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Integration of Geogebra in Teaching and Learning Geometric Transformation

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Abstract

This paper discusses the use of GeoGebra in teaching geometric transformations. As GeoGebra is an interactive geometry, algebra, statistics and calculus application of mathematical software is very essential from school to university level to foster mathematical experiments and discoveries. Also, the contribution of this paper was a reflection of several specific examples of transformation namely reflection, rotation, translation and dilation for teaching mathematics to sixteen students in one of the secondary schools of Kathmandu Valley, Nepal. Subscribing teaching experiment as qualitative research methodology, this paper deals with the implementation of GeoGebra in six episodes, we have used adequate illustrations, pictures and animations of objects using GeoGebra to make abstract concepts of transformation visible to the students. The result of experiment shows that GeoGebra is helpful in teaching and learning the abstract concepts of transformation. Findings of this study show that if GeoGebra is used in mathematics classroom, students could become an active constructor of knowledge. Similarly, they collaborate with each other, visualize the process of transformation, and enjoy their authority in such classes. It acts as an important educational tool so as to support the traditional lecture-method of teaching mathematics that shifts education system from teacher centered to learner centered.

Keywords: Constructor of knowledge, GeoGebra, teaching of mathematics, transformation, mathematical experiment, teaching experiment.

1. Introduction

Technology is a powerful tool for engaging students in learning mathematics. No doubt, technology has become one of the powerful resources of learning and teaching mathematics. In this ethos, new technological applications such as GeoGebra, Google Sketch Up, and Sketch Pad, etc. are becoming more useful to improve and enhance teaching and learning mathematics to visualize mathematical concepts. Even more, the importance of using technology in mathematics teaching has been advocated by the National Council of Teachers of Mathematics (NCTM) for many years (NCTM, 2015, 2018). One of the convincing tools that can be used to teach mathematics is GeoGebra. In addition, GeoGebra is an interactive geometry, algebra, statistics and calculus application of mathematical software for the basic level to the university as the best application designed to construct and illustrate mathematical concepts. It was created by Markus in 2001/2002 as part of his master’s thesis in mathematics education and computer science at the University of Salzburg in Austria. Last but not least, GeoGebra is available on multiple platforms such as Android, ISO, and desktop.

2. Statement of Research Problem

21st century learners are familiar to the pictorial culture of mathematics as well as other contents through a wide access to social media. In this regard, traditional lecture-based method of teaching mathematics is still dominant in most of the mathematics teaching and learning in Nepalese mathematics classes which is not aligned to the pictorial culture of teaching and learning mathematics. Further, the main obstacles in teaching mathematics could be concepts without an adequate illustration followed by mathematical graphs which are static in classical way of teaching mathematics i.e. drawing it on a piece of paper, where static objects do not allow for generalization of the concept (Dahal, Luitel & Pant, 2019). Hence, the purpose of this research article was to explore the possible use of dynamic geometric software GeoGebra in teaching and learning geometric transformation as a great tool for teaching and learning mathematics. This research attempts to determine whether the use of GeoGebra helps the students to understand the concepts of geometric transformation.

3. Literature Review

3.1 ICTs in Mathematics Classroom

Technology has a great supporting role in mathematics education. If technologies are used in an appropriate way, it can accelerate the effective learning of the students. Teachers and students can investigate more on mathematical ideas with the help of proper use of technologies. So it greatly influences on what mathematics is being taught and what is being learned by students (Bismarck, 2009). New innovations on technologies have made the lifestyle of human beings more comfortable and easier. These technologies can also be incorporated into mathematics education to make teaching and learning activities effective and live. It can be a supporting tool for making education system more effective. In addition, use of ICTs in mathematics classroom can help teachers to make class interactive and to motivate students to learn mathematics. Instead, if ICTs are used in mathematics teaching, objects can be moved any time to show movement of objects by simply dragging 'free' objects around the plane of the drawing. With the use of ICTs, students can generate changes using a technique or manipulating free objects and they can learn how the dependent objects are affected. In this way, students will have the opportunity to solve problems by investigating mathematical relations. Although drawings on paper or on whiteboard can make ideas visible, these static drawing often fail to convey principles of mathematics. For example, it is easier to see how the variable m in $f(x)= mx + c$ represents the rate of change when the function is graphed (where 'm' represents slope of a line, 'c' represents y-intercept) and students can explore the connection between 'm' and the gradient (slope) of the line (Roschelle et al., 2007). Many other concepts such as in mathematics can be made visible using ICTs in the classroom.

