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Constructional tasks of conics with GeoGebra

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Abstract

This paper is devoted to the teaching of conics at the grammar school and at the college in Czech Republic. There are presented GeoGebra worksheets which can to help teach constructional tasks about conic sections both at grammar school and solving more challenging tasks at college.

1 Introduction

Conics we meet in many fields, for example architecture, astronomy, optics and so on. We use properties of conics to construct buildings or to made technical components. We can find conics in the universe – trajectories of planets and the other solids. To use conics in practise we must know properties and construction of conics. Some architecs see conics in spiritual dimense. One church in Ostrava has the elliptical platform, [4]. The architect perceives the ellipse as set of points, where man can meet God because one focus of this ellipse is God and the second focus is a man. On the contrary if anybody omit some properties of conics an accident can become. It is demonstrated in the case of unfinished skyscraper Walkie Talkie in London in 2013, [3]. The parabolic form of this building has reflected rays to near streets and it has smelt down parked cars.

Students of secondary schools studies basic properties of conics in mathematics and more details they study in descriptive geometry. Unfortunately, we often can see that geometry (and so conics) are not favorite part of mathematics for some teachers. Students then have a formal knowledges and they can not use these knowledges in practise. It is also the common phenomenon that students in the age level until 15 years have misconception about conics because their teachers incorrectly presented conics, and older students can not then understand the other (deeper) properties and constructions of conics. Therefore is usual that many students like geometry during their studies at primary school but when they leaving grammar school they thinking about geometry as unnecessary part of mathematics and they dislike geometry.

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Students of mathematics teaching (or descriptive geometry teaching) learn to teach geometry in a research and constructivist way. They use GeoGebra to mak dynamic worksheets. These worksheets can be prepared for any geometric topic and students can use them in their pedagogical practice.

2 Construction tasks about conics

At secondary schools in descriptive geometry lessons, students first define conics as loci of points with given properties and the they construct conics using definition and using derived focus properties. Then they construct conics using some projection methods as sections of a conical surface of revolution (a cone of revolution). This conical surface has a director circle (a base circle) in the horizontal plane. Students are introduced in basics of collineation. They usually do not learn improper elements of collineation, they only learn a construction of images of points and they construct section as set of images of points of director circle. Students determine a sort of conic section according to position of vertex plain which is parallel to cut plane, and conical surface.

Students of the program "Training of teachers of descriptive geometry for secondary schools" (and lesser extent students of the program "Training of teachers of mathematics for secondary schools") learn also projective properties of conics. They construct conics as collinear image of some circle (in the plane) and, last but not least, they also construct conics as section of an arbitrary conical surface and also as an image of some circle in a certain collineation in space.

Students prepare worksheets using acquired knowledge in descriptive geometry seminars. These worksheets they use in their teaching practice at grammar school in extended seminars. They start with easy worksheets which pupils can solve without knowledge of projective geometry. Finally students make harder worksheets that are focussed on university curriculum of projective geometry. These hard worksheets are also used in lessons and homework preparation of projective geometry in lower grades at college.

Students deepen and consolidate their knowledge during preparation of worksheets. They beside find out that they must thoroughly think out each step of a construction and each displayed construstruction so that their pupils understand it.

Students choose various form of progressively display of construction. There are using animation in some tasks. The construction is displayed step by step as you can see at image 1. There are using also checkboxes in another construction or if we want display only one of more solutions, viz image 1 or 3. Last but not least, there are using also sliders in worksheets so that they display process of construction task, viz image 4.

Each of tasks are entereded such that pupils (students) can change positions of points, lines etc. They can observe change of the solution in depence on the

position of given objects. But they can not change position of given objects completely arbitrary. First, because they would not to know solution of the task with their existing knowledge, partly, because the solution of such changed task can be a different conic. For example, the teacher give a task: "Find an ellipse ..." and a solution of changed task would be a hyperbola.

Pupils (students) can complement objects to find the solution of the task. Changes of positions of given objects (changes allowed by the teacher in the worksheet) leads pupils (students) to finding number of solutions a they can diskus solutions.

Next example shows one of easy task for which students to make worksheets and this task they use at grammar schools.

Task 1 Construct a conic, if you know its focus F and three point A, B, C different to each other.

Solution using colineation (GeoGebra worksheet: https://ggbm.at/zjfjzq8a) Students first solve tasks using a center collineation. They find center collineation and a circle such that its image is the wanted conic. There is an example of GeoGebra worksheet shown at the image number 1. There is an animation of construction step by step selected in this worksheet. The circle k' (which is an image of wanted conic k) at first will be displayed to students. Then there are displayed images of given points and also we can see the task has more solutions. Next the axis of collineation is displayed and finally the conic k as the image of the circle is displayed. There are listed both solutions, students can choose one of them. For this there are displayed two check boxes. Finally students research types of conics depending on positions of given points. They also to conduct a finally discussion about number of solutions using worksheet. At the same time, during construction, there is displayed a description of a construction procedure.

