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Parameter Identification of Nonlinear Dynamic Systems of Industrial Processes

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Abstract: the problem of parameter identification of nonlinear dynamic systems of industrial processes on the set of continuous block-oriented models, the elements of which are different modifications of the Hammerstein and Wiener models, is considered. Method of parameter identification in steady state based on the observation of the system's input and output variables at the input sinusoidal influences is proposed. The solution of the problem of parameter identification is reduced to the solution of the systems of algebraic equations by using the Fourier approximation. The parameters estimations are received by the least squares method. Reliability of the received results, at the parameter identification of the nonlinear system in industrial conditions at the presence of noise, depends on the accuracy of the measurement of system output signals and mathematical processing of the experimental data at the approximation.

Key words: nonlinear system, block-oriented model, control, identification, parameter, sinusoidal.

1. Introduction

Basis of automation of any system of industrial processes is the presence of the information on a condition of the system in the form of mathematical model.

One of the main trends of control theory - system identification is supposed to build optimal system model based on observations system input and output variables at the time of real operation of the system.

It should be noted that identification methods are widely used in practice, particularly in metallurgy, energy, chemical production, physics, biology and medicine, aviation, communication systems, etc.

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The systems identification is connected with the solution of different problems depending on the a priori information about the system. The construction of the system's adequate model in many respects is determined by solving the structure and parameter identification problems.

Usually model structure is determined on the basis of a priori information or depending on the physical laws of the processes that take place in the system [1]. However, the structure of the model determined in this way often has high dimension, and its application is not expedient for the solution of practical problems. Therefore, it is necessary to rely on the identification and estimation.

The problem of parameter identification of systems is solved based on a posteriori information by different approaches and methods.

The majority of the real industrial dynamic systems are nonlinear and possess many "inconvenient" features from the point of view of their control. Such systems can be adequately characterized only by nonlinear models on all working area of change of variables.

The nonlinear systems are generally represented by the block-oriented models [2] consisting of different modifications of the Hammerstein and Wiener models, or general models, in particular, the Volterra [3] and Wiener [4] series and the Kolmogorov-Gabor continuous and discrete polynomials [5-6].

Different modifications of the Hammerstein and Wiener models consist of different combinations of connections of linear dynamic and nonlinear static blocks. Despite their simplicity, such models are successfully used in many fields of the industrial processes (modelling of ball-tube mills of the concentrating factory, distillation columns, hydraulic actuators, electrical water-heater process, heat transfer process, etc.).

At the representation of nonlinear systems by the block-oriented models, most of the existing developed methods of parameter identification is developed for the simple Hammerstein and Wiener models (e.g. [7-10]). Comparatively small quantity of works is devoted to the identification of parameters of other block-oriented models (e.g. [11-12]). This can be explained by the fact that the majority of such models, except for the Hammerstein models (simple and generalized) are nonlinear relative to the parameters, and also because of the big number of estimated parameters. So, for example, the number of estimated parameters of the simple Wiener-Hammerstein cascade model with the nonlinear elements in the form of polynomial functions of n degree and linear dynamic elements described by the differential equations of m_1 and m_2 order, is equal to $n+m_1+m_2+3$. Therefore, the solution of the problem of parameter identification is analytically possible only for some block-oriented low order models.

In this work the problem of parameter identification is considered on the set of models, elements of which are generalized Hammerstein and Wiener models, Wiener-Hammerstein and Hammerstein-Wiener cascade models with nonlinear elements in the form of polynomial functions of second degree and linear elements described by the ordinary differential equation of first and second order. Despite their simplicity, such models are widely used for the modelling of industrial processes.

It should be noted that the problem of parameter identification considered in this work can be connected directly with the problem of structure identification using the experimental data, received for solving the problem of structure identification based on a posteriori information application (e.g. [13-14]).

2. Classes of Models and Input Signals

The problem parameter identification is considered on the following class of continuous block-oriented models:

$$L = \{s_i \mid i = 1,2,3.4\}, \tag{1}$$

where s_1 is the generalized Hammerstein model, s_2 - the generalized Wiener model, s_3 - Wiener-Hammerstein cascade model and and s_4 - Hammerstein-Wiener cascade model (Fig. 1).

