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To Theory One Class of Three-Dimensional Integral Equation with Super-Singular Kernels by Tube Domain

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Abstract

In this work, we investigate one class of three-dimensional integral equation by tube domains, are in power basis and lateral surface and way have super-singularity. In depend of the roots of the characteristic equations (2), (3) integral representation manifold solution is obtained in an explicit form. In the case, when parameters present in kernels, such that general solution integral equation contain arbitrary functions, invers formula is found. On basis obtained integral representation and its invers formula , in the case when general solution integral equation contain arbitrary functions, determined correct stand Dirichlet boundary valued problem and found its solution.

Keywords: Integral representation, super-singular kernels, invers formula, three-dimensional integral equations, Dirichlet type boundary value problem.

1. Integral Representation Manifold Solution

Let Ω denote the cylindrical domain $\Omega = \{(t,z): a < t < b, |z| < R\}$. The base of this cylinder will be denoted by $D = \{t = a, |z| < R\}$ and the lateral surface will be denoted by $S = \{a < t < b,\ |z| = R\}$, $z = x + iy$.

We consider the integral equation in the domain Ω of the form

$$\varphi(t,z) + \int_a^t \frac{K_1(t,\tau)}{(t-a)^\alpha}\varphi(\tau,z)d\tau + \frac{1}{\pi}\iint_D \frac{exp[i\theta\,]K_2(r,\rho)}{(R-\rho)^\beta(s-z)}\varphi(t,s)ds +$$

$$\frac{1}{\pi}\int_a^t \frac{dt}{(\tau-a)^\alpha}\iint_D \frac{exp[i\theta]K_3(t,\tau;r,\rho)}{(R-\rho)^\beta(s-z)}\varphi(\tau,s)ds = f(t,z)\ , \qquad (1)$$

where $\theta = args,\ s = \boldsymbol{\xi + i\eta},\ \boldsymbol{ds = d\xi d\eta},\ \boldsymbol{\rho^2 = \xi^2 + \eta^2, r^2 = x^2 + y^2},$

$K_1(t,\tau) = \sum_{j=1}^n A_j\left(\omega_a^\alpha(t) - \omega_a^\alpha(\tau)\right)^{j-1}$, $K_2(r,\rho) = \sum_{l=1}^m B_l\left(\omega_R^\beta(r) - \omega_R^\beta(\rho)\right)^{l-1}$, $K_3(t,\tau;r,\rho) = K_1(t,\tau)K_2(r,\rho)$, $A_j(1 \le j \le n)$, $B_l(1 \le l \le m)$ −, are given constants , $f(t,z)$ −are given functions, $\varphi(t,z)$ −uncnoun functions, $\omega_R^\beta(r) = \left[(\beta-1)(R-\rho)^{(\beta-1)}\right]^{-1}$, $\alpha = const > 1, \beta = const > 1$.

The solution of the integral equation (1) is sought in a class on functions $\varphi(t,z) \in C(\overline{\Omega})$, $\varphi(a,z) = 0$, $\varphi\left(t, Re^{i\theta}\right) = 0$, $\theta = argz$, and its asymptotic behavior for $t \to a$ and $r \to R$ is given by the formula $\varphi(t,z) = o\left[(t-a)^{\delta_1}\right]$ $\delta_1 > (n+1)(\alpha-1)$ at $t \to a$, $\varphi(t,z) = o\left[(R-r)^{\delta_2}\right]$ $\delta_2 > (m+1)(\beta-1)$ at $r \to R$.

In this work in depend of the roots of the characteristic equations

$$\lambda^n + \sum_{j=1}^{n} A_j (j-1)!\,\lambda^{n-j} = o \tag{2}$$

and

$$\mu^m + \sum_{j=1}^{m} B_j (j-1)!\,\mu^{m-j} = o\ , \tag{3}$$

integral representation manifold solution is obtained. The research in this directions has a history , see [1- 12].

Let in integral equation (1) $K_1(t,\tau), K_2(r,\rho), K_3(t,\tau;r,\rho)$ among themselves by formula $K_3(t,\tau;r,\rho) = K_1(t,\tau)K_2(r,\rho)$. Then introducing in consideration new function $\Psi(t,z)$ by formula

$$\Psi(t,z) = \varphi(t,z) + \int_a^t \frac{K_1(t,\tau)}{(t-a)^\alpha}\varphi(\tau,z)d\tau \equiv K_\alpha(\varphi), \tag{4}$$

we arrive to solution following I. N. Vecua type integral equation [14]

$$\Psi(t,z) + \frac{1}{\pi}\iint_D \frac{exp[i\theta\,]K_2(r,\rho)}{(R-\rho)^\beta (s-z)}\psi(t,s)ds = f(t,z). \tag{5}$$

So, in this case the problem found solution integral equation (1) reduce to problem found solution splitting system integral equation (4) and (5).

Investigating special cases, integral equation (3) and (4) was dedicate in [1]-[11].

In the case, when the roots of characteristic equation (2) is different , real and positive , and exist solution integral equation (4), corresponding to [3] , general solution integral equation (4) represent able in form

$$\varphi(t,z) = \sum_{j=1}^{n} C_j(z)exp\left[-\lambda_j\omega_a^\alpha(t)\right] + \ \psi(t,z) +$$

$$+\frac{1}{\Delta_0}\int_a^t \left\{\sum_{j=1}^{n}(-1)^{n+j}\Delta_{jn}exp\left[\lambda_j(\omega_a^\alpha(\tau)-\omega_a^\alpha(t))\right]\right\}\frac{\psi(\tau,z)}{(\tau-a)^\alpha}d\tau \equiv$$

$$\sum_{j=1}^{n} C_j(z)exp\left[-\lambda_j\omega_a^\alpha(t)\right] + \ (K_\alpha)^{-1}(\psi)\ , \tag{6}$$

where Δ_0 is a Vandermond determinant,corresponding parameters , $\lambda_j (1 \le j \le n)$, Δ_{jn} is minor of $(n-1)$ −order , which is obtained from Δ_0 by dividing n −th lines and j −th column ,$C_j(z)(1 \le j \le n)$ are arbitrary function the domain D.

Integral in right part formula (6) converges, if $\psi(t,z) \in C(\overline{\Omega})$, $\psi(a,z) = 0$ with asymptotic behavior

$$\psi(t,z) = o[exp[-\lambda\omega_a^\alpha(t)](t-a)^\gamma]\ ,\gamma > \alpha - 1 \text{ at } t \to a\ , \tag{7}$$

where $\lambda = max(\lambda_1, \lambda_2, \dots\dots\dots \lambda_n)$.