3.2 Abstractness to Concreteness: Visualization

Mathematical concepts can be applied in fields like engineering, astronomy, finance, economics, statistics etc. So, these concepts can be useful in real life to discuss real-life knowledge. These concepts can be discussed on the basis of concrete materials like physical, virtual and pictorial objects. This practice of using concrete materials in education has been a great support in psychology and education (Bruner, 1966; Piaget, 1970). The benefits to use of concrete objects are that it provides a practical context for learners to understand real-world knowledge. Similarly, concrete objects allow learners to discuss and construct knowledge of abstract concepts (Brown et al., 2009). ICT tools (or mathematical software) such as a GeoGebra helps to relate abstract concepts of transformation of geometry with tangible objects with the help of a picture and its animation, thus, GeoGebra promotes other software to be used which helps to manipulate physical objects by changing the variables. Hence, students will find it easy to visualize mathematics while GeoGebra is used to explain such concepts. Thus, GeoGebra can help to change the abstractness of mathematics to concreteness and it can helpful for visualization as well.

3.3 Difficulties in Learning Geometry

Inadequate understanding in learning geometry causes discouragement among students and leads to poor performance in geometry (Idris, 2006). In addition, some factors have been identified as causing difficulties in geometry learning such as geometry language, visualization abilities and ineffective instruction of the tutor(s) (Idris, 2006). In this regard, Idris (2006) highlighted that some spatial visualization has been linked to geometric achievement with the reason that geometry is visual in nature as well. Geometry is the study of shape and space, it requires visualizing abilities but many students cannot visualize (Guven & Kosa, 2008). Hence, learning geometry may not be easy, and a large number of students fail to develop an adequate understanding of its concepts, its reasoning, and its problem-solving skills, whilst there exist both opportunities and challenges to incorporate ICT software (Dahal & Dahal, 2015) like GeoGebra in Nepalese mathematics education.

4. Purpose and Research Question

The purpose of this study was to explore possible use of dynamic geometric software GeoGebra in teaching and learning geometric transformation. To study more systematically and completely, the following research question works as the guideline for this study: how does use of GeoGebra help students to understand concepts of geometric transformation?

5. Theoretical Referents

5.1 Cognitive Learning Theory

Cognitive theory is concerned with the development of a person’s thought processes. It can be broadly defined as the act or process of knowing (Belbase & Sanzenbacher, 2016). According to this theory, learning happens when knowledge in environment is transformed into a human’s mind and is stored there. Knowledge is gained through experience or modification of pre-existing knowledge to adapt in the changing environment. In addition to that, this theory focuses on the mind and attempts to show that usually the information is received, assimilated, stored and recalled. According to cognitivist, learning can be acquired through listening, watching, touching, studying and then processing and remembering the information. There are various software which can support such learning processes of cognitivist learning, among various software such as GeoGebra, Google Sketch Up, and Sketch Pad, etc. that are used in teaching mathematics, GeoGebra is one of them.

5.2 Radical Constructivism

The basic assumption of radical constructivism is that mathematics is not an “out there” pre-existing body which has to be discovered, but it is something which is constructed by individuals in an active way (Ellerton & Clements, 1992). Constructivism motivates students to share their own ideas, expand their knowledge by utilizing their experiences, think critically on new ideas and experiences, reflect upon changing some of their ideas and create meaningful learning environment (Huang, 2002). In constructivist view of learning, learners construct, reconstruct and deconstruct their own understanding of the world by experiencing things and reflecting on those experiences. Also, constructivist believes that there is always more than one solution to a problem, and students must try to solve problems from more than one perspective (Ellerton & Clements, 1992). For this, students must ask questions, explore and assess what they know. Radical constructivism states that knowledge cannot be transmitted from one person to another person but instead learning occurs by the process "learning by doing".