The nonlinear static elements, which are included in the models are described by finite degree polynomial functions:

$$f_1(u) = \sum_{i=0}^{n} c_i u^i(t), \tag{2}$$

$$f_2(x) = \sum_{j=0}^{m} d_j x^j(t), \tag{3}$$

where $c_i \ (i = 0,1,\cdots,n)$, $d_j \ (j = 0,1,\cdots,m)$- constant coefficients.

$W(p)$ and $W_i(p)$ $(i = 1,2)$ are transfer functions of the linear dynamic systems in the operational form, i.e. p denotes the differentiation operation - $p \equiv d/dt$. $u(t)$ and $y(t)$ are input and output variables, correspondingly.

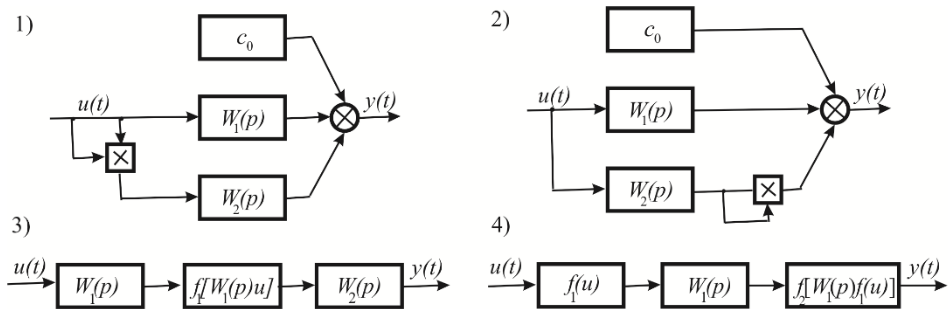

Fig.1. Block-oriented models: 1) generalized Hammerstein model; 2) generalized Wiener model; 3) simple Wiener-Hammerstein cascade model; 4) simple Hammerstein -Wiener cascade model.

The linear dynamic elements included in the class of block-oriented models, are assumed to be steady, i.e. the roots of their characteristic equations are placed in left half plane of the roots plane.

Models of the set L are generally described by the following equations:

- **Generalized Hammerstein model**

$$y(t) = c_0 + W_1(p)u(t) + W_2(p)u^2(t); \tag{4}$$

- **Generalized Wiener model**

$$y(t) = c_0 + W_1(p)u(t) + [W_2(p)u(t)]^2; \tag{5}$$

- **Simple Wiener-Hammerstein cascade model**

$$y(t) = W_2(p)\sum_{i=0}^{n} c_i [W_1(p)u(t)]^i; \tag{6}$$

- **Simple Hammerstein -Wiener cascade model**

$$y(t) = \sum_{i=1}^{m} d_j \left[W(p) \sum_{i=1}^{n} c_i u^i(t) \right]^j. \tag{7}$$

For solving the problem of parameter identification of nonlinear systems on the basis of the active experiment it is supposed, that the input variable of the system $u(t)$ is a sinusoidal function:

$$u(t) = A\cos\omega t, \tag{8}$$

where A is an amplitude and ω is an angular frequency.

3. The Mathematical Description of the Forced Oscillation

When the harmonic signal acts on the input of the nonlinear block-oriented system, the forced oscillation with features for different models is obtained at the output of the system in the steady state after the termination of the transient process.

When a sinusoidal signal acts on the input of the linear element**,** then at the output of the element in the steady state the sinusoidal signal with same frequency and with the amplitude and phase, which depends on the frequency, is obtained. In this case, the periodic signal is obtained on the output of nonlinear static element, which is the sum of the sinusoidal signals with multiple frequencies. These facts were used to determine analytical expressions of the forced oscillations on the output of models.

