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Application of Critical Path Method (CPM) and Project Evaluation Review Techniques (PERT) in Project Planning and Scheduling

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Abstract

In the field of project management today, most projects face cost and time-runs which increase the complexity of project involved. This study presents an analysis of the critical path method (CPM) and the project evaluation review technique (PERT) in projects planning (a case study of FUTA post graduate building). This study used secondary data collected from SAMKAY Construction Company comprised of list on project activities. It highlights the means by which a network diagram is constructed from a list of project activities and the computation involved for each method. The CPM will enable project managers to evaluate the earliest and latest times at which activities can start and finish, calculate the float (slack), define the critical path. PERT will enable the project manager to approximate the probability of completing the project within estimated completion time. Then we apply both methods with the data collected, where the earliest time, latest time, and slack are calculated to get the completion day of 207days. Finally, we estimate the probability that the project will be completed within the estimated completion days to be 68.8% using PERT.

Keywords: Network Analysis, CPM and PERT, Project Construction.

Introduction

A project can be defined as a set of a large number of activities or jobs that are performed in a certain sequence determined logically or technologically and it has to be completed within a specified time to meet the performance standards. The project could be the development of a software program, the building of a house or an office building, development of a new drug, and many others. For many years, two approaches that have been proven to be useful for planning, scheduling and controlling construction projects have been the Critical Path Method (CPM) and the Project Evaluation and Review Technique (PERT).

These techniques enable project managers to evaluate the earliest and latest times at which activities can start and finish and it also estimate the probability for the completion day. Therefore, this project is aimed at exploring the time of various activities involved in a construction building at Federal university of technology Akure, Ondo state, Nigeria in order to determine the optimal completion time using CPM and PERT techniques. The aim of this research is to study the construction of building using CPM & PERT techniques to satisfy the following objectives:

- To estimate the earliest and latest time for each activity to be completed without delay.
- To predict the completion day for construction of the building.
- To obtain the probability that the project will be completed within the predicted completion day.

Methodology

Critical Path Method Approach

The critical path method is a mathematically based algorithm for scheduling a set of project activities. It is an important tool for effective project management. Any project with interdependent activities can apply this method of scheduling. The essential technique for using CPM is to construct a model of the project that includes the following:

- A list of all activities required to complete the project (also known as work breakdown structure).
- The time (duration) that each activity will take to completion.
- The dependencies between the activities.
- Calculate the longest path of planned activities to the end of the project. **–** The earliest and latest that each activity.

Definition of terms

Earliest Event Time (*ETi*)**:** is the earliest time at which the event corresponding to node (i) can occur.
Latest Event Time (*LTi*)**:** is the latest time at which the event corresponding to node (i) can occur without delaying the completion of the project.
Total Float (*TFij*) **of activity** (*i*, *j*)**:** ($i < j$) is the amount by which the starting time of activity (i, j) can be delayed beyond its possible starting time without delaying the completion of the project.

The critical path is computed in two parts: For the starting event, we set the time at zero, $ESo = 0$. Let the duration of activity (i, j) be denoted by *dij* ($i < j$).

Earliest time formula: $max\ [ETi, ETi + dij]$

For the terminal node we set the latest event time equal to the earliest time, i.e. $LTT = ETT$, and compute Latest time formula:

$$min\ [LTi, LTi - dij]$$

The final calculation is the total float (*TFij*) for activity (*i, j*)

$$\text{FLOAT} = |ET - LT|$$

The PERT Approach

This is called programmable Evaluation and review techniques. This deals with problem of uncertain activities. The statistical analysis to apply in order to estimate or determine time of each activity concerning the project used 3-time estimates. Namely are:

$t_{o:}$ The most optimistic time

$t_{p:}$ The most pessimistic time

$t_{l:}$ The most likely time

Optimistic estimate: the time the activity would take if things did go well.
Pessimistic estimate: the time the activity would take if things did not go well.
Most likely estimate: the consensus best estimate of the activity's duration.