The function $\psi(t,z)$ have property (7), if $f(t,z) \in C(\overline{\Omega})$, $f(a,z) = 0$ with asymptotic behavior

$$f(t,z) = o[exp[-\lambda\omega_a^\alpha(t)](t-a)^\gamma]\ ,\gamma > \alpha - 1 \text{ at } t \to a\ , \tag{8}$$

where $\lambda = max(\lambda_1, \lambda_2, \dots\dots\dots \lambda_n)$.

Note that

$$\frac{\partial}{\partial\overline{\zeta}}\left[\frac{K_2(r,\rho)\psi(t,\rho)}{(R-\rho)^\beta}\right] = -\frac{\partial}{\partial\rho}\left[\int_\rho^R \frac{K_2(r,\rho_1)\psi(t,\rho_1)}{(R-\rho_1)^\beta}d\rho_1\right]e^{i\theta} = \frac{K_2(r,\rho)\psi(t,\rho)}{(R-\rho)^\beta}e^{i\theta}$$

From here

$$\frac{K_2(r,\rho)\psi(t,\rho)}{(R-\rho)^\beta}e^{i\theta}=\frac{\partial}{\partial\overline{\zeta}}\left[\frac{K_2(r,\rho)\psi(t,\rho)}{(R-\rho)^\beta}\right].$$

Then in integral equation (5) $\Psi(t,z) = \Psi(t,r)$, we have

$$\frac{1}{\pi}\iint_D \frac{exp[i\theta\,]K_2(r,\rho)}{(R-\rho)^\beta(s-z)}\psi(t,s)ds = \frac{1}{\pi}\iint_D \frac{exp[i\theta\,]K_2(r,\rho)}{(R-\rho)^\beta(s-z)}\psi(t,\rho)ds =$$

$$\frac{1}{\pi}\iint_D \frac{\partial}{\partial\overline{\zeta}}\left[\int_\rho^R \frac{K_2(r,\rho_1)}{(R-\rho_1)^\beta}\psi(t,\rho_1)d\rho_1\right]\frac{ds}{s-z} = -\int_r^R \frac{K_2(r,\rho)\psi(t,\rho)}{(R-\rho)^\beta}d\rho$$

Then integral equation (5) has the following form

$$\psi(t,r) - \int_r^R \frac{K_2(r,\rho)\psi(t,\rho)}{(R-\rho)^\beta}d\rho = f(t,r)\ , \tag{9}$$

If $f(t,z) = f(t,r)$.

Assume that, the solution of the equation (9) is the function $\psi(t,r) \in C^{(m)}(D)$ at variable r. Besides let the function $f(t,r) \in C^{(m)}(D)$ at variable r in equation (9). Then differentiating integral equation (9) m times and every time multiplying by $(R-r)^\beta$, we obtained the mth order degenerate differential equation

$$(D_r^\beta)^m\psi(t,r) + B_1(D_r^\beta)^{m-1}\psi(t,r) + B_2(D_r^\beta)^{m-2}\psi(t,r) + 2!\,B_3(D_r^\beta)^{m-3}\psi(t,r) + \dots\dots\dots\dots + (m-1)!\,B_m = 0, \tag{10}$$

where $D_r^\beta = (R-r)^\beta\frac{\partial}{\partial r}$.

The homogeneous differential equation (10) is corresponding to the characteristic equation(3).

The case of equation (9), when the parameters $B_j(1 \le j \le m)$ are such that, the roots of the characteristic equations (3) are real, different and positive, the solution homogenous differential equation (10) is given by formula

$$\psi(t,r) = \sum_{j=1}^m exp\left[\mu_j\omega_R^\beta(r)\right]\psi_j(t)\ , \tag{11}$$

where $\psi_j(t)(1 \le j \le m)$ −arbitrary function variable t , μ_j −the roots of characteristic equation (3).

Function of the type (11) also will be general solution of the homogeneous integral equation (9) . For obtained general solution non homogeneous integral equation (9), necessary is found particular solution non homogenous integral equation (9).

Assume that in the integral equation (9) , the function $f(t,r)$ on the right hand side can be represented in the following uniformly convergent functional series form

$$f(t,r) = \sum_{k=0}^\infty exp\left[-(k+\gamma)\omega_R^\beta(r)\right]f_k(t) \tag{12}$$

where $f_k(t)(k = 0,1,2,\dots\dots) -$ are known functions on S. The solution function $\psi(t,r)$ to the integral equation (9) will be sought in the class of function $\psi(t,r)$ representable in the generalized convergent functional series form

$$\Psi(t,r) = \sum_{k=0}^\infty exp\left[-(k+\gamma)\omega_R^\beta(r)\right]\Psi_k(t), \tag{13}$$

where $\Psi_k(t)(k = 0,1,2,\dots\dots)$ are unknown function on S.

Let us substitute these expressions $f(t,r)$ and $\Psi(t,r)$ using equalities (12) and (13) into integral equation (9), after calculating the corresponding integrals , we arrive to following equality

$$\sum_{k=q}^{\infty} exp\left[-(k+\gamma)\omega_R^{\beta}(r)\right]\Psi_k(t)\left[1+\sum_{l=1}^{m}\frac{B_l(l-1)!}{(k+\gamma)^l}\right]=\sum_{k=q}^{\infty} exp\left[-(k+\gamma)\omega_R^{\beta}(r)\right]f_k(t) .$$

From here we equate the coefficients in $exp\left[-(k+\gamma)\omega_R^{\beta}(r)\right]$ at $k=0,1,2,\dots.$, for determining unknown functions $\Psi_k(t)(k=0,1,2,\dots\dots)$, so that we obtained the following equality we

$$\left[1+\sum_{l=1}^{m}\frac{B_l(l-1)!}{(k+\gamma)^l}\right]\Psi_k(t)=f_k(t).$$

From here get

$$\Psi_k(t)=\left[\frac{(k+\gamma)^m}{(k+\gamma)^m+\sum_{l=1}^{m}B_l(l-1)!}\right]f_k(t) , (k=0,1,2,\dots\dots) \tag{14}$$

Substituting value $\Psi_k(t)$ in (13) we have

$$\Psi(t,r)=\sum_{k=0}^{\infty} exp\left[-(k+\gamma)\omega_R^{\beta}(r)\right]\left[\frac{(k+\gamma)^m}{(k+\gamma)^m+\sum_{l=1}^{m}B_l(l-1)!}\right]f_k(t)\equiv K_\beta(f) \tag{15}$$

This function will be particular solution integral equation (5), at $f(t,z)=f(t,r)$.