Let's consider the case when the transfer functions of the model's linear dynamic parts are defined by the expression:

$$W_i(p)=\frac{1}{T_{0i}p^2+T_ip+1}\quad(i=1,2), \tag{9}$$

where $T_{0i}>0\,(i=1,2)$ has a dimension of time square, and $T_i>0\,(i=1,2)$ - a dimension of time.

The forced oscillations, obtained at the output of the models in the steady state, as in case when $T_i>2\sqrt{T_{0i}}\ \ (i=1,2)$ and $T_i<2\sqrt{T_{0i}}\ \ (i=1,2)$, are determined by the expressions:

- **Generalized Hammerstein model**

$$y(t)=c_0+\frac{1}{2}A^2+\frac{A}{\sqrt{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2}}\cos(\omega t+\varphi_{11})+\frac{A^2}{2\sqrt{\left(1-4\omega^2T_{02}\right)^2+4\omega^2T_2^2}}\cos(2\omega t+\varphi_{22}), \tag{10}$$

where

$$\varphi_{1i}=-arctg\frac{\omega T_i}{1-\omega^2T_{0i}},\ \varphi_{2i}=-arctg\frac{2\omega T_i}{1-4\omega^2T_{0i}}\ (i=1,2). \tag{11}$$

From (10), taking into account (11), after a series of transformations we obtain:

$$y(t)=c_0+\frac{1}{2}A^2+\frac{A\left(1-\omega^2T_{01}\right)}{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2}\cos\omega t+\frac{A\omega T_1}{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2}\sin\omega t+$$

$$+\frac{A^2\left(1-4\omega^2T_{02}\right)}{2\left[\left(1-4\omega^2T_{02}\right)^2+4\omega^2T_2^2\right]}\cos 2\omega t+\frac{A^2\omega T_2}{\left(1-4\omega^2T_{02}\right)^2+4\omega^2T_2^2}\sin 2\omega t\,. \tag{12}$$

- **Generalized Wiener model**

$$y(t)=c_0+\frac{1}{2}A^2+\frac{A\left(1-\omega^2T_{01}\right)}{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2}\cos\omega t+\frac{A\omega T_1}{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2}\sin\omega t+$$

$$+\frac{A^2\left(1-4\omega^2T_{02}\right)}{2\left[\left(1-4\omega^2T_{02}\right)^2+4\omega^2T_2^2\right]}\cos 2\omega t+\frac{A^2\omega T_2}{\left(1-4\omega^2T_{02}\right)^2+4\omega^2T_2^2}\sin 2\omega t \tag{13}$$

- **Simple Wiener-Hammerstein cascade model**

To simplify the calculations, we will suppose that in expression (2): $n=2,\ c_0=c_1=0,\ c_2=c$ and in (9): $T_{0i}=0\ (i=1,2)$, then after calculation we will receive:

$$y=\frac{cA^2}{2(1+\omega^2T_1^2)}+\frac{cA^2\left(1-\omega^2T_1^2-4\omega^2T_1T_2\right)}{2(1+\omega^2T_1^2)^2(1+4\omega^2T_2^2)}\cos 2\omega t+\frac{cA^2\left[\left(1-\omega^2T_1^2\right)\omega T_2+\omega T_1\right]}{2(1+\omega^2T_1^2)^2(1+4\omega^2T_2^2)}\sin 2\omega t\ ; \tag{14}$$

- **Simple Hammerstein -Wiener cascade model**

After supposing that in expressions (2), (3): $n=2,\ c_0=c_1=d_0=d_1=0,\ c_2=d_2=1$ and in (9): $T_{0i}=0\ (i=1)$ and $T_1=T$, then we will receive:

$$y=\frac{A^4}{4}+\frac{A^4+4A^4\omega^2T^2}{8\left(1+4\omega^2T^2\right)^2}+\frac{A^4}{2\left(1+4\omega^2T^2\right)}\cos 2\omega t+\frac{A^4\omega T}{1+4\omega^2T^2}\sin 2\omega t+$$

$$+\frac{A^4-4A^4\omega^2T^2}{8\left(1+4\omega^2T^2\right)^4}\cos 4\omega t+\frac{A^4\omega T}{2\left(1+4\omega^2T^2\right)^2}\sin 4\omega t\ . \tag{15}$$

4. Parameter Identification

Let's consider the features for the parameters estimation of models by the method using the Fourier approximation by the method of the least squares.