The roles of the teacher's shifts from being a source of knowledge to facilitating learning, so, as students co-operate with each other rather than compete. In second authors' research class, students were free to express their opinion on the efficiency of experimental classes in their study. The second author, allowed students to work on mathematical problems and then solve those problems on their own ways, they were enthusiastic during the whole process. Even more, they were eager to share their work. Sometimes, though they made mistakes, they did not hesitate to come forward with their work. It was good to know that they were learning from their mistakes too. Second author tried to make the experimental classes students centered to the extent possible. It helped students to become active constructor of knowledge and not only a passive recipient of knowledge. There were meaningful discussions in the classroom where all the students collaborated with each other to construct knowledge of transformation.

5.3 Vygotsky's Social Learning Theory

Vygotsky (1978) believed that the community plays an important role in meaning-making. According to it, knowledge is socially constructed. As per this theory, some of the social aspects like language, culture, every day practices, material objects, interpersonal interaction, peer interaction, tools, and symbols are key features in learning. Further discussions, conversations, listening, speaking and other interactions conrtibute constructively in learning in classroom, students are encouraged to participate in interaction and discussion among themselves. For Vygotsky, cognitive development occurs as a result of social interaction. Cooperation, collaboration, and group investigation methods allow students to discuss ideas, beliefs, conceptions and misconceptions with their peers and teachers. This enhances the learning of the students. According to Vygotsky (1978), a difference exists between what a student can do on his own and what the student can do with help on the zone of proximal development. With the help of peers and teacher(s), students are capable to perform tasks that they are incapable of completing on their own. In our experiment classes, GeoGebra helped students the same way. Students already had some pre-existing knowledge and thereby GeoGebra helped in scaffolding the students' level of understanding of transformation. For this, we interacted with students to access the knowledge they already had about transformation and then tried to help them gain new knowledge with the help pictorial images, animations and adequate illustrations in GeoGebra. Similarly, social constructivism guided us to make research classes collaborative. We tried to create an environment where students actively discussed in the subject matter. For this, we have separated few minutes every day for discussion on the efficacy of the previous session on their understanding of transformation. Similarly, we gave some questions for students to solve. They helped each other in calculating the correct solution. While doing so, some of them constructed new knowledge through interaction with their peers. On other days, second author allowed students to operate GeoGebra software to explore properties of transformation. Students helped each other while operating GeoGebra. Similarly, while playing games in GeoGebra, students helped each other to play the game. All of these social activities helped in strengthening students to construct new knowledge.

6. Methodology

We adopted teaching experiment methodology in this research study to explore the use of GeoGebra in teaching geometric transformation (Steffe & Thompson, 2000). In this regard, we have used teaching experiment method to critically analyzing the emotions and behaviors shown by research participants during research study. As teaching experiment method is specifically designed for mathematics education, we have chosen teaching experiment over qualitative methods of research. The purpose of choosing this methodology was to experience students' learning process. We conducted experiment classes for two weeks. In experiment classes, second author taught reflection, rotation, translation and dilation of geometric transformation by using GeoGebra in six episodes with the help of first author and third author. Second author also made use of adequate illustrations, pictures and animations of objects using GeoGebra in order to make abstract concepts of transformation visible to the students. For that, we selected sixteen students of grade IX from one of the private institutions of Kathmandu Valley as research participants and research site. Steffe and Thompson (2000) explained that in teaching experiment methodology, a teacher or an educationist, as a researcher, can experience students' first-hand mathematical learning and reasoning. In addition, teaching experiment occurs only when we get to actively participate with students. One of the data collection tools in our research was responsive and intuitive interactions with the students. As it is hard in a short duration of time to understand how students constructed knowledge, we engaged intuitively with the same students for two weeks. We tried to uncover the processes by which students learn geometric transformation. A teaching experiment involves a sequence of teaching episodes (Steffe & Thompson, 2014). We divided whole study period into six teaching episodes. We included a witness-researcher as well in the study to record anecdotal behavior of research participants. In this research, second author have involved in teaching the concepts of geometrical transformation by using GeoGebra (Shrestha, 2017) where as first and third author spent some other days in interacting with them to collect data using field note and interview to experience for meaning-making process in relation to the students. Retrospective analysis is a critical part of teaching experiment methodology. It is even more labor-intensive than the activity of teaching (Steffe & Thompson, 2000). As a researcher and teacher, we analyzed the collected information retrospectively. We reviewed teaching experiment method critically by analyzing the emotions and behaviors shown by research participants during research study. Then, we analyzed the activities of classes watching the video recordings of second authors' experiment classes. The actions included participants' body gestures, facial expressions and their behaviors during classes. In addition to that, we also wrote down the information collected from the witness-researcher. Similarly, we transcribed interviews with the students. Then, we have developed a theme from research findings. After analyzing and interpreting the collected data, we finally discussed findings of the study.

