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DT - Space

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Abstract

In this search we define a new type of topological space which is a DT-space, Define a DT-open, DT-closed sets, DTi-space, DTd-space and study the relation between topological space and DT-space, give examples, define dual subspace of (X, *, DTx), DT- neighbourhood system, then generalize the concept of continuity to DT- continuity between two DT-spaces, define a DT - homeomorphism with example.

Keywords: General topology, dual topology (DT- topology), dual open set, dual continuous, dual homeomorphism.

1. Introduction

In 1965 Zadeh was the first publication in fuzzy set theory, Change In1968 used fuzzy set theory to define a fuzzy topological space, soft sets was introduced by Demetry Molodtsove 1999 as a general mathematical tool for dealing with uncertain objects, operations on soft set was introduced by P.K. maji, R. Biswas and A.R. Roy 2003, Muhammad Shabir and Munazza Naz 2011 introduce and study the concept of soft topological.

For many years, scientists have been trying to search for new topological structures to be used in real life applications and for structures to be more practical and adaptable to life. While our world may does not contain a single structure, for these reasons we introduce a new topological structure "The DT-space" (dual topological space) which is a general mathematical tool for dealing with real life applications (physics, quantum theory, engineering …), in this paper we use a general topology as base for construction a DT-space , we replace the union and the intersection operations with any operation "*" and its dual "°" .

Define a DT-open, DT-closed sets, DTi-space, DTd-space and study the relation between topological space and DT-space and give examples, define dual subspace of (X, *, DTx), DT- neighbourhood system, then generalize the concept of continuity to DT-continuity between two DT-spaces, define a DT-homeomorphism with example.

2. Construction a dual topological space

we will define dual topology , dual open set, dual continuous ,dual homeomorphism with examples and study the relation between general topology and dual topology .

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Notification 2.1 The null set Φ is the set of all identity elements ,where each identity element in Φ has a corresponding with the given set , X is the universal set , $\Re$ is the set of real numbers .

Definition 2.2 Let X be a set and let DT be a collection of subsets satisfying the following conditions :

1- $\Phi \in DT$, $X \in DT$.

2- If G_1 and G_2 belongs to DT then $G_1 * G_2 \in DT$ (where * is a closed operation on subsets G of X).

3- If $G_\lambda \in DT \quad \forall \lambda \in \Lambda$ where Λ is arbitrary then $\circ \{G_\lambda , \lambda \in \Lambda \} \in DT$,

(where $\circ$ is a dual closed operation of * on subsets G of X).

Then DT is called a DT - topology for X, the members of DT are called
DT-open their complements are called DT-closed and the triple (X ,*, DT) is called dual topological space simply DT-space the elements of X will be called points of the DT- space .

Remark 2.3 In a DT-space we will study many cases separated each other's where each case depending on a given operator and its dual , and the identity .

Preposition 2.4 Let X be any set with closed operation "*" and dual closed operation " $\circ$ "if X contain all elements with their inverse element then the collection DTi = $\{\Phi,X\}$ consist the empty set and the universal set is always a DTi- topology called the indiscrete (or trivial) dual topology the triple (X , *, DTi) is called the DTi-space

Proof

1- $\Phi \in DTi$, $X \in DTi$.

2- If $\Phi \in DTi$ and $X \in DTi$ then $\Phi * X \in DTi$

also $X * \Phi \in DTi$, (since * is a closed operation on subsets of X).

3- If $G_\lambda \in DTi$,$\forall \lambda \in \Lambda$ where Λ is arbitrary then $\circ \{G_\lambda , \lambda \in \Lambda \} \in DTi$

Since the only sets are Φ,X and $\circ$ is a closed dual operation of * on sets Φ,X then $\Phi \circ X \in DTi$ similarly $X \circ \Phi \in DTi$.

Notice 2.5 From the previous proposition the condition " X contain all elements with their inverse element " we notice that there is a different from indiscrete dual topology and indiscrete topology .

