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Liquid Vibrations in Cylindrical Quarter Tank Subjected to Harmonic, Impulse and Seismic Lateral Excitations

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Abstract

Containers and storage tanks for oil, flammable and poisonous liquids are widely used in various fields of engineering practice, such as aircraft engineering, chemical, oil and gas industries, power engineering, and transport. These tanks function in conditions of high technological loads and are often filled with oil, flammable or poisonous substances. As a result of sudden actions of loadings caused by earthquakes, other force majeure, the intensive sloshing of liquid stored in tanks is occurred. This can lead to dangerous phenomena associated with the filler spraying. Therefore, studying the dynamic behavior of liquids in reservoirs is an urgent task. In this paper, we propose methods for solving fluid vibration problems in rigid tanks with partitions. The numerical method for modeling the external influence upon liquid storage tanks is proposed. It is assumed that the fluid is incompressible and ideal one, and its motion, caused by the action of external loading, is vortex-free. In these conditions, there exists a velocity potential that satisfies the Laplace equation. The mixed boundary value problem is formulated for defining the velocity potential. This is the base to obtain the own modes of free liquid vibrations in the cylindrical tanks, that are considered as basic functions for studying the force liquid vibrations in the baffled cylindrical tank. The lateral excitations caused by harmonic, impulse, and seismic loading are considered and their influence on the free surface elevation is examined.

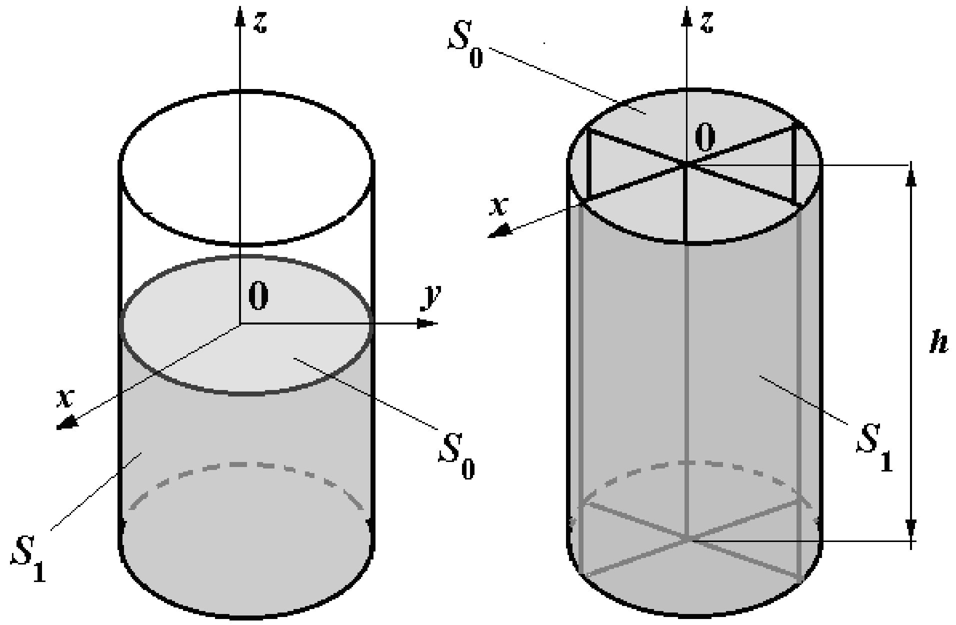
Keywords: cylindrical tank, baffle, free and forced vibration, sloshing.

Introduction

Sloshing is a phenomenon associated with the intense movement of fluid in partially filled tanks. This phenomenon can lead to negative consequences caused by the action of suddenly applied loads (earthquakes, fall of planes, etc.). The intensive liquid sloshing is usually caused by external container lateral excitations. The filler sloshing is accompanied by an intense movement of fluid inside the reservoir that can lead to dangerous environmental effects. To reduce the amplitude of the sloshing, different devices in the form of partitions of various shapes [1-4] were proposed and investigated. Most research works were limited to the study of horizontal partitions in reservoirs. In [3], the approach to the analysis of influence of conical partitions on the frequencies of fluid vibrations was proposed. In [1, 3], it was found that the shape of the partition and its location are significant in the design of reservoirs with optimal parameters, taking into account geometric and strength constraints. Analysis of studies on the problems of fluid sloshing in reservoirs is given in [5-13]. In these works the shells filled with a liquid, with or in without horizontal partitions were considered. The problem of influencing the vertical baffles on a free surface elevation was considered for prismatic tanks only [8].