Directness verification it is possible, that in the case when. the roots of the algebraic equation (3) $\mu_j(1\le j\le m)$ real, different and positive , then function

$$\Psi_0(t,z)=\sum_{j=1}^{m} exp\left[\mu_j\omega_R^{\beta}(r)\right]\Phi_j(t,z),$$

where $\Phi_j(t,z)(1\le j\le m)-$ arbitrary functions two variable continuously by variable t and analytical by variable z will be general solution homogeneous integral equation (5).

Then function

$$\Psi(t,z)=\sum_{j=1}^{m} exp\left[\mu_j\omega_R^{\beta}(r)\right]\Phi_j(t,z)+K_\beta(f) \tag{16}$$

will be general solution integral equation (5) in the case , when function $f(t,z)=f(t,r)$ representable in form (16).

Substituting the value $\Psi(t,z)$ from formula (16) to formula (6) we have

$$\varphi(t,z)=\sum_{j=1}^{n}C_j(z)exp\left[-\lambda_j\omega_a^{\alpha}(t)\right]+\sum_{l=1}^{m}exp\left[\mu_l\omega_R^{\beta}(r)\right](K_\alpha)^{-1}(\Phi_l(t,z))+$$

$$(K_\alpha)^{-1}K_\beta(f) \tag{17}$$

Integrals in second same at the right part expression (17) converge, if $\Phi_l(t,z)\in C\left(\overline{\Omega}\right)$, $\Phi_l(a,z)=0(1\le l\le m)$ with asymptotic behavior

$$\Phi_l(t,z)=o[exp[-\lambda\omega_a^{\alpha}(t)](t-a)^{\gamma}] \text{ , at} t\to a \tag{18}$$

where $\lambda=max(\lambda_1,\lambda_2,\dots\dots.\lambda_n),\gamma>\alpha-1$.

Integrals in third expression the right part of (17) converge, if $f_k(t)\in C\left(\overline{S}\right)$, $f_k(a)=0$ with asymptotic behavior

$$f_k(t) = o[exp[-\lambda\omega_a^{\alpha}(t)](t-a)^{\gamma}] \text{, at } \quad t \to a \tag{19}$$

where $\lambda = max(\lambda_1, \lambda_2, \dots \dots . \lambda_n), \gamma > \alpha - 1$, for $k = 0,1,2, \dots \dots \dots$

So, we proof the following confirmation

Theorem 1.*Let in integral equation* ***(1) functions present in kernels connected among themselves by formula***$K_3(t,\tau;r,\rho) = K_1(t,\tau)K_2(r,\rho)$. *In* $K_1(t,\tau)$ *parameters* $A_j(1 \leq j \leq n)$ *such that, the roots of the characteristic equation (2)* $\lambda_j(1 \leq j \leq n)$ *real, different and positive and parameters* $B_j(1 \leq j \leq m)$ *such that the roots of the characteristic equation (3) real, different and negative. Function* $f(t,z) = f(t,r)$ *representable in the uniformly convergent functional series form (12), where* $f_k(t)$ *are given functions, where* $f_k(a) = 0$ *with asymptotical behavior (19). Then any solution of the integral equation (1) from class* $C(\overline{\Omega})$*representable in form*

$$\varphi(t,r) = \sum_{k=0}^{\infty} exp\left[-(k+\gamma)\omega_R^{\beta}(r)\right]\varphi_k(t), \tag{20}$$

we can be represented in the form (17)*, where* $C_j(z)(1 \leq j \leq n)$ *is an arbitrary continuous functions in the domain* $\overline{D}$ *, moreover* $C_j(Re^{i\theta}) = 0 \ (1 \leq j \leq n)$ *, with asymptotic behavior*

$$C_j(z) = o[(R-r)^{\gamma}](1 \leq j \leq n) \text{ , } \gamma > \beta - 1 at\, r \to R,$$

$\Phi_l(t,z)(1 \leq l \leq m)$ *−arbitrary functions domain* $\overline{\Omega}$ *analytically by variables* z *in* D *, continuous by* t *in* $\overline{S}$ *, moreover* $\Phi_l(a,z) = 0$ *with asymptotical behavior (18) and* $\Phi_l(t,Re^{i\theta}) = 0$ *with asymptotical behavior*

$$\Phi_l(t,z) = o[(R-r)^{\gamma}](1 \leq j \leq n) \text{ , } \gamma > \beta - 1 at\, r \to R.$$

Remark 1. Statement similar of the theorem 1, obtained in the following cases:

1. When the parameters $A_j(1 \leq j \leq n)$ such that , the roots of the characteristic equation (2) real, different and positive, besides of one , which have negative sign , and parameters $B_j(1 \leq j \leq m)$ such that, the roots of the characteristic equation (3) real, different and negative, besides of one, which have positive sign ;

2. When the parameters $A_j(1 \leq j \leq n)$ such that the roots of the of the characteristic equation (3) real, different and positive, besides of two, which have negative sign and so on.

Moreover in the case, when all the roots of characteristic equation (2) have positive signs besides one, which have negative signee and the roots of characteristic equation (3) have negative sign, besides one, which have positive sign, general solution integral equation (1) depend from $(n-1)$ arbitrary function $C_j(z)(1 \leq j \leq n-1)$ and from $(m-1)$ arbitrary functions $\Phi_l(t,z)(1 \leq l \leq m-1)$, continuously by variable t and analytically by variable z and so on.