Solution using spatial properties:(GeoGebra worksheet: https://ggbm.at/aydw4wvn) Then, students solve the same task using spatial properties. The teacher shows them (for example using 3D model in GeoGebra, viz image 2) that the wanted conic can be an orthogonal projection of some cut of some cone of revolution. Such cone of revolution is displayed in dimensioned projection, its axis of revolution is perpendicular to a projection plane, its director circle lies in the projection plane, the orthogonal projection of vertex of this cone is the focus of the wanted conic and the distance of vertex and its orthogonal projection is the same as a radius of the direct circle. Given points are orthogonal projection of points of a cut plane and at the same time they are points of this cone of revolution. The worksheet in on the image 3. Students first find dimensions of given points using tipping of projective plane planes of elements of the cone of revolution. Then they can construct a trace line of the cut plane. Then they can construct a trace line of the cut plane and its steepest line. Now, they can construct vertices of the projection of the cut (using tipping of a projection plane of the steepest line) and, finally, orthogonal projection of the cut which is the wanted conic.

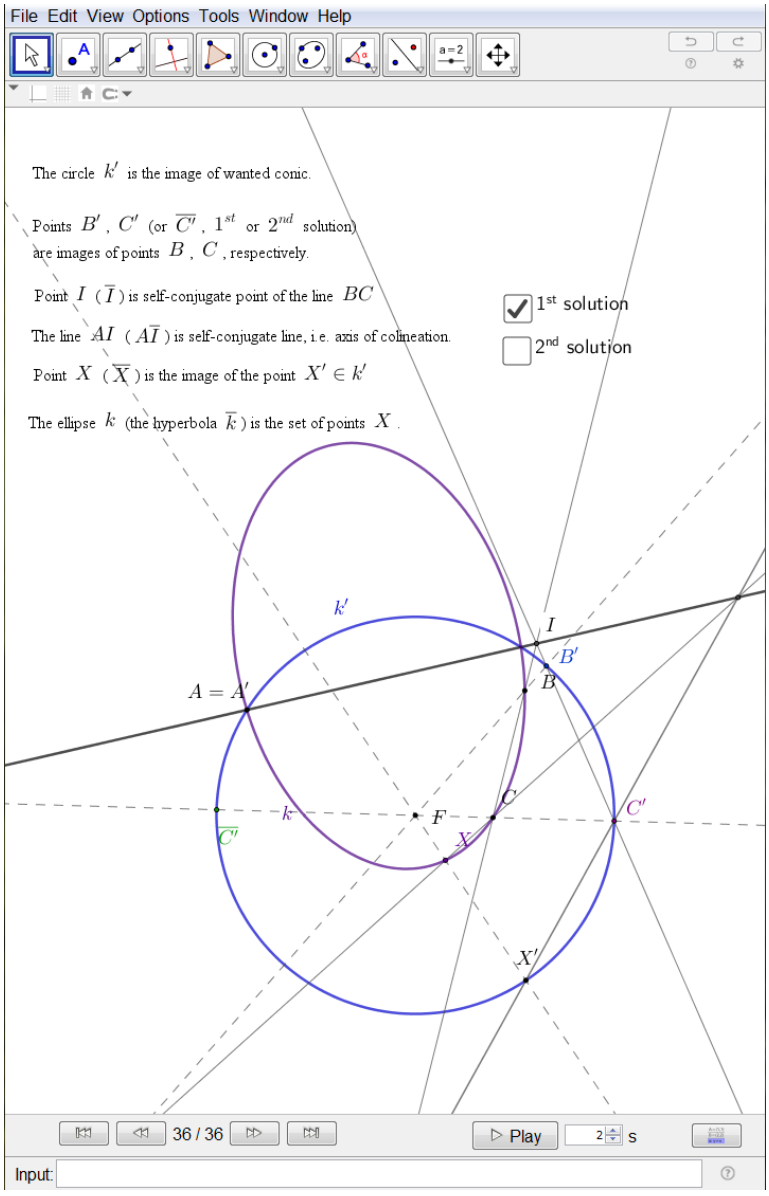

Figure 1: Construction of conic, solution using collineation

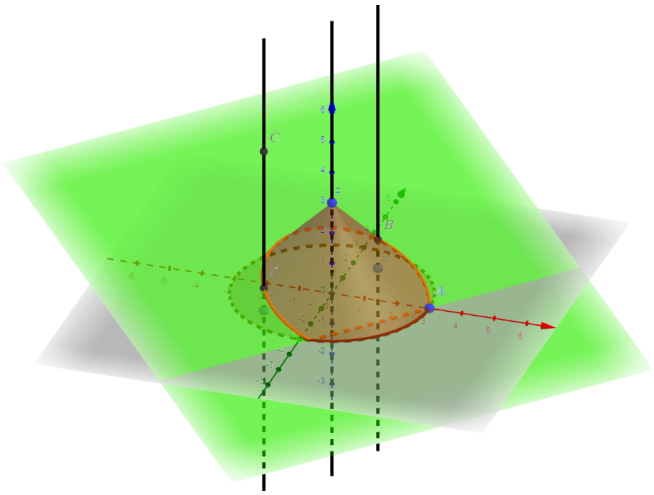

Figure 2: Construction of conic, solution using spatial properties

The construction is again displayed step by step using the animation panel. Check boxes enable display one of both solutions and conduct a discussion about number of solutions depending on positons of given objects. There is displayed step by step description of a construction procedure also.

Finally, an example of more demanding task. Students needs to know projective properties of conis to solve this tasks.

Task 2 Construct a conic, if you know its focus F, its tangent line u and its polar line p of the point P.

Solution using colineation (GeoGebra worksheet: https://ggbm.at/jj3mscbt)

The animation there is implemented by slider, because the construction is more complex and the navigation bar for construction steps might not be clear, The illustration of the worksheet is on the image 4.

The center of the collineation is focus of athe wanted conic k. We must find an axis o of collineation and a circlek' which is an image of the wanted conic. The center of that cirle is focus F. First, the polar p' of the point P' (the image of the given polar p) is found. Next, students can follow the construction of the other tangent of the conic, such that lines u, v, p, $q = \overleftrightarrow{PQ}$, where $q = u \cap p$ are projective harmonic conjugates. The purely projective construction of projective harmonic conjugates is displayed only than the tangent is constructed, because it is auxiliary construction. There is deduced finding of images of lines u, v using properties of dual ratio. The derivation of the construction of such images (by using properties of dual ratio and ratio of division) is display as description next to construction and it is only displayed for as long as necessary. Finally, there are gradually constructed the axis of the collineation, the circle and the wanted conic. It has been said it is not a simple task, therefore this worksheet can not be