It also assumes that the duration of each activity has a density function approximated by the beta distribution. The beta distribution is unimodal and takes the shape of the form

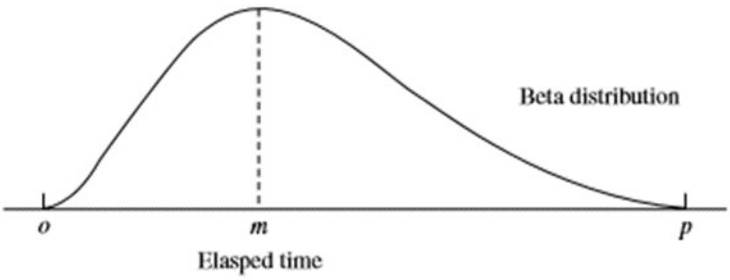

Fig.1. Shape of Beta Distribution

The duration of an activity is calculated using the following formula:

$$t_e = \frac{(t_0 + 4t_m + t_p)}{6}$$

Where

t_e is the expected time,

t_o is the optimistic time,

t_m is the most probable activity time and

tp is the pessimistic time.

The standard deviation, which is a good measure of the variability of each activity is calculated by the rather simplified formula:

$$s = \frac{(t_p - t_0)}{6}$$

The variance is the square of the standard deviation.

Application and Results

The computation of the earliest and latest time to get the completion day for the SAMKAY project using the critical path method (CPM). This is done by getting the earliest and latest time for each activity and calculating the float (slack) by subtracting latest time from earliest time i.e. LT-ET and the activity with zero float (slack) gives the critical path which constitute the completion day. Also, the project evaluation review technique (PERT) will be used to estimate the probability that the work will be completed within the estimated completion day. The PERT make use of three estimates; optimistic, most likely and pessimistic estimate to get the expected value and variance of the duration of each activity for the project. The company estimated 140days for the project. This means the construction of the building must be completed within 140days. We looked at past data and present data on each activity to be carried out on the site and found the best and worst case of each activity.

The Project Network Diagram

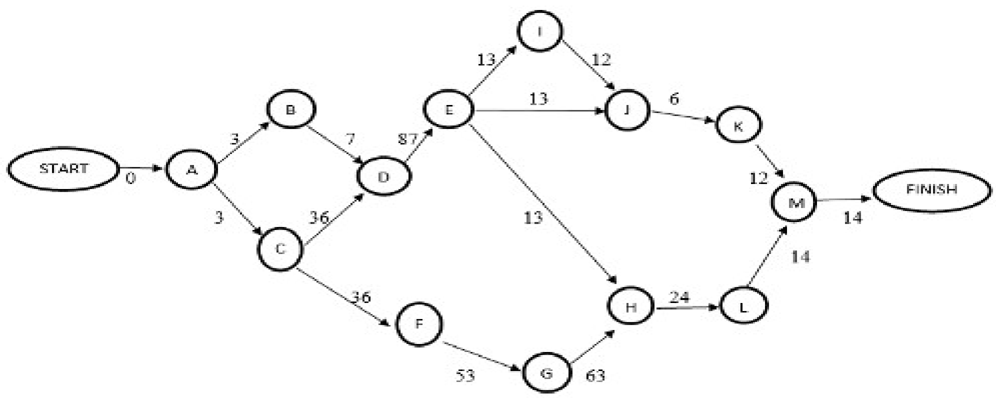

Fig.2. SAMKAY project network diagram

CPM CALCULATIONS

The earliest and latest time for each activity will be derive from figure. 2 (the project network diagram) above. The critical path is the paths that make up the project completion day. The activities with zero slack make up the critical path.