Example 2.6 ($\Re$, + , DTi) is DTi-space , DTi **=** **{** Φ , $\Re$ } , $\Re$ is the set of real numbers , Φ = the set of all zero elements ,where each zero element associate to element in $\Re$**.**

Example 2.7 Let X =$\{1,2,3\}$ with operations
* and its dual $\circ$ defined as follows :

$\{a, b ,c\}*\{d ,e ,f\}= \{ a + d , b + e , c + f \}$

$\{a, b ,c\}\circ\{d ,e ,f\}= \{ a - d , b - e , c - f \}$

$\Phi = \{0, 0 ,0\}$then DT **=** $\{\Phi ,\{1,2,3\}\}$ is not DTi-space since

$\Phi *\{1,2,3\}= \{0, 0 ,0\}*\{1,2,3\}=\{1,2,3\}\in DTi$

but $\Phi \circ\{1,2,3\}= \{0, 0 , 0\}\circ\{1,2,3\}=\{-1,-2,-3\}\notin DTi$.

Proposition 2.8 Let DTd be a collection of all subsets of X , with closed operation "*" and dual closed operation " $\circ$ "

then DTd is called the discrete dual topology the triple

(X ,*, DTd) is called the DTd-space .

Proof

1. $\Phi \in$DTd , X$\in$DTd .
2. If $G_1 \in$DTd and $G_2 \in$DT then $G_1 * G_2 \in$DTd (since * is a closed operation on subsets G_1 , G_2 of X).
3. If $G_\lambda \in$DTd ,$\forall\lambda\in\Lambda$ where Λ is arbitrary then
   $\circ\{G_\lambda , \lambda\in\Lambda \}\in$DTd .(since $\circ$ is a closed dual operation of * on subsets G_λ of X).

Example 2.9 Let * and its dual $\circ$ defined as follows (for finite and infinite case)

For each arbitrary elements a , b in $\Re$ a*b= a+b , a$\circ$b= a-b

Then ($\Re$,+, DTd) is DTd-space since

1- $\forall$ a$\in\Re$ a is arbitrary $\exists$ 0$\in\Re$ such that a+0=0+a=a therefore

Φ = the set of all zero element each one of them associated to the arbitrary element a so $\Phi \in$DTd .

2- $\forall$ a,b $\in\Re$ a,b are arbitrary $\exists$ k $\in\Re$ such that a+b =k$\in\Re$ so a*b$\in$DTd .

3- $\forall$ $a_\lambda\in\Re$,$\forall\lambda\in\Lambda$ where Λ is arbitrary then - $a_\lambda \in\Re$ $\forall\lambda$ so $a_\lambda \circ - a_\lambda \in$DTd .

Example 2.10 Consider (X, $\wedge$,DT) where X={T,F} is a set of any two statements where the first statement is true the second is false , with two operations and "$\wedge$", or "$\vee$"

$\varnothing$ = X^c ={F,T}

$\varnothing \wedge$ X ={F,T}$\wedge${T,F} = {F$\wedge$T, T$\wedge$F}={F,F}

$\varnothing \wedge \varnothing$ ={F,T}$\wedge${F,T} = {F$\wedge$F , T$\wedge$T}={F,T}=$\varnothing$

X $\wedge \varnothing$ ={T,F} $\wedge${F,T} = {T$\wedge$F, F$\wedge$T}={F,F}

X $\wedge$ X ={T,F} $\wedge${T,F} = {T$\wedge$T, F$\wedge$F}={T,F}=X

$\varnothing \vee$ X ={F,T}$\vee${T,F} = {F$\vee$T, T$\vee$F}={T,T}

$\varnothing \vee \varnothing$ ={F,T}$\vee${F,T} = {F$\vee$F, T$\vee$T}={F,T}=$\varnothing$

X $\vee \varnothing$ ={T,F}$\vee${F,T} ={T$\vee$F, F$\vee$T}={T,T}

X $\vee$ X ={T,F}$\vee${T,F} = {T$\vee$T, F$\vee$F}={T,F}=X

Then DT = {$\varnothing$, X,{F,F},{T,T}} .

Example 2.11 Let X =$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ set of all arbitrary 2×2 matrices , where all of its elements *a*,*b*,*c*,*d* are arbitrary elements belongs to $\Re$,

DT = { $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$}

Then (X, + , DT) is a DT-space on a set of 2×2 matrices .

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+2 \\ 0+3 & 0+4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \in \mathrm{DT}$

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1+a & 1+b \\ 1+c & 1+d \end{bmatrix} \in$ DT since X arbitrary.

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1-a & 1-b \\ 1-c & 1-d \end{bmatrix} \in$ DT since $1-a$, $1-b$, $1-c$, $1-d$ are arbitrary element belongs to $\Re$.