In the case, when all the roots of the characteristic equation (2) is real, different and negative and all the roots of characteristic equation (3) is real, different and positive, we have the following confirmation

Theorem 2. *Let in integral equation* ***(1) functions present in kernels connected among themselves by formula***$K_3(t,\tau;r,\rho) = K_1(t,\tau)K_2(r,\rho)$. *In* $K_1(t,\tau)$ *parameters* $A_j(1 \leq j \leq n)$ *such that, the roots of the characteristic equation (2)* $\lambda_j(1 \leq j \leq n)$ *real, different and negative and in* $K_2(r,\rho)$ *parameters* $B_j(1 \leq j \leq m)$ *such that, the roots of the characteristic equation (3) real, different and positive. Function* $f(t,z) = f(t,r)$ *representable in the uniformly convergent functional series form (12), where* $f_k(t)$ *are given functions, where* $f_k(a) = 0$ *with asymptotical behavior*

$$f_k(t) = o[(t-a)^{\gamma}]\ ,\gamma > \alpha - 1 \text{at} \quad t \to a,$$

for any $k = 0,1,2, \dots \dots \dots$ *Then any solution of the integral equation (1) from class* $C(\overline{\Omega})$ *representable in form (20) is uniquely and given by formula*

$$\varphi(t,z) = (K_{\alpha})^{-1} K_{\beta}(f).$$

The case of equation (9) when the parameters $B_j (1 \le j \le m)$ are such that the roots of the characteristic equation (3) are real , different and negative we have the following confirmation

Lemma 1. *Let in integral equation (9) the parameter* $B_j (1 \le j \le m)$ *such that the roots of the characteristic equation (3) real, different and negative, let the function* $f(t,r) \in C(\overline{\Omega})$, $f(t,R) = 0$ *with asymptotic behavior*

$$f(t,r) = o\left[exp\left[-\mu\omega_R^{\beta}(r)\right](R-r)^{\delta}\right] ,\delta > \alpha - 1 \; at\, r \to R \tag{21}$$

where $\mu > max(|\mu_1|, |\mu_2|, \dots \dots \dots |\mu_m|)$.

Then integral equation (9) in the class of function $\Psi(t,z) \in C(\overline{\Omega})$, *vanishing in lines r=R is always solvability and its solution is given by the formula*

$$\Psi(t,z) = \sum_{j=1}^{n} \Phi_j(t,z) exp\left[-\mu_j\omega_R^{\beta}(r)\right] \ + \ f(t,r) +$$

$$+\frac{1}{\Delta_0^1}\int_r^R \left\{\sum_{j=1}^{m}(-1)^{m+j}\Delta_{jm}^1 exp\left[\mu_j(\omega_R^{\beta}(\rho) - \omega_R^{\beta}(r)\right]\right\}\frac{f(t,\rho)}{(R-r)^{\beta}}d\rho,$$

where Δ_0^1 *is Vandermond determinant for parameters* $\mu_j (1 \le j \le m)$, Δ_{jm}^1 *is minor of (m-1)-order, which obtained from* Δ_0^1 *by dividing m-th lines and j-th column ,* $\Phi_j(t,z)(1 \le j \le m)$ *are arbitrary function two variables continuously by variables* t *and analytically by variables* z.

Note that

$$\int_r^R \frac{exp\left[\mu_j\omega_R^{\beta}(\rho)\right]f(t,\rho)}{(R-\rho)^{\beta}}d\rho = -\frac{1}{\pi}\iint_D \frac{exp\left[i\theta+\mu_j\omega_R^{\beta}(\rho)\right]}{(R-\rho)^{\beta}(s-z)}f(t,\rho)ds,$$

we have

$$\Psi(t,z) = \sum_{j=1}^{m} \Phi_j(t,z) exp\left[-\mu_j\omega_R^{\beta}(r)\right] \ + \ f(t,r) -$$

$$\frac{1}{\Delta_0^1}\frac{1}{\pi}\iint_D \ \{\sum_{j=1}^{m}(-1)^{m+j}\Delta_{jm}^1 exp\left[\mu_j(\omega_R^{\beta}(\rho) - \omega_R^{\beta}(r)\right]\}\frac{exp[i\theta]f(t,\rho)}{(R-\rho)^{\beta}(s-z)}ds \equiv$$

$$\sum_{j=1}^{n} \Phi_j(t,z) exp\left[-\mu_j\omega_R^{\beta}(r)\right] \ + \ T_{\beta}(f) \quad , \tag{22}$$

At $\mu_j > 0 (1 \le \mathrm{j} \le \mathrm{m})$ the solution type (21) exist, if $\mathrm{f(t,r)} \in \mathrm{C}(\overline{\mathrm{D}})$, $\mathrm{f(t,R)} = 0$ with asumptotic behavior

$$f(t,r) = o\left[exp\left[-\mu\omega_R^{\beta}(r)\right](R-r)^{\delta_1}\right], \delta_1 > \alpha - 1 \; at\, r \to R, \tag{23}$$

where $\mu = max(\mu_1\mu_2, \dots \dots . \mu_m \ )$.

Substituting obtained value $\Psi(t,z)$ from equality (21) to formula (6), we have

$$\varphi(t,z) = \sum_{j=1}^{n} C_j(z) exp\left[-\lambda_j \omega_a^{\alpha}(t)\right] + \sum_{j=1}^{m} exp\left[\mu_j \omega_R^{\beta}(r)\right] (K_\alpha)^{-1}\left(\Phi_j(t,z)\right) + (K_\alpha)^{-1} T_\beta(f) \ . \tag{24}$$

Integrals in equality (22) is converges, if $f(\mathrm{a},\mathrm{r}) = 0$ with asymptotic behavior (8).

So, we proof the following confirmation

Theorem 3 . *Let in integral equation* ***(1) functions present in kernels connected among themselves by formula*** $K_3(t,\tau;r,\rho) = K_1(t,\tau)K_2(r,\rho)$. *In* $K_1(t,\tau)$ *parameters* $A_j (1 \le j \le n)$ *such that, the roots of the characteristic equation (2)* $\lambda_j (1 \le j \le n)$ *real, different and positive and parameters* $B_j (1 \le j \le m)$ *such that , the roots of the characteristic equation (3) real , different and negative, let the function* $f(t,r) \in C(\overline{\Omega})$, $f(t,R) = 0$ *with asymptotic behavior (21),* $f(a,r) = 0$ *with asymptotic behavior (8). Then any solution of the integral equation (1) from class* $C(\overline{\Omega})$*representable in form (23), where* $C_j(z)(1 \le j \le n)$ *is an arbitrary continuous functions in the domain* $\overline{D}$, *moreover* $C_j(Re^{i\theta}) = 0 \ (1 \le j \le n)$, *with asymptotic behavior*

$$C_j(z) = o[(R-r)^{\gamma}](1 \le j \le n) \ , \gamma > \beta - 1 \ \ at \ \ r \to R, \tag{A}$$

$\Phi_j(t,z)(1 \le j \le m)$ *are arbitrary function two variables continuously by variables* t *and analytically by variables z. Moreover* $\Phi_j(a,z) = 0$ *with asymptotic behavior*

$$\Phi_l(t,z) = o[(t-a)^{\gamma}] \ , \gamma > \alpha - 1 \qquad \text{at} \qquad t \to a \tag{B}$$

In depend from signs the roots of t*he characteristic equations* (2) and (3) we obtained different integral representation manifold solution, containing various number arbitrary function $C_j(z)$ domain $\overline{D}$ and different various number arbitrary function two variables $\Phi_j(t,z)$, continuously by variables t and analytically by variables z.

