_{error} = SS_{total} - SS_{rows} - SS_{Latin} - SS_{Greek} - SS_{columns}$.

Table 3: ANOVA table for Graeco-Latin square design

Source	**Sum of squares**	**Degrees of freedom**	**Mean square₁**	**F₂**
Rows	SS_{rows}	$n-1$	MS_{rows}	F_{rows}
Columns	$SS_{columns}$	$n-1$	$MS_{columns}$	$F_{columns}$
Latin	SS_{Latin}	$n-1$	MS_{Latin}	F_{Latin}
Greek	SS_{Greek}	$n-1$	MS_{Greek}	F_{Greek}
Error	SS_{error}	$(n-1)(n-3)$	MS_{error}	

1 Mean square = Sum of squares/degrees of freedom

2 F = Mean square/Mean square error

The tests for main effects are performed by inspecting the p-values associated with the F-statistics in the last column of the ANOVA table.

4. Example

A food processor wants to determine the effect of package design on the sale of one of his products. He has five designs to be tested: A, B, C, D, E. There are a number of sources of variation. These include: (1) day of the week, (2) differences among stores, and (3) effect of shelf height. He decided to conduct a trial using a Graeco-Latin square design with five weekdays corresponding to the row classification, five different stores assigned to the column classification, and five shelf heights corresponding to the Greek letter classification. The following table contains the results of his trial (observations in brackets).

Table 4: Sales of product

Day/Store	**1**	**2**	**3**	**4**	**5**
Mon.	E α (238)	C δ (228)	B γ (158)	D ε (188)	A β (74)
Tues.	D δ (149)	B β (220)	A α (92)	C γ (169)	E ε (282)
Wed.	B ε (222)	E γ (295)	D β (104)	A δ (54)	C α (213)
Thur.	C β (187)	A ε (66)	E δ (242)	B α (122)	D γ (90)
Fri.	A γ (65)	D α (118)	C ε (279)	E β (278)	B δ (176)

The R code for producing the output below is shown in the appendix.

```
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)
day        4   1545   386.2  0.4177    0.7919
store      4   6139  1534.6  1.6595    0.2510
design     4 115462 28865.5 31.2148 6.256e-05 ***
s_height   4   8852  2213.0  2.3931    0.1366
Residuals  8   7398   924.7
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = g)

$design
     diff         lwr      upr     p adj
B-A 109.4   42.955898 175.8441 0.0030451
C-A 145.0   78.555898 211.4441 0.0004580
D-A  59.6   -6.844102 126.0441 0.0813035
E-A 196.8  130.355898 263.2441 0.0000502
C-B  35.6  -30.844102 102.0441 0.4101358
D-B -49.8 -116.244102  16.6441 0.1624912
E-B  87.4   20.955898 153.8441 0.0119182
D-C -85.4 -151.844102 -18.9559 0.0135959
E-C  51.8  -14.644102 118.2441 0.1412942
E-D 137.2   70.755898 203.6441 0.0006730
```

Only A-D, B-C, B-D and C-E are not significantly different at the 5% level. A-D is significantly different at the 10% level.

5. Constructing a Graeco-Latin Square Design

Houston (1967) explained a method according to which an 18×18 Graeco-Latin square can be constructed. Federer et al (1971) discussed various methods for constructing sets of mutually orthogonal Latin squares. Das and Dey (1990) and Subramani (1996) gave methods for the construction of Graeco-Latin squares of an odd order ≥ 3. Preece and Vowden (1995) explained the construction of 10×10 Graeco-Latin squares by incorporation of superimpostions of 3×7 Youden squares. Youden squares are Latin squares with some rows deleted. Their method is a modification of that used by Parker (1959).

Byers (1993) wrote a computer program (in BASIC) that can generate Graeco-Latin squares of orders 3, 4, 5, 7 and 9 (see website address to get design). The R package called "agricolae" has a built in function that can design Graeco-Latin squares for odd numbers and even numbers 4, 8,10 and 12. As an example, the code below will generate a Graeco-Latin square of order 8.

```
library(agricolae)
T1 <- letters[1:8]
T2 <- 1:8
outdesign <-  design.graeco(T1,T2,serie=2)
print(outdesign$sketch)
```

6. Tukey's Test for Non-Additivity

The Graeco-Latin square design is performed under the assumption of no interaction effects (model is additive). Kohli (1988) presents some examples that show that the presence of interaction effects, when assuming their absence, can distort the results of such experiments. Therefore the assumption of additivity should be checked. The non-additivity sum of squares can be calculated from

$$SS_{nonadd} = \frac{\sum_{ij}[(x_{ijkl} - \hat{x}_{ijkl})(y_{ijkl} - \hat{y}_{ijkl})]^2}{\sum_{ij}(x_{ijkl} - \hat{x}_{ijkl})^2},$$

where $\hat{y}_{ijkl}$ is the fitted value according to the Graeco-Latin model, $x_{ijkl} = \hat{y}_{ijkl}^2$ and $\hat{x}_{ijkl}$ is the fitted value when using x_{ijkl} as response in the model.