Model and Analysis

In this paper, the problem of free vibrations of a liquid in a rigid cylindrical reservoir with radius R, and with two vertical partitions, is considered. The scheme of the tank is shown in Fig. 1b). Let S_1 is the wetted shell surface, and S_0 is the liquid free surface. For comparing results we also consider un-baffled cylindrical tank, Fig 1a). It is necessary for clarifying the effect of partitions on the change in the level of free surface elevation. The lateral excitations caused by periodic, impulse, and seismic loadings are under consideration. It is supposed that these lateral excitations are acted along the 0*x* axis, Fig.1,



a) b)

Figure 1. Cylindrical reservoirs with and without baffles

We suppose that the fluid is an inviscid and incompressible one, and its movement cause by reservoir exitation is potential. In these conditions, there exist a velocity potential $\varphi(x, y, z, t)$ defined as

$$V_x = \frac{\partial\varphi}{\partial x}; V_y = \frac{\partial\varphi}{\partial y}; V_z = \frac{\partial\varphi}{\partial z}. \quad (1)$$

In above formulated suppositions this potential satisfies the Laplace equation. A mixed boundary value problem for this equation is formulated below. In this case, the conditions of non-penetration are given on the lateral surfaces and the bottom of the reservoir (surface S_1), and the kinematics and dynamic conditions are stated on a free surface (surface S_0). The kinematics condition consists in following. If the point belongs to the free surface of the liquid at the initial time, it will remain on this surface throughout the whole process of motion. A dynamic condition characterizes the equality of atmospheric and fluid pressures on the free surface. Unknown functions here are the velocity potential $\varphi(x, y, z, t)$ and a function $\zeta = \zeta(x, y, t)$, describing the changes in the free surface level in time. The connection between these two unknown functions is obtained from the following boundary dynamic condition:

$$\frac{\partial \varphi}{\partial t} + g\zeta = 0. \tag{2}$$

Here g is the gravity acceleration. Let the equation of free surface at the initial moment of time look like

$$\zeta(x, y, 0) = 0\,, \tag{3}$$

i.e. we suppose that the surface z=0 corresponds to the liquid free surface, and the surface $z= - h$ is associated with the tank bottom. The zero initial conditions for finding unknown functions are selected that corresponded to the assumption that at the initial moment the liquid in the reservoir was in a state of rest. The boundary conditions of the mixed boundary-value problem for the quarter tank acquire the next form [13]:

$$\left.\frac{\partial \varphi}{\partial r}\right|_{r=R} = 0, \quad \left.\frac{\partial \varphi}{\partial z}\right|_{z=-h}, \quad \left.\frac{1}{r}\frac{\partial \varphi}{\partial \theta}\right|_{\theta=0,\theta=\frac{\pi}{2}}. \tag{4}$$

The liquid pressure on the surfaces of the reservoir is received from the linearized Bernoulli's integral. This allows us to express the fluid pressure through the velocity potential. An expression for the Bernoulli integral is obtained considering the presence of external lateral influences

$$p - p_0 = -\rho_l\left(\frac{\partial \varphi}{\partial t} + a_s(t)x + gz\right),$$

where p_0 is an atmospheric pressure, ρ_l if for the liquid density, $a_s(t)$ is an acceleration corresponding to external influence, z is the vertical coordinate of the point inside the liquid volume.

Cylindrical reservoirs are considered here. For cylindrical tanks without partitions (un-baffled tanks) the modes of liquid vibrations are obtained using the method of integral equations [4]. This made it possible to carry out the study of fluid vibrations in both un-baffled cylindrical reservoirs and in presence of internal horizontal and conical baffles (baffled tanks) [3].