Table 1. Table showing the earliest and latest time with their slack values

Activity	Earliest time (ET)	Latest time (LT)	Slack (ET - LT)	Activity on critical path?
A	0	0	0	Yes
B	3	48	45	No
C	3	3	0	Yes
D	39	55	16	No
E	126	142	16	No
F	39	39	0	Yes
G	92	92	0	Yes
H	155	155	0	Yes
I	139	163	24	No
J	151	175	24	No
K	157	181	24	No
L	179	179	0	Yes
M	193	193	0	Yes

DISCUSSION: The summation of the estimated duration of each activity on the critical path will give the project completion day. Since A, C, F, G, H, L, M are the activities on the critical path, therefore the critical path is

$$A - C - F - G - H - L - M = 3 + 36 + 53 + 63 + 24 + 14 + 14 = 207 days$$

Table 2. Table showing the paths and lengths through the project network diagram

Path	Length
START - A - B - D - E - I - J - K - M - FINISH	3+7+87+13+12+6+12+14 = 154days
START - A - B - D - E - J - K - M - FINISH	3+7+87+13+6+12+14 = 142days
START - A - B - D - E - H - L - M - FINISH	3+7+87+13+24+14+14 =162days
START - A - C - F - G - H - L - M - FINISH	3+36+53+63+24+14+14 =207days
START - A - C - D - E - I - J - K - M - FINISH	3+36+87+13+12+6+12+14 =183days
START - A - C - D - E - J - K - M - FINISH	3+36+87+12+6+12+14 =170days
START - A - C - D - E - H - L - M - FINISH	3+36+87+13+24+14+14 =191days

Thus, for the SAMKAY Construction Co. project, we have

$$START - A - C - F - G - H - L - M - FINISH$$

(Estimated) project duration is 207days

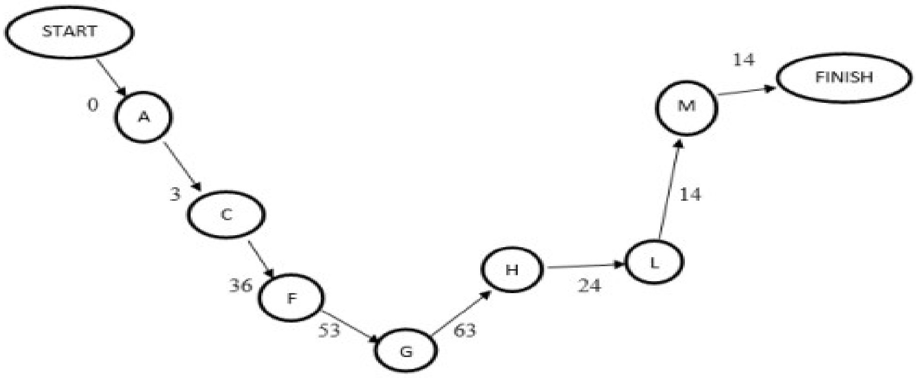

Fig. 3. The Critical Path Network Diagram of the Project

DISCUSSION: The schematic diagram above is the critical path because it is the path with the highest duration on the network diagram. Summing up the estimated duration of each activity on this path gives 207days. Therefore, the project completion day is 207days. Also, from table 2. by using all the possible path of the network diagram, we are able to conclude that the project completion day is 207days since it has the longest path.

PERT CALCULATIONS

Using PERT to estimate the probability, PERT uses three estimates; optimistic, most likely and pessimistic estimate. Three observations were collected from the SAMKAY construction company which is classified under the three estimates to compute the expected value and variance of the duration of each activity for the project.

Table 3. Table showing the Expected value and Variance of the duration of each activity for the project

Activity	**Optimistic estimate**	**Most likely Estimate**	**pessimistic estimate**	**Mean**	**Variance**
A	3	3	5	3.33	0.11
B	6	7	10	7.33	0.44
C	30	36	42	36	4
D	77	87	97	87	11.11
E	11	13	18	13.5	1.36
F	45	53	58	52.5	4.69
G	60	63	72	64	4
H	20	24	29	24.17	2.25
I	10	12	15	12.17	0.69
J	5	6	8	6.17	0.25
K	10	12	16	12.33	1
L	11	14	20	14.5	2.25
M	12	14	20	14.67	1.77

DISCUSSION: The mean and variance for the project are computed by summing up the mean of the activities on the critical path and also, summing up the variance of the activities on the critical path.