7. Discussion of the Findings

In this section, we present the major findings of our research study. From the continuous conversation with participants and going through the retrospective analysis of the records that we collected in experiment classroom, following are the findings we have summarized.

7.1 GeoGebra is easy to use Software

We experienced that our participants found GeoGebra software easy to operate. A glow could be seen on their faces to see GeoGebra screen and the tools to operate. During research period, most students operated GeoGebra software without any major difficulties. Sometimes, we had to guide them but most often they operated GeoGebra like an expert. It was a proud moment for us to see participants operating GeoGebra and creating mathematical concepts by themselves. It was easy for us to explain about the GeoGebra tools and their uses.

During the research classes, second author allowed participants to play games and puzzles in GeoGebra. They were very excited to know that games can also be played on GeoGebra. While solving the puzzle and playing the game, they moved the objects in GeoGebra like an expert does. This made us to realize that students are very familiar with today's technology. During conversation with participants, second author asked them whether they were confident on operating GeoGebra to which most of them answered that they could operate it. First and third author could actually feel the confidence in their voices during observation. As it is easy for students to operate GeoGebra to learn geometric transformation, it helps students to explore new concepts of mathematics even without the presence of a teacher. It helps students in strengthening their thorough processes. They develop new concepts of transformation linking new knowledge with their existing one. Hence, its user-friendly interface helps students to explore different characteristics of geometric transformation by students themselves.

7.2 GeoGebra Assists for Active Learners

We found students actively participating in classroom activities. They regularly interacted with second author during the classes and tried to learn by themselves. We found them enthusiastic watching second author using technology in the classroom. It seems that it was a new classroom experience for them. Second author research classes were student-centered which supported constructivism (Bruner, 1966). In most of the classes, we worked on different geometrical objects. We constructed geometrical objects, changed variables of objects to see the effect on their properties while animated objects, reflected, rotated and dilated them. After some demonstration, second author allowed students to construct geometrical objects in GeoGebra application. Students operated GeoGebra, constructed objects, changed variables of objects, reflected and rotated and dilated the objects and then observed the changes in the position of objects and their images. While operating GeoGebra, students were active enough. While one of the participants was busy in operating GeoGebra, other participants discussed on the correct method to use GeoGebra tools. They were helping each other. It rarely happened in regular classes as second author was the only one who spoke in those classes. As students remained passive listener most of the time, after we finished demonstration of transformation each day, second author asked students to find an image of an object after transformation followed by reflection, rotation, translation and dilation thereby told them to plot those object and image in a graph paper, this too they did it actively. During the tasks assigned to them, they did not hesitate to interact and to show their tasks. They made mistakes but that helped them to learn too. Second author did not feel any difficulty in guiding them to solve problems of the textbook. Further, visualization of transformation motivated them to do the task actively. Similarly, when second author used GeoGebra software to construct an object and then transformed it, students were at ease to explore the relationship between position of an object and an image. Most of the time, they constructed the relation (formula) between the position of an object and an image by themselves. They were constructing knowledge. Students interacted with us frequently in experiment classes than in regular classes, by now they had already become active learners rather than passive receivers. From our observation and the information collected from the research field, we can conclude that GeoGebra assists students to learn geometric transformation actively.