The following mixed boundary value problem is formulated for finding unknowns functions $\varphi(x, y, z, t)$ and $\zeta(x, y, z, t)$ as in [5]:

$$\Delta\varphi = 0\,; \quad \left.\frac{\partial \varphi}{\partial \mathbf{n}}\right|_{S_1} = 0; \quad \left.\frac{\partial \varphi}{\partial t} + g\zeta\right|_{S_0} = 0; \quad \left.\frac{\partial \varphi}{\partial \mathbf{n}}\right|_{S_0} = \frac{\partial \varsigma}{\partial t}. \tag{5}$$

The zero own values obviously exist for problem (5), but they are excluded by the next orthogonality condition [14]:

$$\iint_{S_0} \frac{\partial \varphi}{\partial \mathbf{n}} dS_0 = 0$$

Supposing $\varphi(t,x,y,z)=e^{i\omega t}\varphi(x,y,z)$ we obtain the next boundary condition on the free surface as in [15]:

$$\frac{\partial\varphi}{\partial\mathbf{n}}=\frac{\omega^2}{g}\varphi \tag{6}$$

In [13], the solution of above formulated boundary-value problem (5) for was obtained for the potential φ in the cylindrical coordinate system. It takes the following form

$$\varphi(r,z,t,\theta)=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}A_{mn}\cos 2m\theta\sin(\omega_{mn}t)\frac{\cosh[\xi_{mn}(z+h)/R]}{\cosh(\xi_{mn}h)}\mathrm{J}_{2m}\left(\frac{\xi_{mn}r}{R}\right). \tag{7}$$

Frequencies of free liquid vibrations are calculated by the formula

$$\omega_{mn}^2=\frac{g}{R}\xi_{mn}\tanh\left(\frac{\xi_{mn}h}{R}\right),\quad m=0,1...;\quad n=1,2,... \tag{8}$$

The function ζ for describing the variable level of free surface is given in the following form

$$\zeta(r,z,t,\theta)=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}A_{mn}\omega_{mn}\cos 2m\theta\cos(\omega_{mn}t)\mathrm{J}_{2m}\left(\frac{\xi_{mn}r}{R}\right). \tag{9}$$

In the above expressions J_{2m} are Bessel functions of the first kind, and ξ_{mn} are the roots of the equation below

$$\left.\frac{d\,\mathrm{J}_{2m}(\xi_{mn}r/R)}{dr}\right|_{r=R}=0.$$

This equation is a consequence of the first equation in boundary conditions (4).

So the following systems of basic functions for considering the dynamical conditions at free surface for each harmonics $\alpha=0,1,2$ for quarter reservoir

$$\begin{array}{lll}
\alpha=0, & \mathrm{J}_0\left(\frac{\xi_{0n}r}{R}\right), & n=1,2,...,\\
\alpha=1, & \mathrm{J}_2\left(\frac{\xi_{1n}r}{R}\right)\cos 2\theta, & n=1,2,...,\\
\alpha=2, & \mathrm{J}_4\left(\frac{\xi_{2n}r}{R}\right)\cos 4\theta, & n=1,2,...,\\
... & & \\
\alpha=m, & \mathrm{J}_{2m}\left(\frac{\xi_{mn}r}{R}\right)\cos 2m\theta, & n=1,2,...,
\end{array}$$

are obtained that will be used for analyzing forced vibrations of the liquid in quarter reservoirs.

In [5, 15] the analogical systems were received for forced vibration analysis of the linear liquid sloshing both in cylindrical reservoirs with circular rigid baffles and in un-baffled ones.

We now turn to the problem of forced vibrations of the liquid in baffled and un-baffled reservoirs. For this purpose, the boundary value problem is formulated

$$\nabla^2\varphi = 0\,;\ \left.\frac{\partial\varphi}{\partial\mathbf{n}}\right|_{S_1} = 0\,;\ \left.\frac{\partial\varphi}{\partial\mathbf{n}}\right|_{S_0} = \frac{\partial\zeta}{\partial t};\quad \left.p - p_0\right|_{S_0} = 0\,;\ \left.\frac{\partial\varphi}{\partial t} + g\zeta + a_s(t)x\right|_{S_0} = 0. \tag{10}$$

Here **n** is an external unit normal to the corresponding surfaces.

Let us suppose that the velocity potential can be presented as a following series [14]

$$\varphi = \sum_{k=1}^{M}\dot{d}_k\varphi_k \tag{11}$$

where functions φ_k are own modes of liquid vibrations described above, and $d_k(t)$ are unknown coefficients depending on time only.