In particular, if the parameter $A_j (1 \le j \le n)$ such that, the roots of the characteristic equation (2) $\lambda_j (1 \le j \le n)$ real, different and negative and parameters $B_j (1 \le j \le m)$ such that the roots of the characteristic equation (3) real, different and positive have the following confirmation

Theorem 4. *Let in integral equation* ***(1) functions present in kernels connected among themselves by formula*** $K_3(t,\tau;r,\rho) = K_1(t,\tau)K_2(r,\rho)$. *In* $K_1(t,\tau)$ *, parameters* $A_j (1 \le j \le n)$ *such that, the roots of the characteristic equation (2)* $\lambda_j (1 \le j \le n)$ *real, different and negative and in* $K_2(r,\rho)$ *, parameters* $B_j (1 \le j \le m)$ *such that the roots of the characteristic equation (3) real, different and positive, function*$f(t,z) = f(t,r) \in C(\overline{\Omega})$, $f(a,z) = 0$ *with asymptotic behavior (8),* $f(t,R) = 0$ *with asymptotic behavior*

$$f(t,r) = o\left[(R-r)^{\delta_1}\right] , \delta_1 > \beta - 1 \text{ at } r \to R,$$

Then, integral equation (1) in class $C(\overline{\Omega})$ *have unique solution is given by formula*

$$\varphi(t,z) = (K_\alpha)^{-1} T_\beta(f).$$

2. Invers Formula

New let us assume that in integral representation (22) function $\Psi(t,z) = K_\alpha(\varphi)$ is well-known function, than behave how [12], we found

$$\Phi_k(t,z) = \frac{exp\left[\mu_k \omega_R^\beta(r)\right]}{\Delta_0^1} \sum_{j=1}^{m} (-1)^{k+j} \Delta_{jk}^1 D_{\bar{z}}^{j-1}\left[K_\alpha(\varphi) - T_\beta(f)\right], (1 \le k \le m), \qquad (25)$$

where $\boldsymbol{D_{\bar{z}} = 2exp[-i\theta]\frac{\partial}{\partial \bar{z}}}$, Δ_0^1 is Vandermond determinant for parameters $\mu_j (1 \le j \le m)$, Δ_{jm}^1 is minor of (m-1)-order , which obtained from Δ_0^1 by dividing m-th lines and j-th column.

Theorem 5. ***Let fulfillment any condition of the theorem 3 and function*** $f(t,z)$ *,unknown function* $\varphi(t,z)$ *differentiability* $(m-1)$ *time. Then in integral representation (22) , the functions* $\Phi_k(t,z)(1 \le k \le m)$ *by valued* $\varphi(t,z), f(t,z)$ *and its derivatives* $(m-1)$ *–th order is found by formula (25).*

Now in integral representation (24) functions $f(t,z)$, $\Phi_k(t,z)(1 \le k \le m)$, $\varphi(t,z)$ well-known, we found valued $C_j(z)(1 \le j \le n)$.

To this end of the formula (24) we represented in following form

$$\sum_{j=1}^{n} C_j(z) exp\left[-\lambda_j \omega_a^\alpha(t)\right] = \varphi(t,z) - \sum_{j=1}^{m} exp\left[\mu_j \omega_R^\beta(r)\right](K_\alpha)^{-1}\left(\Phi_j(t,z)\right) - (K_\alpha)^{-1} T_\beta(f) \equiv$$

$$\mathbf{T}\left[\boldsymbol{f(t,z)}, \varphi(t,z), \Phi_1(t,z), \ldots\ldots\ldots, \Phi_m(t,z)\ \right] \qquad (26)$$

Assume that, in (26) known function $f(t,z)$, unknown function $\varphi(t,z)$ and function $\Phi_j(t,z)(1 \le j \le m)$ by variables t, differentiable $(n-1)$ –time, and every time obtained expression to $(t-a)^\alpha$ for finding functions $C_j(z)(1 \le j \le n)$, we obtained following algebraic system

$$\sum_{j=1}^{n} C_j(z) exp\left[-\lambda_j \omega_a^\alpha(t)\right] = \mathbf{T}\left[\boldsymbol{f(t,z)}, \varphi(t,z), \Phi_1(t,z), \ldots\ldots\ldots, \Phi_m(t,z)\ \right]$$

$$\sum_{j=1}^{n} C_j(z) exp\left[-\lambda_j \omega_a^\alpha(t)\right]\lambda_j = D_\alpha^t\left[\ \mathbf{T}\left[\boldsymbol{f(t,z)}, \varphi(t,z), \Phi_1(t,z), \ldots\ldots\ldots, \Phi_m(t,z)\right]\right]$$

$$\sum_{j=1}^{n} C_j(z) exp\left[-\lambda_j \omega_a^\alpha(t)\right]{\lambda_j}^2 = (D_\alpha^t)^2\left[\ \mathbf{T}\left[\boldsymbol{f(t,z)}, \varphi(t,z), \Phi_1(t,z), \ldots\ldots\ldots, \Phi_m(t,z)\right]\right]$$

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$$\sum_{j=1}^{n} C_j(z) exp\left[-\lambda_j \omega_a^\alpha(t)\right]{\lambda_j}^{n-1} = (D_\alpha^t)^{n-1}\left[\ \mathbf{T}\left[\boldsymbol{f(t,z)}, \varphi(t,z), \Phi_1(t,z), \ldots\ldots\ldots, \Phi_m(t,z)\right]\right],$$

where $D_\alpha^t = (t-a)^\alpha \frac{d}{dx}$.