7. 3 Discovery Learning through GeoGebra

We experienced that GeoGebra can help students in discovery learning of mathematics. It helps in exploration of the mathematical concepts. Students can create geometric figure by themselves and explore different properties of that geometric object by simply dragging the variables (Bakar, Ayub & Tarmizi, 2010). We no longer represented the traditional teacher who used only lecture-method in teaching mathematics. During second authors' experiment classes; we got our self involved with them to explore mathematical concepts as second author was a co-learner along with students. We experienced that students were enthusiastic in learning by doing. Most students during research period came forward to operate GeoGebra software to explore different properties of geometrical objects. It gives a hint that in preliminary schooling, students could be encouraged to explore the basic properties of geometrical figures with the help of GeoGebra. During classes, participants discovered the relation between the position of an object and an image after the object is transformed. So, if students are given opportunity to use GeoGebra, they can discover mathematical concepts by themselves. It gives them feeling of ownership of their own creations. GeoGebra helped first, second and third authors as a teacher-researcher to create an environment for students to learn by doing on their own pace, time and styles. As second author acted as a coach but not as a knowledge transmitter, students developed some of the concepts of transformation by sharing of knowledge. In this way, it can be said that GeoGebra helps in constructive learning. Finally, we found that GeoGebra helps in discovery learning which obviously helps to improve quality of teaching and learning mathematics.

7.4 Better Understanding of Geometric Transformation by Using GeoGebra

While interviewing some of the students before second authors' experiment classes, we found that students feared in mathematics. In response to our curiosity regarding their learning of geometric transformation in earlier grade, one of them shared that she used to understand but could not retain it for a longer period of time. She recalled that her teacher did not show her the process of reflection and rotation in grade VIII. That might be one of the reasons she was unable to understand the process of reflection and rotation. But, during study, we found participants enjoying in learning geometric transformation. Their actions and facial expressions showed that they were able to understand transformation better than they did in grade VIII. When second author demonstrated some abstract concepts of mathematics in GeoGebra, we noticed glow in their faces. On day one, one of the participants said to second author, *"Sir, I think I will retain the mathematical concepts for longer period of time if I am taught like this."* During research classes, we called some students to operate GeoGebra software. It gave them feeling of being the creator of their own work. They enjoyed using GeoGebra software. It helped them to learn transformation in a better way. They constructed knowledge by themselves. During an interview with the students after episode two, we found that participants could easily visualize the process of reflection, rotation and translation. They told us that the visualization of geometric transformation helped them to understand transformation in a better way. According to one of the students, they were involved and active in experiment classes than in regular classes. Other students also felt that there was interaction in experiment classroom than in regular classroom. So, from the collected information, we can say that the use of GeoGebra helped participants to understand geometric transformation in a better way.

7.5 GeoGebra Encourages Cooperative Learning

We experienced regular interaction. There was friendly environment in the classroom. Students guided each other while playing games and while operating GeoGebra software. They also supported each other while solving puzzle in GeoGebra. When some participants came in front of classroom to operate GeoGebra to construct an object and to transform it and made a mistake, other participants guided their peer(s) to operate GeoGebra. While doing the class work too, they supported each other in finding solution to the problems. Students felt free to ask anything to the second author and to show their work. During interview after each teaching episode, students told that the classroom was more interactive than regular classroom. There was collaborative learning among students in experiment classroom. The research classes supported Vygotsky's social learning theory as there were interactions among the students and knowledge were socially constructed. Hence, we thought the help of GeoGebra, uplifted (scaffolded) students understanding of geometric transformation from their level of understanding to a new level.