Substituting expression (11) to the dynamical boundary condition at the free surface in (10), we obtain the next differential relationship:

$$\sum_{k=1}^{M}\ddot{d}_k\varphi_k + g\sum_{k=1}^{M}d_k\frac{\partial\varphi_k}{\partial\mathbf{n}} + a_s(t)x = 0 \tag{12}$$

Consider now the forced vibrations of liquids in cylindrical shells under different loadings.

It would be noted that in main relation (11) there is

$$x=\rho\cos\theta,$$

because the only lateral exitations are considered.

So to solve the problem in the linear approximation, it suffices to consider only the eigen modes corresponding to the first harmonic (m=1). Using the orthogonality of the eigen modes of oscillations of the liquid in cylindrical tanks [14] and the boundary condition on the free surface (4), we obtain after the dot product of relation (11) by $\varphi_m\left(m = 1,...M\right)$ the following system of second-order differential equations:

$$\ddot{d}_k + \omega_{1k}^2 d_k + a_s(t)F_k = 0;\quad F_k = \frac{(r,\varphi_k)}{(\varphi_k,\varphi_k)};\quad k = \overline{1,M}\ . \tag{13}$$

It allows us to carry out the investigation of changing in the free surface levels in baffled and un-baffled tanks.

Analysis and Discussion

As an example of numerical simulation, consider the cylindrical shell with two vertical partitions, as well as the cylindrical shells without partitions. The radius of the shell R=1m, and the level of liquid filling is h=1m According to formula (8) we obtain the following values of free liquid oscillation frequencies as

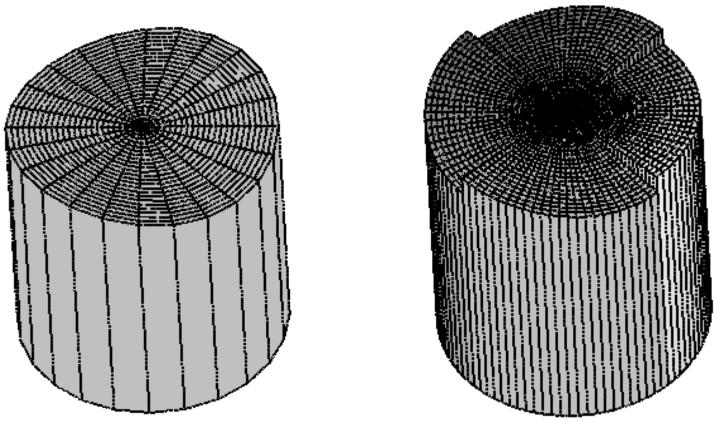
$$\omega_{11} = 5.461\ \omega_{12} = 8.11\ \omega_{13} = 9.889\ \omega_{14} = 11.45\,.$$

If there are no partitions, then the values of the frequencies are as follows [5]:

$$\omega_{11} = 4.14, \ \omega_{12} = 7.22, \ \omega_{13} = 9.14, \ \omega_{14} = 10.7$$

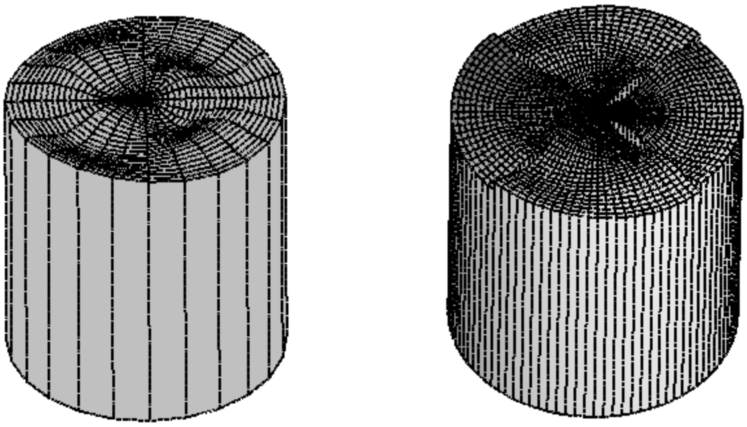
Thus, the installation of partitions leads to increasing the lowest own frequencies.