Solving this system, we found

$$C_j(z)=\frac{exp\left[\lambda_j\omega_a^{\alpha}(t)\right]}{\Delta_0}$$

$$\sum_{k=1}^{n}(-1)^{j+k}\Delta_k^{j}((D_\alpha^t))^{k-1}\left[\,\mathbf{T}\left[\boldsymbol{f(t,z)},\varphi(t,z),\Phi_1(t,z),\dots\dots\dots,\Phi_m(t,z)\right]\right], \qquad (27)$$

$1\leq j\leq n$, where

$$\Delta_0=\begin{vmatrix} 1,1,1,\dots\dots..1 \\ \lambda_{1,}\lambda_{2,}\lambda_{3,}\dots\lambda_n \\ \lambda_1^2,\lambda_2^2,\lambda_3^2,..\lambda_n^2 \\ \dots\dots\dots\dots\dots.. \\ \lambda_1^{n-1},\lambda_2^{n-1},..\lambda_n^{n-1}\end{vmatrix},\ \Delta_k^j=\begin{vmatrix} 1,1,1,\dots,1\dots\dots\dots.1 \\ \lambda_{1,}\lambda_{2,}\lambda_{3,}\dots.\lambda_{j-1,}\lambda_{j+1,}\dots\lambda_n \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots. \\ \lambda_1^{k-1},\lambda_2^{2},..\lambda_{j-1}^{k-1},\lambda_{j+1}^{k-1},..\lambda_n^{k-1} \\ \dots\dots\dots\dots\dots.. \\ \lambda_1^{n-1},\lambda_2^{n-1},.\lambda_{j-1,,}^{n-1}\lambda_{j+1,,}^{n-1}\dots.\ \ \lambda_n^{n-1}\end{vmatrix},\ 1\leq k,j\leq n.$$

Theorem 6. ***Let fulfillment any condition of the theorem 3 and function*** $f(t,z)$, *unknown function $\varphi(t,z)$ differentiability $(n-1)$ time by variables t. Then in integral representation (24), the functions $C_k(z)(1\leq k\leq n)$ by valued $\varphi(t,z),f(t,z)$, $\Phi_j(t,z)(1\leq k\leq n)$ and its derivatives $(m-1)-$th order is found by formula (25).*

Integral representation obtained in theorem 3 and its invers formula obtained in theorems 5 and 6 give possibility for integral equation (1) explain correct stand different boundary value problems and its investigation.

Problem D. Is required found solution integral equation (1) from class $C(\overline{\Omega})$ **at fu**lfillment any condition of the theorems 3,5,6 by boundary condition

$$\boldsymbol{Re}\left[\boldsymbol{exp}\left[\boldsymbol{\mu_k\omega_R^{\beta}(r)D_{\bar z}^{j-1}}[\boldsymbol{K_\alpha(\varphi)}]\right]\right]_{\boldsymbol{r=R}}=\boldsymbol{E_k^j(t,\theta)}\ \boldsymbol{(1\leq j,k\leq m),\ 0\leq\theta\leq 2\pi} \qquad (28)$$

In boundary domain D, condition

$$\left[\boldsymbol{exp}\left[\boldsymbol{\mu_k\omega_R^{\beta}(r)D_{\bar z}^{j-1}}[\boldsymbol{K_\alpha(\varphi)}]\right]\right]_{\boldsymbol{z=0}}=\boldsymbol{F_k^j(t)}\ \boldsymbol{(1\leq j,k\leq m)} \qquad (29)$$

in principal axis cylinder and conditions

$$\left[\boldsymbol{exp}\left[\boldsymbol{\lambda_k\omega_a^{\alpha}(t)(D_\alpha^t)^{j-1}}[(\boldsymbol{\varphi})]\right]\right]_{\boldsymbol{t=a}}=\boldsymbol{W_k^j(z)}\ \boldsymbol{(1\leq j,k\leq n)} \qquad (30)$$

in lateral surface cylinder , where $E_k^j(t,\theta)$ $(1\leq j,k\leq m)$- are given functions the boundary lower ground of the cylinder, $F_k^j(t)$ $(1\leq j,k\leq m)$ $-$are given functions in principal axis cylinder and $W_k^j(z)$ $(1\leq j,k\leq n)$ $-$ are given functions lower ground of the cylinder.

Solution problem D. Let fulfillment any condition of the theorem 5 and 6. Then from formula (25) we found

$$\left[\,Re\Phi_k(t,z)\right]_{\boldsymbol{r=R}}=\frac{\boldsymbol{1}}{\Delta_0^1}\sum_{\boldsymbol{j=1}}^{\boldsymbol{m}}(\boldsymbol{-1})^{\boldsymbol{k+j}}\Delta_{\boldsymbol{jk}}^{\boldsymbol{1}}\boldsymbol{K_\alpha}\left[\boldsymbol{Re}\left[\boldsymbol{exp}\left[\boldsymbol{\mu_k\omega_R^{\beta}(r)}\right]\boldsymbol{D_{\bar z}^{j-1}(\varphi)}\right]_{\boldsymbol{r=R}}\right]=$$

$$\frac{\mathbf{1}}{\Delta_0^1}\sum_{j=1}^{m}(-\mathbf{1})^{k+j}\Delta_{jk}^1 \boldsymbol{K_\alpha}\left[\boldsymbol{E_k^j(t,\theta)}\right] \equiv \boldsymbol{W_k(t,\theta)} \quad (\mathbf{1}\leq \boldsymbol{k} \leq \boldsymbol{m}). \tag{30}$$

So, for determined unknown functions $\Phi_k(t,z)(\mathbf{1} \leq \boldsymbol{k} \leq \boldsymbol{m})$, we have $\boldsymbol{m}$ Shivers problems the complex analysis theory [13]. According to [13] solution problems (30) is given by formula

$$\Phi_k(t,z) = \frac{1}{2\pi}\int_\gamma \frac{\tau+z}{\tau(\tau-z)}W_k(t,\theta)d\tau + i\Phi_k(t) \quad (\mathbf{1}\leq \boldsymbol{k} \leq \boldsymbol{m}), \tag{31}$$

where $\Phi_k(t)$ −arbitrary functions point $\boldsymbol{t}$**.**

From formula (25) we have

$$\Phi_k(t,0) = \frac{1}{\Delta_0^1}\sum_{j=1}^{m}(-1)^{k+j}\Delta_{jk}^1 \left[K_\alpha\left[\left\{exp\left[\mu_k\omega_R^\beta(r)\right]D_{\bar{z}}^{j-1}(\varphi)\right\}_{z=0}\right]\right] =$$

$$\frac{1}{\Delta_0^1}\sum_{j=1}^{m}(-1)^{k+j}\Delta_{jk}^1 \left[K_\alpha\left[F_k^j(t)\right]\right] \equiv E_k(t), (1 \leq k \leq m) \tag{32}$$

From formula (31) we find

$$\Phi_k(t,0) = \frac{1}{2\pi}\int_\gamma \frac{1}{\tau}W_k(t,\theta)d\tau + i\Phi_k(t) \quad (\mathbf{1}\leq \boldsymbol{k} \leq \boldsymbol{m}), \tag{33}$$