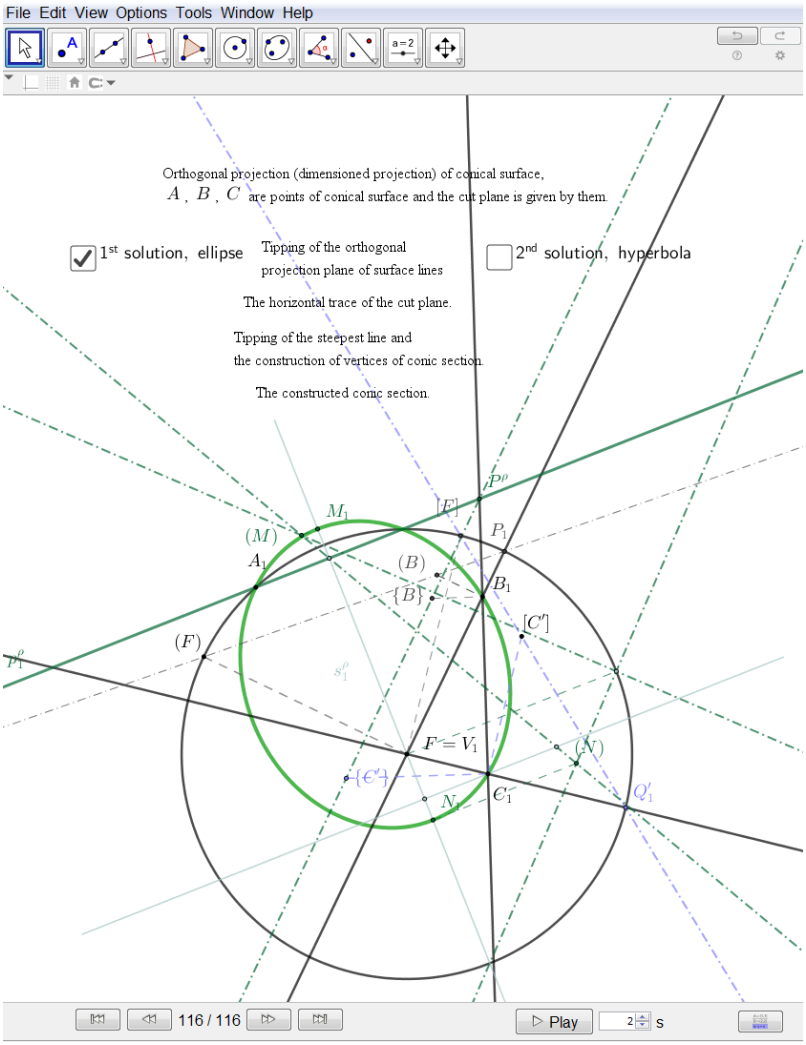

Figure 3: Construction of conic, solution using spatial properties

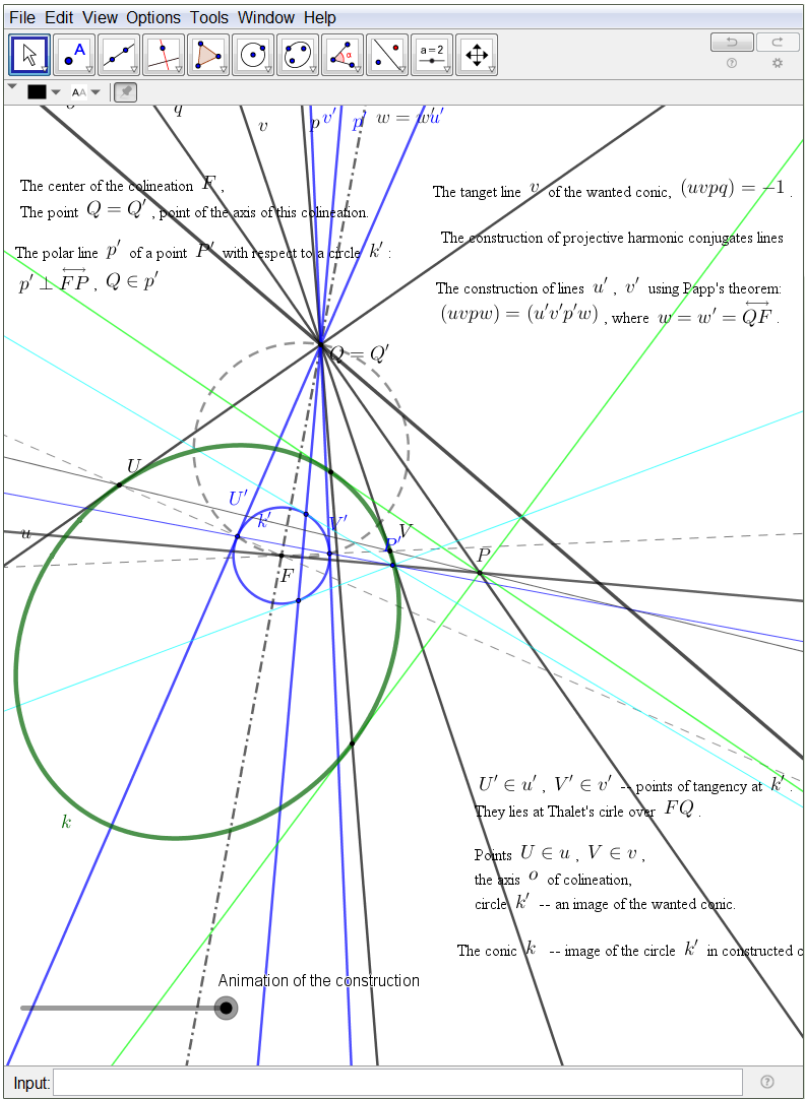

Figure 4: Construction of conic, solution using collineation

used at the grammar school. But it is very useful in studying projective geometry at college.

3 Conclusion

Our students learn at college following facts: most of the informations obtained permanently by man are obtained eyes. Significantly smaller part of informations obtained by man is obtained the other senses, [5]. The most of informations forwarded at schools nevertheless are forwarded by ear, the minority of informations is forwarded by eyes and the other senses. The exception is descriptive geometry, students obtain more informations by eye, but it is not yet sufficient.

We try to our graduates know and use old Chinese proverb: "Everything which you see you forget. (To see means to forget.) Everything which you see and hear you know. (To see and to hear means to know.) Everything which you see, hear and do, you know and understand. (To see, to hear and to do means to know and to understand.)" Dynamic worksheets can be one of way to help teachers to teach their students so that students understand and like geometry, and so that students obtain permanent knowledges.

Students of the program "Training of teachers of descriptive geometry for secondary schools" (and lesser extent students of the program "Training of teachers of mathematics for secondary schools") at college learn also projective properties of conics including improper elements.

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