Fluctuations of the free surface at wave number m = 1 are depicted in Fig. 2, 3 for the first and third forms of oscillations, respectively.



a) b)

Figure 2. The first modes of the liquid free surface oscillation in cylindrical shells, m=1



a) b)

Figure 3. The third modes of the liquid free surface oscillations in cylindrical shells, m=1

Figures 2a) and 3a) correspond to fluid fluctuations in a cylindrical shell without partitions, and Figures 2b) and 3b) are for the shell with vertical partitions.

Suppose that the shell is subjected to the harmonic loading $a_s(t) = \cos\omega t$ applied along the axis *Ox*.

The solution of the differential equations (12) in this case is obtained in the form

$$d_k(t) = \frac{aF_k}{\omega_{1k}^2 - \omega^2}\left(\cos\omega t - \cos\omega_{1k} t\right), \quad k = \overline{1, M} \tag{14}$$

For the analysis of the method convergence, the calculations of the change in the free surface level with a different number of eigenmodes in expression (11) are carried out. Figure 4 shows the results obtained for M = 1 and M = 2. Dot line corresponds to M = 1, and the solid line to M = 2.

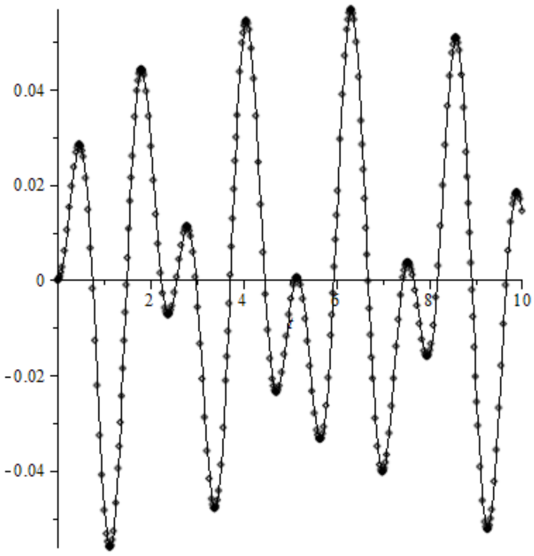
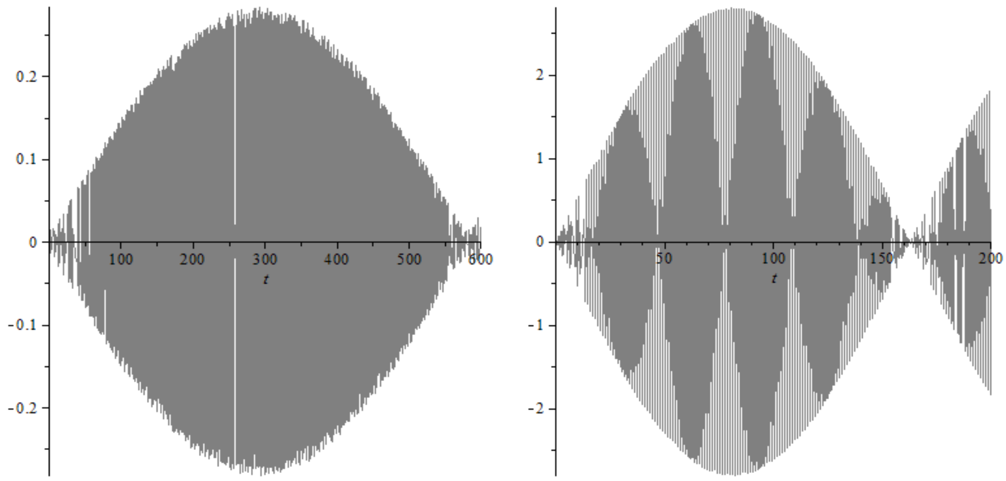


Figure 4. Convergence of the numerical method

We see that using even one mode of own oscillations in series (11) is sufficient for a satisfactory description of the process of changing in the free surface level.

Figures 5a) and 5b) show changing the liquid free surface elevation in quarter cylindrical tank via time at the following forced oscillation frequencies: ω=9.9Hz and ω=5.5Hz, respectively



a) b)

Figure 5. Time-history of the liquid free surface level at ω=9.9Hz and ω =5.5Hz

It would be noted that these frequencies of lateral harmonic excitations are near values of free liquid oscillations. So we see the enormous.