Compared formula (32) and (33) we have

$$\frac{1}{2\pi}\int_\gamma \frac{1}{\tau}W_k(t,\theta)d\tau + i\Phi_k(t) = E_k(t) \quad (\mathbf{1}\leq \boldsymbol{k} \leq \boldsymbol{m}).$$

From here, we found

$$i\Phi_k(t) = E_k(t) - \frac{1}{2\pi}\int_\gamma \frac{1}{\tau}W_k(t,\theta)d\tau.$$

Substituting obtained valued $\Phi_k(t)$ in formula (31), we found explicit for**m** $\Phi_k(t,z)$ In following form

$$\Phi_k(t,z) = \frac{1}{2\pi}\int_\gamma \frac{\tau+z}{\tau(\tau-z)}W_k(t,\theta)d\tau + E_k(t) - \frac{1}{2\pi}\int_\gamma \frac{1}{\tau}W_k(t,\theta)d\tau \; (\mathbf{1}\leq \boldsymbol{k} \leq \boldsymbol{m}). \tag{34}$$

For discover $C_j(z)$**(1**$\leq \boldsymbol{j} \leq \boldsymbol{n}$**), we use formula (27) and conditions (30). From this formula we have**

$$C_j(z) = \frac{1}{\Delta_0}\sum_{k=1}^{n}(-1)^{j+k}\Delta_k^j \left[exp\left[\lambda_j\omega_a^\alpha(t)\right](D_a^t)^{k-1}(\varphi)\right]_{t=a} =$$

$$\frac{1}{\Delta_0}\sum_{k=1}^{n} (-1)^{j+k}\Delta_k^j \boldsymbol{W_k^j(z)} \equiv \boldsymbol{\Psi_j(z)}(\mathbf{1} \leq \boldsymbol{j} \leq \boldsymbol{n}) \tag{35}$$

So, if solution of the problem **D exist, then it may be represented in form (24), (34), (35). In this connection property from equality (A) and (B) follows that functions $\boldsymbol{\Psi_j(z)}(\mathbf{1} \leq \boldsymbol{j} \leq \boldsymbol{n})$ vanishing in boundary domain D with following asymptotic behavior**

$$\boldsymbol{\Psi_j(z)} = o[(R-r)^\gamma](1 \leq j \leq n)\,, \gamma > \beta - 1 \;\; at\, r \to R, \tag{36}$$

and functions $E_k(t)$, $W_k(t,\theta)$ vanishing in lateral surface of the cylinder with following asymptotical behavior

$$E_k(t) == o[(t-a)^{\gamma_1}] , \gamma_1 > \alpha - 1 \qquad \text{at} \qquad t \to a, \tag{37}$$

So, we proof the following confirmation

Theorem 7. ***Let any condition of the theorems 3,5,6 fulfill. Besides in problem*** **D** ***functions*** $\boldsymbol{E_k^j(t,\theta), F_k^j(t)}$ **(1≤** $\boldsymbol{j,k \le m}$**,** $\boldsymbol{W_k^j(z)}$ **(1≤** $\boldsymbol{j,k \le n}$**),** ***such that*** $\boldsymbol{\Psi_j(Re^{i\theta}) = 0}(1 \le j \le n)$*with asymptotical behavior (36) and* $E_k(a) = 0$ ***(1≤*** $\boldsymbol{k \le m}$) ***with asymptotical behavior (37).Then problem*** **D** ***have unique solution, which is given by formulas (24), (34), (35).***

3. One Special Case

Let $\boldsymbol{m = n = 2}$ **and** $K_3(t,\tau;r,\rho) = K_1(t,\tau)K_2(r,\rho)$. Then, corresponding characteristic equations have the following form

$$\lambda^2 + A_1\lambda + A_2 = 0 , \tag{I}$$

$$\mu^2 + B_1\mu + B_2 = 0. \tag{II}$$

Lower, we reduce to the results, in the case when the roots of the characteristic equation (II) real, different and positive, but the roots of the characteristic equation (I) may be real, different and negative; real, equal and negative; complex-conjugate with negative real part ($A_1 < 0, 4A_2 - A_1^2$ >0).

Theorem 8. *Let* in int*egral equation (1) the roots of the characteristic equation (II) real, different and positive, that is* $\mu_1 > 0$, $\mu_2 > 0(B_1 > 0,\ B_2 > 0)$ *and the roots of the characteristic equation (I) real, different and negative , that is* $\lambda_1 < 0, \lambda_2 < 0(p < 0,\ q > 0)$. *Function*$f(t,z) \in C(\overline{\Omega})$, $f(a,z) = 0$ *with asymptotic behavior*

$$f(t,z) = o\left[exp[\lambda_1\omega_a^\alpha(t)](t-a)^{\delta_1}\right], \delta_1 > \alpha - 1 \quad at\, t \to a$$

and, $f(t,Re^{i\theta}) = 0$ *with asymptotical behavior*

$$f(t,z) = o\left[exp\left[\mu_{11}\omega_b^\beta(r)\right](R-r)^{\delta_2}\right], \delta_2 > \beta - 1 \quad at\, r \to R. \tag{III}$$

Then integral equation (1) always solvability, and its solution is given by formula

$$\varphi(t,z) = exp[\lambda_1\omega_a^\alpha(t)]C_1(z) + exp[\lambda_2\omega_a^\alpha(t)]C_2(z) + exp\left[-\mu_1\omega_b^\beta(r)\right].$$

$$(T_\alpha^1)^{-1}[C_1(t,z)] + exp\left[-\mu_2\omega_b^\beta(r)\right].(T_\alpha^1)^{-1}[C_2(t,z)] + \left(\Pi_\beta\right)^{-1}(T_\alpha^1)^{-1}[f],$$

where

$$\left(\Pi_\beta\right)^{-1}(f) = f(t,z)$$
$$+\frac{1}{\pi}\iint\limits_{D} \frac{exp[i\theta]}{(R-\rho)^\beta(s-z)}\left[\mu_2^2 exp\left[\mu_2\left(\omega_b^\beta(r)-\omega_b^\beta(\rho)\right)-\mu_1^2 exp\mu_1\left(\omega_b^\beta(r)-\omega_b^\beta(\rho)\right)\right]\right],$$

$$(T_\alpha^1)^{-1}[f] = f(t,z) - \frac{1}{\sqrt{p^2-4q}}\int_a^t \left[\lambda_2^2 exp[\lambda_2(\omega_a^\alpha(t)-\omega_a^\alpha(\tau))] - \lambda_1^2 exp[\lambda_1(\omega_a^\alpha(t)-\omega_a^\alpha(\tau))]\right]\frac{f(\tau,z)}{(\tau-a)^\alpha}d\tau,$$