7.6 GeoGebra Assists in Visualization of Geometric Transformation

When interviewed with some of the students, we found that they were confused to visualize some of the mathematical concepts (Dahal, 2019). Some of them were unable to understand mathematics because they could not visualize it confidently. Our findings of research study suggest that GeoGebra can be widely used to make abstract concepts of mathematics tangible for students. During experiment classes, when second author moved objects to show different properties of geometrical objects, students were surprised. When second author showed them the process of reflection, rotation, translation and dilation, they were very excited and pleased. It was new experience for them to visualize the process of reflection, rotation, translation and dilation. It was fun for students to see objects moving in irregular path during transformation. Later on, during the discussion on the relation between position of an object and an image after transformation, most of them agreed that they could remember the relation for longer period of time due to visualization shown in GeoGebra. This supported that the cognitive theory proposed by Piaget (1970) that students first develop ideas concretely and then only progress to abstractions later on. On day five, we allowed participants to play a game in which they had to find the axis of reflection of the object and the image given in GeoGebra worksheet. They made some mistakes but they learned from those mistakes. Later, almost all participants found correct axis of reflection. The game helped them to visualize the position of the axis of reflection, the object and the image. In this way, it can be said that most of the participants who were in iconic stage, developed concepts of transformation with the help of pictures, animated objects and games. On the ninth day of the class, one of the participants assured that he could easily visualize the process of reflection and rotation and could remember the direction of rotation. He said that he used to feel difficulty in finding positive and negative direction of rotation but after attending research class, with the help of GeoGebra, he overcame that difficulty. During the interview, one of the participants told that she can easily solve the problems of transformation. According to her, visualization helped her in it as she can recall relation between the object and the image after visualizing their position. All of them agreed that GeoGebra helped them in visualizing process of reflection, rotation, translation, and dilation. This fact supported cognitive learning theory as cognitivists believe that learning occurs through listening, watching, studying and then processing and retaining the information. So, it can be said that GeoGebra assists in visualization of geometric transformation.

8. Conclusions

This study was focused on exploring possible use of GeoGebra in teaching geometric transformation. In this study, we used teaching experiment methodology to understand students' mathematical realities. Similarly, we put effort to understand students' meaning making process of geometric transformation through questioning (Dahal, 2017). During the study, we experienced students' active participation in experiment classes; they also seemed to be motivated to learn transformation through GeoGebra software. In this context, Luitel (2017) mentioned that to motivate the students, teachers need to shift their traditional pedagogies such as lecture method to modern pedagogies such as activity based, collaborative and cooperative, etc. We realized that using GeoGebra enhanced one (or few) of those modern pedagogies as well as helped them to motivate to learn mathematics. From the experiment of the learner, the study shows that the use of GeoGebra in teaching and learning geometric transformation can have a positive effect on students' learning of geometric transformation. Similarly, students' behavior in the classroom and their responses after the classes exposed that GeoGebra can help students to visualize the abstract concepts of mathematics. In addition, GeoGebra can help students and teachers to make mathematics classroom interesting through mutual teacher-student relationship (Dahal, 2013). Also, students, from any level of mathematical knowledge can be encouraged to study mathematics by using this application. One of the tremendous conclusions is learning and teaching of mathematics should not be focused on purely theoretical, but also a variety of learning approaches that involve the use of teaching aids proven to help stimulate students' interest in mathematics. This software should be introduced to mathematics teacher(s) so that students can explore the world of mathematics in a wider horizon and make the students able to think critically and creatively. Some components of ICTs are Mathematica, Maple, Geometers' Sketch Pad, Autograph, and others. Among them, GeoGebra is one of the mathematical software. So, if other applications of ICT tools too can be used in teaching mathematics, it can help students and teachers to make mathematics classroom interesting.

9. Implications

We hope our research work is not extremely implacable for all. In contrast, it can offer some insights to the readers, novice teachers, novice teacher trainers and educational researchers to integrating GeoGebra in mathematics teaching and learning. No doubt, mathematics teacher(s) would be highly benefited from our research process to strengthening their knowledge of integrating GeoGebra in order to focus and improve student learning in mathematics while teaching transformation. Our research is highly implacable to the mathematics teacher for classroom practices while teaching geometric transformation. Those mathematics teachers, who are unaware of GeoGebra, this research work will be eye-opening for them and even those started their teaching career recently. Our research indicates, teachers have a rich mastery of mathematical concepts and the interconnectedness between different representations and topics to be able to integrate GeoGebra that promote thinking. Similarly, our research will support policymakers and curriculum designers to make some provisions of the ICT integrated pedagogy for 21st century learners.

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