4.1. Generalized Hammerstein model

The application of the Fourier approximation [15] for the output periodic signal of the system enables to obtain the estimates of the Fourier coefficients $\frac{\hat{a}_0}{2}$, $\hat{a}_k$, $\hat{b}_k$, $(k=1,2)$. Equating such estimates with their theoretical values we'll get:

$$\frac{\hat{a}_0}{2}=c_0+\frac{1}{2}A^2, \tag{16}$$

$$\hat{a}_1=\frac{A\left(1-\omega^2T_{01}\right)}{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2},\quad \hat{b}_1=\frac{A\omega T_1}{\left(1-\omega^2T_{01}\right)^2+\omega^2T_1^2}, \tag{17}$$

$$\hat{a}_2=\frac{A^2\left(1-4\omega^2T_{02}\right)^2}{2\left[\left(1-4\omega^2T_{02}\right)^2+4\omega^2T_2^2\right]},\quad \hat{b}_2=\frac{c_2A^2\omega T_1}{\left(1-4\omega^2T_{02}\right)^2+4\omega^2T_2^2}. \tag{18}$$

From expression (16) we obtain the estimates of the coefficient c_0:

$$\hat{c}_0 = \frac{1}{2}\left(\hat{a}_0 - A^2\right). \tag{19}$$

Using the expressions (17) and (18) we obtain the equations for determining the estimates of the parameters T_{01}, T_1, T_{02} and T_2:

$$\omega_i^2 T_{01} + \frac{\hat{a}_{1i}}{\hat{b}_{1i}}\omega_i T_1 + \varepsilon_{1i} = 1 \ (i = 1,2,...,n), \tag{20}$$

$$4\omega_i^2 T_{02} + 2\frac{\hat{a}_{2i}}{\hat{b}_{2i}}\omega_i T_2 + \varepsilon_{2i} = 1 \ (i = 1,2,...,n), \tag{21}$$

where $\hat{a}_{1i}, \hat{b}_{1i}, \hat{a}_{2i}, \hat{b}_{2i} \ (i = 1,2,\dots,n)$ - values of the Fourier coefficients at the frequency ω_i, $\varepsilon_{1i}, \varepsilon_{2i} \ (i = 1,2,\dots,n)$ - errors of measurements and approximations.

Let's consider the features for T_{01} and T_1 parameters estimation by the method of least squares using the expression (20).

The error squared sum is

$$S = \sum_{i=1}^{n}\varepsilon_{1i}^2 = \sum_{i=1}^{n}\left(1 - \omega_i^2 T_{01} - \frac{\hat{a}_{1i}}{\hat{b}_{1i}}\omega_i T_1\right)^2. \tag{22}$$

Now we'll determine the values of the estimates $\hat{T}_{01}$ and $\hat{T}_1$ so that their substitution for T_{01} and T_1 should give the minimal value S in the equation (22). For that purpose differentiating (22) at first by T_{01} and then by T_1, and equating the received results to zero, we'll obtain the following expressions for estimating $\hat{T}_{01}$ and $\hat{T}_1$:

$$\begin{gathered} \sum_{i=1}^{n}\omega_i^4 T_{01} + \sum_{i=1}^{n}\frac{\hat{a}_{1i}}{\hat{b}_{1i}}\omega_i^3 T_1 = \sum_{i=1}^{n}\omega_i^2, \\ \sum_{i=1}^{n}\frac{\hat{a}_{1i}}{\hat{b}_{1i}}\omega_i^3 T_{01} + \sum_{i=1}^{n}\frac{\hat{a}_{1i}^2}{\hat{b}_{1i}^2}\omega_i^2 T_1 = \sum_{i=1}^{n}\frac{\hat{a}_{1i}}{\hat{b}_{1i}}\omega_i. \end{gathered} \tag{23}$$