$C_j(t,z)(j=1,2)$ – arbitrary functions domain $\overline{\Omega}$, continuously by variables t and analytically by variables z, moreover $C_j(a,z)=0(j=1,2)$ with asymptotically behavior

$$C_j(t,z) = o\left[exp[\lambda_1\omega_a^\alpha(t)(t-a)^{\delta_3}]\right](j=1,2)\ \ \delta_3>\alpha-1 \quad at \quad t\to a,$$

$C_j(z)(j=1,2)$ –arbitrary continuously functions domain $\overline{D}$. Moreover $C_j(Rexp[i\theta])=0(j=1,2)$ with asymptotically behavior

$$C_j(z) = o[(R-r)^{\delta_4}](j=1,2),\ \delta_4>\beta-1 \quad at \quad r\to R.$$

Theorem 9. *Let in integral equation (1) the roots of the characteristic equation (II) real, different and positive, that is $\mu_1>0, \mu_2>0(B_1>0,\ B_2>0)$ and the roots of the characteristic equation (I) real, equal and negative, that is $\lambda_1=,\ \lambda_2=\lambda=\frac{p}{2}<0(p<0,\ q>0)$. Function $f(t,z)\in C(\overline{\Omega})$, $f(a,z)=0$ with asymptotic behavior*

$$f(t,z) = o\left[exp\left[\frac{p}{2}\omega_a^\alpha(t)\right](t-a)^{\delta_4}\right],\ \delta_4>2(\alpha-1)\ \ at\ \ t\to a$$

and $f(t,Re^{i\theta})=0$ with asymptotic behavior (III). Then integral equation (1) always solvability, and its solution is given by formula

$$\varphi(t,z) = exp\left[\frac{p}{2}\omega_a^\alpha(t)\right]\{C_5(z)+\omega_a^\alpha(t)\,C_6(z)\} + \ exp\left[-\mu_1\omega_b^\beta(r)\right].$$

$$(T_\alpha^2)^{-1}[C_1(t,z)] + \ exp\left[-\mu_2\omega_b^\beta(r)\right].(T_\alpha^2)^{-1}[C_2(t,z)] + \left(\Pi_\beta\right)^{-1}(T_\alpha^2)^{-1}[f],$$

where $C_j(t,z)(j=1,2)$ – arbitrary functions domain $\overline{\Omega}$, continuously by variables t and analytically by variables z, moreover $C_j(a,z)=0(j=1,2)$ with asymptotically behavior

$$C_j(t,z) = o\left[exp\left[\frac{p}{2}\omega_a^\alpha(t)(t-a)^{\delta_5}\right]\right](j=1,2)\ \ \delta_5>2(\alpha-1\_ \quad at \quad t\to a,$$

$C_j(z)(j=1,2)$ –arbitrary continuously functions domain $\overline{D}$. Moreover $C_j(Rexp[i\theta])=0(j=1,2)$ with asymptotically behavior

$$C_j(z) = o[(R-r)^{\delta_6}](j=1,2),\ \delta_6>\beta-1 \quad at \quad r\to R,$$

$$(T_\alpha^2)^{-1}[f(t,z)] = f(t,z) - \int_a^t \left[exp\left[\frac{p}{2}(\omega_a^\alpha(t)-\omega_a^\alpha(\tau))][p+q(\omega_a^\alpha(t)-\omega_a^\alpha(\tau))\right]\right]\frac{f(\tau,z)}{(\tau-a)^\alpha}d\tau,$$

Theorem 10. *Let* in int*egral equation (1) the roots of the characteristic equation (II) is complex and conjugate, that is* $\lambda_1 = A + iB$, $\lambda_2 = A - iB$ *(*$A = \frac{p}{2}$, $B = \frac{\sqrt{4q-p^2}}{2}$,*p<0,* $4q - p^2 > 0$ *) and the roots of characteristic equation (I) real, different and positive , that is* $\mu_1 > 0, \mu_1 > 0 (B_1 > 0,\ B_2 > 0)$. *Function* $f(t,z) \in C(\overline{\Omega})$, $f(a,z) = 0$ *with asymptotic behavior*

$$f(t,z) = o\left[exp\left[\frac{p}{2}\omega_a^\alpha(t)\right](t-a)^{\delta_7}\right], \delta_7 > (\alpha - 1) \quad at \quad t \to a$$

and $f(t, Re^{i\theta}) = 0$ *with asymptotic behavior (III). Then integral equation (1) always solvability, and its solution is given by formula*

$$\varphi(t,z) = exp\left[\frac{p}{2}\omega_a^\alpha(t)\right]\{cos\left[\frac{\sqrt{4q-p^2}}{2}\omega_a^\alpha(t)\right]C_7(z) + \sin\left[\frac{\sqrt{4q-p^2}}{2}\omega_a^\alpha(t)\right]C_8(z)\} + \ exp\left[-\mu_1\omega_b^\beta(r)\right].$$

$$(T_\alpha^3)^{-1}[C_1(t,z)] + \ exp\left[-\mu_2\omega_b^\beta(r)\right].(T_\alpha^3)^{-1}[C_2(t,z)] + \left(\Pi_\beta\right)^{-1}(T_\alpha^3)^{-1}[f],$$

where $C_j(t,z)(j = 1,2) -$ *arbitrary functions domain* $\overline{\Omega}$, *continuously by variables* t *and analytically by variables* z, *moreover* $C_j(a,z) = 0 (j = 1,2)$ *with asymptotically behavior*

$$C_j(t,z) = o\left[exp\left[\frac{p}{2}\omega_a^\alpha(t)(t-a)^{\delta_5}\right]\right](j = 1,2) \ \ \delta_5 > (\alpha - 1) \quad at \qquad t \to a\ ,$$

$C_j(z)(j = 1,2)$ *−arbitrary continuously functions domain* $\overline{D}$. *Moreover* $C_j(Rexp[i\theta]) = 0 (j = 1,2)$ *with asymptotically behavior*

$$C_j(z) = o\left[(R-r)^{\delta_6}\right](j = 1,2)\ , \delta_6 > \beta - 1 \quad at \quad r \to R.$$

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