Example 2.12 Let X $= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$,

DT = { $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 5 \\ 3 & 4 \end{bmatrix}$ }

Then (X, + , DT) is not a DT-space on X

Since $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 7 \\ 6 & 8 \end{bmatrix} \notin$ DT .

Example 2.13 Let X $= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ set of all arbitrary 2×2 matrices , where all of its elements *a*,*b*,*c*,*d* are arbitrary elements belongs to $\Re$,

DTi = { $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} a & b \\ c & d \end{bmatrix}$} Then (X, + , DTi) is DTi-space on a set X of an arbitrary of 2×2 matrices .

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0+a & 0+b \\ 0+c & 0+d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in$ DTi

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0-a & 0-b \\ 0-c & 0-d \end{bmatrix} \in$ DTi .

Remark 2.14 Each topology is a DT- topology but the converse is not necessary true **.**

Examples 2.15

(1) Each topological space (X,T) can be written as a DT-space as follows :
(X, * ,DT) were "*" here represent the intersection operator .

(2) From the previous example (X, $\wedge$,DT) is a DT-space but not a topological space .

Definition 2.16 Let Y be a subset of X , DTx is dual topology on X then the relation DT_y for Y is the collection

DTy = {G∩Y , G∈DTx } is a DTy – space (Y,*, DTy) is called the dual subspace of (X,*, DTx) .

The next theorem show that DTy ***is a dual topology for* Y *with special conditions on a closed operation* "* " *on* X .**

Theorem 2.17 Let (X,*, DTx) be a DT-space on X and let Y ⊂ X , (with condition : distribution intersection on "*" and distribution intersection on "°") then the collection DTy = { G∩Y : G∈DTx }is a dual topology on Y .

Proof

(1) Since $\Phi \in DTx$ then $\Phi \cap Y \in DTy$.

Since $X \in DTx$, ($Y \subset X$) then $X \cap Y = Y \in DTy$.

(2) Let $H_1, H_2 \in DTy$ Then $H_1 = G_1 \cap Y$ and $H_2 = G_2 \cap Y$ for some $G_1, G_2 \in DTx$

Now $H_1 * H_2 = (G_1 \cap Y) * (G_2 \cap Y) = (G_1 * G_2) \cap Y$

Since $G_1, G_2 \in DTx$ then $G_1 * G_2 \in DTx$

so $(G_1 * G_2) \cap Y = H_1 * H_2 \in DTy$.

(3) Let $H_\lambda \in DTy$ $\forall \lambda \in \Lambda$ where Λ is an arbitrary set

Then there exist sets $G_\lambda \in DTx$ such that $H_\lambda = G_\lambda \cap Y$ $\forall \lambda$

Now $\circ \{ H_\lambda : \lambda \in \Lambda \} = \circ \{ G_\lambda \cap Y, \lambda \in \Lambda \} = \circ \{ G_\lambda , \lambda \in \Lambda \} \cap Y$

Since $G_\lambda \in DTx$ $\forall \lambda$ then $\circ \{ G_\lambda , \lambda \in \Lambda \} \in DTx$

so $\circ \{ G_\lambda , \lambda \in \Lambda \} \cap Y = \circ \{ H_\lambda : \lambda \in \Lambda \} \in DTy$

hence DTy is a dual topology on Y .

3. DT- neighbourhood on DT-space

Definition 3.1 If (X,*, DTx) be a DT-space , x is a point in X , a DT-neighbourhood of x is a subset V of X that includes a DT-open set U containing x (i.e. $x \in U \subseteq V$) .

Notice 3.2 The DT- neighbourhood V need not be a DT- open set itself. If V is DT- open it is called a DT-open neighbourhood.

Definition 3.3 The DT- neighbourhood system for a point x is the collection of all DT- neighbourhoods for the point x.

Remarks and examples 3.4

- Trivially the DT- neighbourhood system for a point is also a DT-neighbourhood for the point.
- Given the DTi-space on X the DT- neighbourhood system for any point x is only the whole space X .
- (X, + , DTi) is DTi-space on a set X of an arbitrary of 5×5 matrices then $\begin{bmatrix} a_{11} & \cdots & a_{15} \\ \vdots & \ddots & \vdots \\ a_{51} & \cdots & a_{55} \end{bmatrix}$ is a DT- neighbourhood of each of its elements.

4. Continuity on a DT-space

Definition 4.1 Let $f : (X,*,DTx) \rightarrow (Y,*,DTy)$ then f is DT- continuous if The preimages of the DT- open (DT- closed) sets in DTy are DT- open (DT- closed) in DTx .