The solution of the system of equations (23) allows obtaining the estimates of the parameters $\hat{T}_{01}$ and $\hat{T}_1$ using the method of the least squares:

$$\hat{T}_{01} = \frac{\left(\sum_{i=1}^{n}\omega_i^2\right)\left(\sum_{i=1}^{n}\frac{\hat{a}_{1_i}^2}{\hat{b}_{1_i}^2}\omega_i^2\right) - \left(\sum_{i=1}^{n}\frac{\hat{a}_{1_i}}{\hat{b}_{1_i}}\omega_i\right)\left(\sum_{i=1}^{n}\frac{\hat{a}_{1_i}}{\hat{b}_{1_i}}\omega_i^3\right)}{\left(\sum_{i=1}^{n}\omega_i^4\right)\left(\sum_{i=1}^{n}\frac{\hat{a}_{1_i}^2}{\hat{b}_{1_i}^2}\omega_i^2\right) - \left(\sum_{i=1}^{n}\frac{\hat{a}_{1_i}}{\hat{b}_{1_i}}\omega_i^3\right)^2}, \tag{24}$$

$$\hat{T}_{1} = \frac{\left(\sum_{i=1}^{n}\omega_i^4\right)\left(\sum_{i=1}^{n}\frac{\hat{a}_{1_i}}{\hat{b}_{1_i}}\omega_i\right) - \left(\sum_{i=1}^{n}\omega_i^2\right)\left(\sum_{i=1}^{n}\frac{\hat{a}_{1_i}}{\hat{b}_{1_i}}\omega_i^3\right)}{\left(\sum_{i=1}^{n}\omega_i^4\right)\left(\sum_{i=1}^{n}\frac{\hat{a}_{1_i}^2}{\hat{b}_{1_i}^2}\omega_i^2\right) - \left(\sum_{i=1}^{n}\frac{\hat{a}_{1_i}}{\hat{b}_{1_i}}\omega_i^3\right)^2}. \tag{25}$$

$\hat{T}_{02}$ and $\hat{T}_2$ estimates by the method of the least squares, obtained using (21), look like:

$$T_{02} = \frac{1}{4}\frac{\left(\sum_{i=1}^{n}\omega_i^2\right)\left(\sum_{i=1}^{n}\frac{\hat{a}_{2i}^2}{\hat{b}_{2i}^2}\omega_i^2\right) - \left(\sum_{i=1}^{n}\frac{\hat{a}_{2i}}{\hat{b}_{2i}}\omega_i\right)\left(\sum_{i=1}^{n}\frac{\hat{a}_{2i}}{\hat{b}_{2i}}\omega_i^3\right)}{\left(\sum_{i=1}^{n}\omega_i^4\right)\left(\sum_{i=1}^{n}\frac{\hat{a}_{2i}^2}{\hat{b}_{2i}^2}\omega_i^2\right) - \left(\sum_{i=1}^{n}\frac{\hat{a}_{2i}}{\hat{b}_{2i}}\omega_i^3\right)^2}, \tag{26}$$

$$T_{2} = \frac{1}{2}\frac{\left(\sum_{i=1}^{n}\omega_i^4\right)\left(\sum_{i=1}^{n}\frac{\hat{a}_{2i}}{\hat{b}_{2i}}\omega_i\right) - \left(\sum_{i=1}^{n}\omega_i^2\right)\left(\sum_{i=1}^{n}\frac{\hat{a}_{2i}}{\hat{b}_{2i}}\omega_i^3\right)}{\left(\sum_{i=1}^{n}\omega_i^4\right)\left(\sum_{i=1}^{n}\frac{\hat{a}_{2i}^2}{\hat{b}_{2i}^2}\omega_i^2\right) - \left(\sum_{i=1}^{n}\frac{\hat{a}_{2i}}{\hat{b}_{2i}}\omega_i^3\right)^2}. \tag{27}$$