Comparing with the results obtained in [15], we see that the installation of vertical partitions moves the spectrum of resonant frequencies towards high frequencies oscillations. It would be noted that the frequency of harmonic excitation increasing of liquid elevations. It is the reason to study these effects in non-linear statements.

Next we have compared the behavior of baffled and un-baffled cylindrical reservoirs under impulse and seismic loadings.

First, the impulse loading is considered. We suppose that the impulse loading has the form

$$a_s(t) = \begin{cases} 1, & t < T \\ 0, & t \geq T \end{cases}$$

The behavior of this impulse loading is shown in Figure 6a).

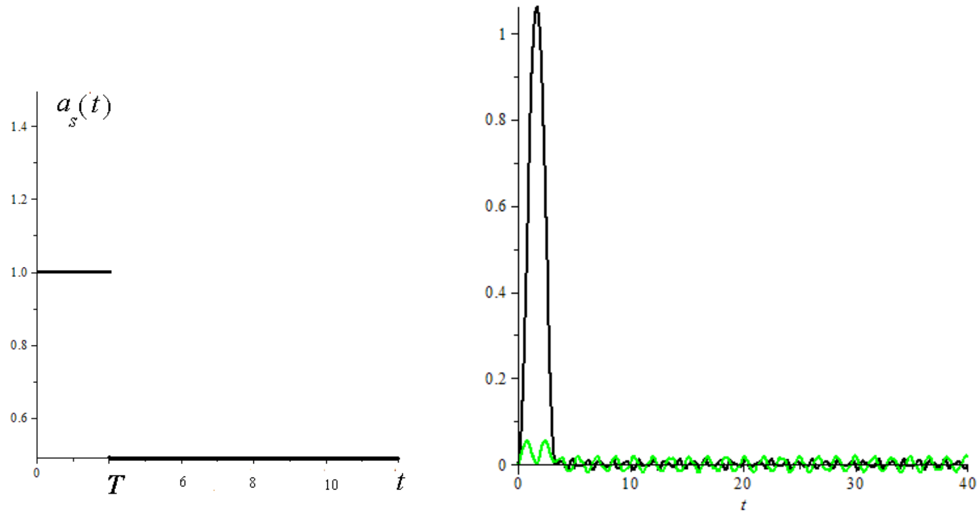
For receiving the solution of system (12) we use Laplace transform. We suppose that

$$d_m(0) = 0; \quad \dot{d}_m(0) = 0.$$

The following expressions for coefficients $d_m(t), \quad m = \overline{1, M}$ are obtained:

$$d_m(t) = \begin{cases} \dfrac{1}{\omega_{1m}^2} - \dfrac{1}{\omega_{1m}^2}\cos(\omega_{1m}t) & 0 \leq t \leq T \\ \dfrac{1}{\omega_{1m}^2} - \dfrac{1}{\omega_{1m}^2}\cos(\omega_{1m}t) - \dfrac{1}{\omega_{1m}^2} + \dfrac{1}{\omega_{1m}^2}\cos\omega_{1m}(t-T) & t > T \end{cases}$$

Supposing *T*=3.41 sec, we received data characterized the changing in liquid free surface level via time. Figure 6b) shows the time-history of liquid free surface levels in baffled and un-baffled cylindrical tanks.



a) b)

Figure 6. Impulse loading and time-history of liquid free surface levels.

Here the black line denotes the free surface elevation of un-baffled tank, and green line is for the tank with the two vertical baffles.

For numerical simulation of seismic influence we consider the model seismic impulse as following

$$a_s(t) = 0.02\sin(\pi t/5)\cos(2\pi t), \quad 0 \le t \le 5$$
$$a_s(t) = 0, \quad t \ge 5 \tag{15}$$

Let $\Omega_1 = \frac{12}{5}\pi, \quad \Omega_2 = \frac{9}{5}\pi.$ Then we obtain for Laplace images

$$p^2 D_m + \omega_{1m}{}^2 D_m = -F_1\left(\frac{\pi 12}{5}\left(p^2 + \left(\frac{\pi 12}{5}\right)^2\right)(1-e^{-pT}) + F_2\left(\frac{\pi 9}{5}\left(p^2 + \left(\frac{\pi 9}{5}\right)^2\right)(1-e^{-pT}\right)$$