Example 4.2 If a set X is given with the DTd , all functions to any DT –space $f : (X,*, DTd) \rightarrow (X,*, DT)$ are DT- continuous.

5. DT - Homeomorphism

Definition 5.1 A function $f : (X,*, DTx) \to (Y,*, DTy)$ is called a DT- homeomorphism if it has the following properties:

- f is a bijection (one-to-one and onto),
- f is DT- continuous,
- the inverse function f^{-1} is DT- continuous (f is a DT- open mapping).

If such a function exist , we say (X,*,DTx) and (Y,*,DTy) are DT-homeomorphic.

Definition 5.2

Let *A be a matrix represented by* $A_{m\times n} = \begin{bmatrix} \eta_{11} & \cdots & \eta_{1n} \\ \vdots & \ddots & \vdots \\ \eta_{m1} & \cdots & \eta_{mn} \end{bmatrix}$, $\eta_{mn} \in [0,1]$

its complement $A^{c}_{m\times n} = \begin{bmatrix} 1-\eta_{11} & \cdots & 1-\eta_{1n} \\ \vdots & \ddots & \vdots \\ 1-\eta_{m1} & \cdots & 1-\eta_{mn} \end{bmatrix}$.

Definition 5.3 Let $A= [a_{ij}]$,$B = [b_{ij}]$ then A is sub matrix of B ,if $a_{ij} \le b_{ij}\ \forall\ i,j$.

Example 5.4 Let $A= [a_{ij}]$, $B= [b_{ij}]$ are two matrices , "*", "$\circ$" of A,B are defined as follow :

$A * B = C = [c_{ij}]$, $c_{ij} = \max\{a_{ij}, b_{ij}\}\ \forall\ i , j$.

$A \circ B = C = [c_{ij}]$, $c_{ij} = \min\{a_{ij}, b_{ij}\}\ \forall\ i , j$.

$$\Phi = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}, \ A = \begin{bmatrix} 0.3 & 0.8 & 0.6 \\ 0.4 & 0.5 & 0.7 \\ 0.5 & 0.9 & 0.4 \\ 0.6 & 0.5 & 0.5 \\ 0.7 & 1 & 0.8 \end{bmatrix}, \ A_1 = \begin{bmatrix} 0.1 & 0.5 & 0.2 \\ 0.2 & 0.1 & 0.4 \\ 0.0 & 0.4 & 0.1 \\ 0.2 & 0.1 & 0.1 \\ 0.4 & 0.0 & 0.4 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.3 & 0.8 & 0.5 \\ 0.4 & 0.4 & 0.7 \\ 0.2 & 0.8 & 0.2 \\ 0.5 & 0.4 & 0.3 \\ 0.6 & 0.9 & 0.8 \end{bmatrix}, A_3 = \begin{bmatrix} 0.2 & 0.7 & 0.4 \\ 0.3 & 0.3 & 0.6 \\ 0.1 & 0.7 & 0.1 \\ 0.3 & 0.3 & 0.2 \\ 0.5 & 0.8 & 0.6 \end{bmatrix}, \ A_4 = \begin{bmatrix} 0.3 & 0.8 & 0.4 \\ 0.3 & 0.3 & 0.7 \\ 0.2 & 0.7 & 0.2 \\ 0.5 & 0.4 & 0.2 \\ 0.6 & 0.8 & 0.6 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} 0.2 & 0.7 & 0.5 \\ 0.4 & 0.4 & 0.6 \\ 0.1 & 0.8 & 0.1 \\ 0.3 & 0.3 & 0.3 \\ 0.5 & 0.9 & 0.8 \end{bmatrix}.$$

Then $DT =\{\Phi,A,A_1,A_2,A_3,A_4,A_5\}$, $(A,\boldsymbol{*},DT_A)$ is a DT-space .

Also (λA, *, $DT_{\lambda A}$) $\lambda \in \Re$ is a DT-space, each matrix is a DT- open set ,

$f : (A, *, DT_A) \rightarrow (\lambda A, *, DT_{\lambda A})$ where $f(x_{ij}) = \lambda x_{ij} \ \forall x_{ij} \in X$

is DT - homeomorphism.

this example describe " topologically by DT-homeomorphism" the Eigen value equation which have many applications in quantum physics .

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