4.2. Generalized Wiener model

Equating the estimates of the Fourier coefficients $\frac{\hat{a}_0}{2}$, $\hat{a}_k$, $\hat{b}_k$, $(k = 1,2)$ of the output periodic signal to their theoretical values from (13), we obtain:

$$\frac{\hat{a}_0}{2} = c_0 + \frac{A^2}{2\left[\left(1-\omega^2 T_{02}\right)^2 + \omega^2 T_2^2\right]}. \tag{28}$$

In that case the coefficients $\hat{a}_1$ and $\hat{b}_1$ are determined by the expressions (17), and

$$a_2 = \frac{A^2\left[\left(1-\omega^2 T_{02}\right)^2 - \omega^2 T_2^2\right]}{2\left[\left(1-\omega^2 T_{02}\right)^2 + \omega^2 T_2^2\right]^2},\; b_2 = \frac{A^2\left(1-\omega^2 T_{02}\right)\omega T_2}{\left[\left(1-\omega^2 T_{02}\right)^2 + \omega^2 T_2^2\right]^2}. \tag{29}$$

In that case $\hat{T}_{01}$ and $\hat{T}_1$ estimates obtained by the method of the least squares are determined by the expressions (24) and (25).

Using the expressions (29) it is possible to get the system of equations linear with respect to T_{02}, T_2, T_{02}^2, T_2^2, $T_{02}T_2$:

$$2\omega_i^2 T_{02} + 2\omega_i \frac{a_{2_i}}{b_{2_i}} T_2 - \omega_i^4 T_{02}^2 - 2\omega_i^2 \frac{a_{2_i}}{b_{2_i}} T_{02} T_2 + \omega_i^2 T_2^2 = 1 \ \left(i = 1,2,\ldots,n\right), \tag{30}$$

and using it to receive the estimates of the parameters $\hat{T}_{02}$ and $\hat{T}_2$ obtained by the method of the least squares. When we know the estimates $\hat{T}_{02}$ and $\hat{T}_2$, it is easy to determine the estimate c_0 using (28).

4.3. Simple Wiener-Hammerstein cascade model

Equating the estimate of the Fourier coefficient $\frac{\hat{a}_0}{2}$ of the output periodic signal to its theoretical value from (14), after series of operations according to the least squares method and transformations we'll get system of equation for estimating $\hat{c}$ and $\hat{T}_0$:

$$\begin{aligned} &A^2 n\hat{c} - \left(\sum_{i=1}^{n} \hat{a}_{0i}\omega_i^2\right)\hat{T}_0 = \sum_{i=1}^{n} \hat{a}_{0i}, \\ &A^2\left(\sum_{i=1}^{n} \hat{a}_{0i}\omega_i^2\right)\hat{c} - \left(\sum_{i=1}^{n} \hat{a}_{0i}\omega_i^4\right)\hat{T}_0 = \sum_{i=1}^{n} \hat{a}_{0i}^2\omega_i^2, \end{aligned} \tag{31}$$

where $T_0 = T_1^2$.

The solution of the system of equations (31) allows obtaining the estimates of the parameters $\hat{c}$ and $\hat{T}_1$ by the least squares method:

$$\hat{c} = \frac{\left(\sum_{i=1}^{n} \hat{a}_{0i}\right)\left(\sum_{i=1}^{n} \hat{a}_{0i}\omega_i^4\right) - \left(\sum_{i=1}^{n} \hat{a}_{0i}\omega_i^2\right)^2}{A^2 n\left(\sum_{i=1}^{n} \hat{a}_{0i}\omega_i^4\right) - A^2\left(\sum_{i=1}^{n} \hat{a}_{0i}\omega_i^2\right)}, \tag{32}$$

$$\hat{T}_1 = \sqrt{\frac{n\left(\sum_{i=1}^{n}\hat{a}_{0i}\omega_i^2\right)-\left(\sum_{i=1}^{n}\hat{a}_{0i}\right)\left(\sum_{i=1}^{n}\hat{a}_{0i}\omega_i^2\right)}{n\left(\sum_{i=1}^{n}\hat{a}_{0i}\omega_i^4\right)-\left(\sum_{i=1}^{n}\hat{a}_{0i}\omega_i^2\right)}}\,. \tag{33}$$