Table 4. Table showing the estimated mean and variance for the project

Activities on mean critical path	**Mean**	**Variance**
A	3.33	0.11
C	36	4
F	52.5	4.69
G	64	4
H	24.17	2.25
L	14.5	2.25
M	14.6	1.77
Project duration	μ **= 209.17**	σ^2 **= 19.07**

Approximating the Probability of Meeting the Completion Day

Let

T = Project duration (in days), which has (approximately) a normal distribution with mean μ = 209.17, variance σ^2 = 19.07 and deadline for the project d = 207*days.*

Since the standard deviation of T is σ = 4.37, the number of standard deviations by which d exceeds μ is

$$K = \frac{(d-\mu)}{\sigma} = \frac{(207-209.17)}{4.37} = 0.49$$

Therefore, using the table for a standard normal distribution (a normal distribution with mean 0 and variance 1), the probability of meeting the completion days (given the three simplifying approximations) is

$P\ (T \leq d) = P\ (\textit{standard normal} \leq K\alpha)$

$= 1 - P\ (\textit{standard normal} < K) = 10.3121 = 0.6879.$

The chance is well above 68.8% that the process duration will not exceed 207days.

Interpretation of Result

After all calculations, we are able to estimate that if there are enough workers the project can be completed in 207days with 68.8% chance that the process duration will not exceed 207days. The activities on the critical path (i.e. those with zero slack) must be started punctually. Also, activity A (bringing plants and equipment to site) must start immediately, activity C (substructure work; foundation) must start after 3days, activity F (erect walls) must start after 39days, activity G (walls finishes) must start after 92days, activity H (floor finishes) must start after 155days, activity L (external works) must start after 179days, activity M (cleaning up and final inspection) must start after 193days. For activities with non-zero slack there is a scope of varying their start times. Activity B (site clearance and site office construction) can be started after 3days, activity D (make and position frame) can be started any time after 39 and 55days, activity E (roofing) can be started any time after 126 and 142days, activity I (ceiling finishes) can be started any time after 139 and 163days, activity J (electrical installation) can be started any time after 151 and 175days, activity K (painting and decoration) can be started any time after 157 and 181days.

We thereby conclude that if all the work is completed on time, this does indeed give a working schedule for the construction of the building in the minimum time of 207days. Also, the solution could well be affected if there is a limit to the number of worker available and some factors. Also, the chance is well above 68.8% that the process duration will not exceed 207days.

Conclusion

This work has shown that CPM and PERT can be used extensively to assist project managers in planning, scheduling, and controlling of projects. Overtime, these two techniques gradually have merged. The CPM and the PERT are both valuable tools that any organization can use successfully to manage their projects. By focusing on the construction of building, a lot can be achieved in planning and scheduling a project.

Nevertheless, CPM/PERT has stood the test of time in providing project managers with most of the help they want. Furthermore, much progress is being made in developing improvements and extensions to CPM/PERT (such as the precedence diagramming method for dealing with overlapping activities) that addresses these deficiencies. Cost can also be included in the analysis to achieve better control over the process.

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Work plan collected for SAMKAY Construction Company

Table 5. List of activities, precedence relation, and their durations.

Activity	**Activity description**	**Immediate predecessors**	**Estimated duration**
A	Bringing plants and equipment to site	-	3days
B	Site clearance and site office construction	A	7days
C	Substructure work (foundation)	A	36days
D	Make and position frame	B, C	87days
E	Roofing	D	13days
F	Erect walls	C	53days
G	Walls finishes	F	63days
H	Floor finishes	E, G	24days
I	Ceiling finishes	E	12days
J	Electrical installation	E, I	6days
K	Painting and decoration	J	12days
L	External works	H	14days
M	Cleaning up and final inspection	K, L	14days