Determining D_m from here and returning to Laplace originals one can received at t<T

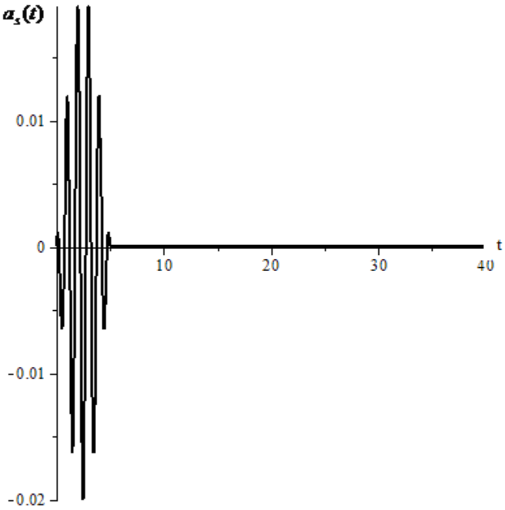
$$d_m = -\frac{F_1}{\Omega_1^2 - \omega_{1m}^2}\left[\frac{\Omega_1 \sin(\omega_{1m} t)}{\omega_{1m}} - \sin(\Omega_1 t)\right] + \frac{F_2}{\Omega_2^2 - \omega_{1m}^2}\left[\frac{\Omega_2 \sin(\omega_{1m} t)}{\omega_{1m}} - \sin(\Omega_2 t)\right]$$

and

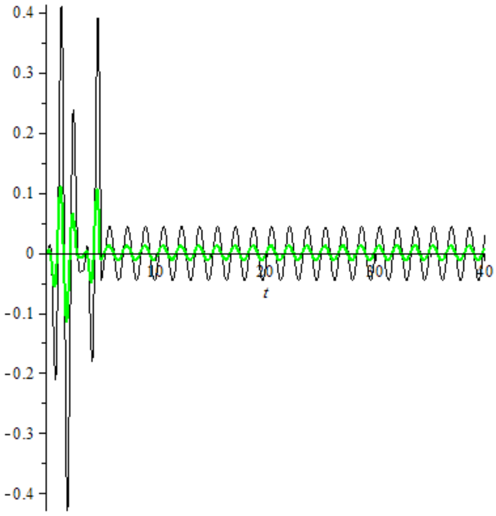
$$d_m = -\frac{F_1}{\Omega_1^2 - \omega_{1m}^2}\left[\frac{\Omega_1 \sin(\omega_{1m}(t-T))}{\omega_{1m}} - \sin(\Omega_1(t-T))\right] + \frac{F_2}{\Omega_2^2 - \omega_{1m}^2}\left[\frac{\Omega_2 \sin(\omega_{1m}(t-T))}{\omega_{1m}} - \sin(\Omega_2(t-T))\right],$$

where t>T.

Figure 7a) below demonstrates the behavior of seismic loading, and figure 7b) shows the time-history of free surface level under the seismic exitation.



a)



b)

Figure 7. Seismic loading and time-history of changing in liquid free surface level

Black line here corresponds to reservoir without baffles, and green one is for cylindrical tank with vertical partitions.

It would be noted that the free surface level does not decrease with time having a periodic behaviour. The reason is in limitations of the proposed model consisting in using the classical dynamics equations for un-damped systems.

From results obtained here one can conclude that baffle installation can be useful for preventing spillage of dangerous fillers. In [15] the impulse loading were studied for cylindrical reservoirs with horizontal baffles. The frequencies of baffled reservoirs are smaller compares with un-baffled ones. But both installation of vertical and horizontal baffles lead to decreasing the level of the free surface elevation. So having data about external force excitation it is possible to tune out unwanted frequencies by installing appropriate partitions.

Conclusions

The behavior of fluid in cylindrical tanks without partitions and with vertical partitions is investigated. The installation of vertical partitions moves the spectrum of resonant frequencies toward high frequency oscillations. This will allow us to set off unwanted excitation frequencies at the design stage and prevent loss of stability. The proposed approach allows us to carry out the numerical simulation for liquid storage tanks with baffles of different forms instead of expensive field experiments.

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