After receiving estimate $\hat{T}_1$ using expression of ratio:

$$\frac{a_2}{b_2} = \frac{1-\omega^2 T_1^2 - 4\omega^2 T_1 T_2}{\left(1-\omega^2 T_1^2\right)\left(\omega T_2 + \omega T_1\right)}\,, \tag{34}$$

it is possible to obtain the estimate of the parameter T_2 by the least squares method:

$$\hat{T}_2 = \frac{\sum_{i=1}^{n}\left(\hat{a}_{2i}\omega_i^3\hat{T}_1^2 - 4b_{2i}\omega_i^2\hat{T}_1\right)\left(\hat{a}_{2i} - \hat{b}_{2i} + \hat{a}_{2i}\omega_i\hat{T}_1 + \hat{b}_{2i}\omega_i^2\hat{T}_1^2\right)}{\sum_{i=1}^{n}\left(\hat{a}_{2i}\omega_i^3\hat{T}_1^2 - 4\hat{b}_{2i}\omega_i^2\hat{T}_1\right)}\,. \tag{35}$$

4.4. Simple Hammerstein -Wiener cascade model

Equating the estimate of the Fourier coefficient $\hat{b}_2$ of the output periodic signal to its theoretical value from (15)

$$\hat{b}_2 = \frac{A^4\omega T}{1+4\omega^2 T^2}\,, \tag{36}$$

after series of operations we'll get system of equation at the different frequencies $\omega = \omega_i \ (i = 1,2,\ldots,n)$ for estimating $\hat{T}$:

$$\begin{aligned} A^4\left(\sum_{i=1}^{n}\omega_i^2\right)T - \left(4\sum_{i=1}^{n}\hat{b}_{2i}^2\omega_i^3\right)\hat{T}_0 &= \sum_{i=1}^{n}\hat{b}_{2i}\omega_i\,,\\ A^4\left(\sum_{i=1}^{n}\hat{b}_{2i}\omega_i^3\right)T - \left(4\sum_{i=1}^{n}\hat{b}_{2i}^2\omega_i^4\right)\hat{T}_0 &= \sum_{i=1}^{n}\hat{b}_{2i}^2\omega_i^2\,. \end{aligned} \tag{37}$$

The solution of the system of equations (37) allows obtaining the estimate of the parameter $\hat{T}$:

$$\hat{T} = \frac{\left(\sum_{i=1}^{n}\hat{b}_{2i}\omega_i\right)\left(\sum_{i=1}^{n}\hat{b}_{2i}^2\omega_i^4\right) - \left(\sum_{i=1}^{n}\hat{b}_{2i}^2\omega_i^3\right)\left(\sum_{i=1}^{n}\hat{b}_{2i}^2\omega_i^2\right)}{A^4\left[\left(\sum_{i=1}^{n}\omega_i^2\right)\left(\sum_{i=1}^{n}\hat{b}_{2i}^2\omega_i^4\right) - \left(\sum_{i=1}^{n}\hat{b}_{2i}^2\omega_i^3\right)^2\right]}\,. \tag{38}$$

Using expression of estimate of $\hat{a}_2$:

$$\hat{a}_2 = \frac{A^4}{2\left(1+4\omega^2 T^2\right)} \tag{39}$$

at the frequencies $\omega = \omega_i \ (i = 1,2,\ldots,n)$ allows to get system of equations for estimating

$$\frac{A^4}{2} - 4\hat{a}_{2i}\omega_i^2 T^2 + \varepsilon_{2i} = \hat{a}_{2i} \quad (i = 1,2,\ldots,n), \tag{40}$$

By solving the system (40) we obtain another estimate $\hat{T}$ by using estimate of Fourier coefficient $\hat{a}_{2i}$:

$$\hat{T} = \sqrt{\frac{2\sum_{i=1}^{n}\hat{a}_{2i}^2\omega_i^2 - A^4\sum_{i=1}^{n}\hat{a}_{2i}\omega_i^2}{8\sum_{i=1}^{n}\hat{a}_{2i}^2\omega_i^4}}\,. \tag{41}$$

5. Investigation of the Identification Method on Accuracy

According to developed method of parameter identification algorithms of parameter identification of nonlinear systems have been designed. In order to use of the designed algorithms of parameter identification in the industrial conditions under noise and disturbances, it is necessary to investigate identification method on accuracy.