The test statistic for performing the test for non-additivity is

$$F = \frac{SS_{nonadd}/1}{(SSE - SS_{nonadd})/[(n-1)(n-3)-1]} \sim F_{1,(n-1)(n-3)-1}.$$

R code for performing a non-additivity test for the sales data.

```
x=g$fitted.values
x=x*x
g2=lm(x ~ day + store + design + s_height)
x=g$fitted.values
x2=x*x
g2=lm(x2 ~ day + store + design + s_height)
x2h=g2$fitted.values
nonadd=sum(((y-x)*(x2-x2h))^2)/sum((x2-x2h)^2)
nonadd
[1] 281.4551
SSE=anova(g)[5,2]
F=nonadd/((SSE-nonadd)/(anova(g)[5,1]-1))
F
[1] 0.2768489
pvalue=pf(q=F, df1=1, df2=anova(g)[5,1]-1, lower.tail=FALSE)
pvalue
[1] 0.6150271
```

The non-additivity test statistic shows a non-significant result. Hence the assumption of no interaction effects is plausible.

7. The Hyper Graeco-Latin Square Design

The **Hyper Graeco-Latin square** design is used to simultaneously control four sources of nuisance variability. An $n \times n$ Hyper Graeco-Latin square is constructed by superimposing three Latin squares (one with Latin letters, one with Greek letters and one with numbers $1, 2, \cdots, n$) so that each (Latin, Greek, number) triplet occurs only once.

In the following example, taken from Box, Hunter & Hunter (2005), two replicates of a 4×4 Hyper Graeco-Latin square design was performed. In the experiment the wearing quality of cloth on a machine was tested. The variables of interest are weight loss of cloth (response), types of cloth (treatment), specimen holder, position on machine, emery paper, machine cycle (nuisance variables). An ANOVA of the experiment is performed in much the same way as explained for the Graeco-Latin square design with two additional sources of variation (additional nuisance variable and replicates). The R code for the experiment is shown below.

```
> cloth=read.table("clipboard",header=T)
> attach(cloth)
> g<- lm(y~treatment+as.factor(rep)+as.factor(position)+as.factor(cycle)+a
s.factor(holder)+as.factor(paper),data=cloth)
> anova(g)

Analysis of Variance Table

Response: y
                    Df  Sum Sq Mean Sq F value    Pr(>F)    
treatment            3  1705.3  568.45  5.3908  0.021245 *  
as.factor(rep)       1   603.8  603.78  5.7259  0.040366 *  
as.factor(position)  3  2217.3  739.11  7.0093  0.009925 ** 
as.factor(cycle)     6 14770.4 2461.74 23.3455 5.273e-05 ***
as.factor(holder)    3   109.1   36.36  0.3449  0.793790    
as.factor(paper)     6  6108.9 1018.16  9.6555  0.001698 ** 
Residuals            9   949.0  105.45                      
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The degrees of freedom for a replicated factor in a replicated Hyper-Graeco-Latin square is $r \times d$, where r is the number of replicates, and d is the degrees of freedom for the factor.

In this example, cycle and paper both have 3 degrees of freedom and were replicated twice. Therefore, the degrees of freedom for both these factors is $2 \times 3 = 6$.

All the factors except holder are significant. The below calculations will give some idea about the differences between the means at the various levels of the significant factors.

```
# Comparison of means
sapply(split(y,treatment),mean) # treatment means
      A       B       C       D 
270.000 275.625 279.875 260.375 
sapply(split(y,position),mean) # position means
      1       2       3       4 
279.000 257.375 274.375 275.125 
sapply(split(y,paper),mean) # paper means
     a      b      c      d      e      f      g      h 
276.00 264.50 278.50 256.00 249.50 295.25 286.75 265.25 
sapply(split(y,cycle),mean) # cycle means
     1      2      3      4      5      6      7      8 
307.25 248.25 245.00 268.00 301.75 268.00 253.75 279.75 
sapply(split(y,rep),mean) # replicate means
       1        2 
267.1250 275.8125 
```

8. Appendix

R code for Graeco-Latin square analysis

```
# Graeco-Latin Square
y=c(238,149,222,187,65,228,220,295,66,118,158,92,104,242,279,188,169,54,
122,278,74,282,213,90,176)
day = as.factor(rep(1:5, each=5))
store = as.factor(rep(1:5, times=5))
design = as.factor(c("E","D","B","C","A","C","B","E","A","D","B","A","D",
"E","C","D","C","A","B","E","A","E","C","D","B"))
s_height = as.factor(c("a","d","e","b","c","d","b","c","e","a","c","a","b",
"d","e","e","c","d","a","b","b","e","a","c","d"))
```

```
data = data.frame(y,day,store,design,s_height)
g = lm(y ~ day + store + design + s_height)
anova(g)

# Tukey HSD tests
 aov(g)
 ( g1 <- TukeyHSD(aov(g), "design") )
```

R code for function that will randomly generate a Latin square of order n ≤ 20

From a Latin square as shown below a Graeco-Latin square can be written down by superimposing a Latin square by using Latin letters with each row in reverse order.

```
> Latin <- function (n)
 {
     LS = matrix(LETTERS[1:n], n, n)
     LS = t(LS)
     for (i in 2:n) LS[i, ] = LS[i, c(i:n, 1:(i - 1))]
     for (i in 1:20) {
         LS = LS[sample(n), ]
         LS = LS[, sample(n)]
     }
     LS
}
# Example
> a=Latin(4) # Generate Latin square of order 4
> b=a[, ncol(a):1] # Reverse order of rows
> a
     [,1] [,2] [,3] [,4]
[1,] "D"  "C"  "B"  "A"
[2,] "A"  "D"  "C"  "B"
[3,] "C"  "B"  "A"  "D"
[4,] "B"  "A"  "D"  "C"
> b
     [,1] [,2] [,3] [,4]
[1,] "A"  "B"  "C"  "D"
[2,] "B"  "C"  "D"  "A"
[3,] "D"  "A"  "B"  "C"
[4,] "C"  "D"  "A"  "B"
```

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