The identification method is investigated by theoretical analysis and computer modelling. The reliability of the received results, at the parameter identification of nonlinear systems of industrial processes conditions in the presence of noise and errors, depends on the measurement accuracy of the systems' input and output signals and on the mathematical processing of the experimental data at the approximation. Besides, as it is known, that used method of the least squares is noiseless.

The investigation of the algorithms of the parameter identification of nonlinear systems was carried out by means of the computer modelling based on using MATLAB.

We used both, the tool of package Simulink-toolbox for the system modelling and tool Symbolic Math Toolbox for the solution of the equations.

For example, result of computer modelling is given here for Generalized Hammerstein model.

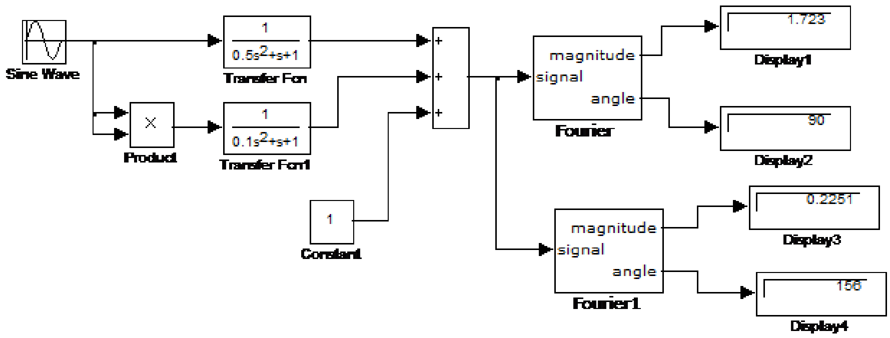

Fig. 2. Scheme for Generalized Hammerstein model

When investigating the algorithms of parameter identification the programs corresponding to such algorithms were designed. Using such programs the diagrams of output variable models and the estimations of unknown parameters have been obtained. As an example, the results obtained for the Generalized Hammerstein model are given below. The experiments were carried out at values of the parameters $c_0 = 2,\ T_{01} = 1{,}5,\ T_1 = 2,\ \ T_{02} = 0{,}5,\ T_2 = 1$. Following estimations of the unknown parameters are obtained: c0 =2, T01 = 1.4709, T1 = 1.9528, T02 = 0.4882, T2 =0.9729.

6. Conclusion

In this work the problem of parameter identification is considered on the set of models, elements of which are generalized Hammerstein and Wiener models, Wiener-Hammerstein and Hammerstein-Wiener cascade models with nonlinear elements in the form of polynomial functions of second degree and linear elements described by the ordinary differential equation of first and second order. Despite their simplicity such models are widely used for the modelling of industrial processes.

The solution of the problem of parameter identification for the majority of block - oriented models is complicated due to the nonlinearity of such models relative to the sought parameters.

Proposed method of parameter identification in steady state based on the observation of the system's input and output variables at the input sinusoidal influences is proposed. The solution of the problem of parameter identification is reduced to the solution of the algebraic equations systems by using the Fourier approximation. The parameters estimations are received by the least squares method. Reliability of the received results depends on the accuracy of the measurement of system output signals and mathematical processing of the experimental data at the approximation.

Developed parameter identification method can be used for the modelling of nonlinear industrial processes when the model structure is known a priori. As the estimations of parametres are received by the least squares method, which is noiseless, it can be used in the industrial conditions in the presence of the noise and measurement errors. The specification of the method of identification allows to use Fourier coefficients of various harmonics to estimate the parametres and compare the received results.

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