Some Issues of Data Sufficiency in Problems

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Abstract

True awareness of every science field includes deep understanding of major concepts and regularities of the field itself. Understanding the concept content includes logical connections with other concepts - a systematic approach. Detailed study of mathematical conditions develops logical thinking, creativeness and applicability of thinking. Actually, this should initially be implemented on school level, then, at the university, specifically, at the Faculty of Pedagogics in order to enable prospective teachers to provide their students with general mathematical skills in school years.

Problem solving occupies huge portion in math studies. Understanding the structure of a problem, identifying the correlations between quantities of its text, whether the problem condition is enough to answer the question stated in the problem itself, appeared to be difficult for the majority of students and a number of teachers during teacher training meetings and throughout a long time of experience with students.

Taking this into account, we think it is important to analyze the problems with conditions and questions regarding the sufficiency of the provided conditions to answer the problems. The present work offers three types of problems that after being stated and resolved will help to achieve the goal.

1. Introduction

Here is the analysis of the problems that have two extra conditions (I and II) in addition to the major one; there is one true among five possibilities to answer the question of the problem:

a) **I sufficient condition , II – not.**

b) **II sufficient condition, I – not.**

c) **I and II conditions are sufficient together; they are not sufficient separately.**

d) **I and II conditions are sufficient separately.**

e) **given conditions are not sufficient**

By solving these types of problems a student will practice the understanding the problem condition to delimitate and again relate quantities which are given or identified, also, point out whether the problem condition is enough for answering the question. It can be illustrated as follows:

Problem 1. *a* **and** *b* **are numbers.**

There are two given conditions:

I. the sum of *a* **and** *b* **is less than** *a*.

II. the sum of *a* **and** *b* **is less than** *b*.

In order to know the sum of *a* ထ *b* **is negative or positive:**

a) I condition is enough, II – not enough.

b) II condition is enough, I – not enough.

c) I and II conditions are sufficient together; they are not sufficient separately.

d) I and II conditions are sufficient separately.

e) given conditions are not sufficient

Let's state three problems and solve each of them:

1) Basic condition + I condition + question;

2) Basic condition + II condition + question;

3) Basic condition + (I + II) condition + question;

Let's discuss 1) how given **problem** can be stated in the (1) case:

Problem 1. *a* **and** *b* **are numbers. Also,** *a* **and** *b* **numbers sum is less than** *a*. **Is the** *a* **and** *b* **numbers sum positive or negative?**

Let's consider: if *a* and *b* are numbers and at the same time, $a + b < a$, this means that $b < 0$, i. e. *b* is a negative number, but the given condition says nothing about the mark of *a*, and we cannot determine the mark of $(a + b)$ sum. Therefore, the I condition of the given problem is not enough to answer the question of the problem.

Let's analyze condition (2) and state the problem:

Problem 2. *a* **and** *b* **are numbers. Also,** *a* **and** *b* **numbers sum is less than** *b*. **Is the** *a* **and** *b* **numbers sum positive or negative?**

Let's start discussion, answer the question, and analyze the problem condition; if *a* and *b* are numbers and at the same time, $a + b < b$, this means that $b < 0$, i. e. *a* is a negative number, but the given condition says nothing about the mark of *b*, and we cannot determine the mark of $(a + b)$ sum. Therefore, the II condition of the given problem is not enough to answer the question of the problem.

Let's analyze case (3) and state the problem:

Problem 3. *a* **and** *b* **are numbers. Also,** *a* **and** *b* **numbers sum is less than** *a* **and** *b*. **Is the** *a* **and** *b* **numbers sum positive or negative?**

Let's start discussion: if *a* and *b* are numbers and at the same time, $a + b < a$, this means that $b < 0$, i. e. *b* is a negative number; also, if $a + b < b$, this means that $a < 0$, i. e. *a* is a negative number. So, if *a* is a negative number and *b* is a negative number, their sum $(a + b)$ is a negative number either. Therefore, there is

an answer to the problem, and we can say: c) I and II conditions are sufficient together; none of them are sufficient separately.

It is interesting to analyze the problem in which, eather taken together or separately, none of the conditions is enough to answer the question stated in the problem.

Problem. Among A, E and N points on the quantitative axis point A lies between the points E and N.

There are two conditions:

I. E point coordinate is positive.

II. N point coordinate in negative.

In order to find out whether A point is positive or not,

a) I condition is enough, II – not enough.

b) II condition is enough, I – not enough

c) I and II conditions are enough together; they are not sufficient separately.

d) I and II conditions are sufficient separately.

e) given conditions are not sufficient.
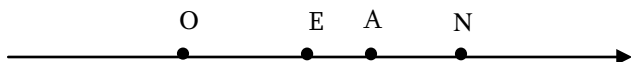
Let's state three problems like we did earlier:

Problem 1. Among A, E and N points on the quantitative axis point A lies between the points E and N, and it is known that E point coordinate is positive. Is A point coordinate positive?

Let's analyze the quantitative axis and represent A, E and N points.

Provided that E point coordinate is positive, E point lies on the right of O point.
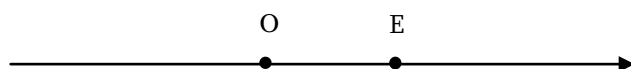
As the task goes, among A, E and N points on the quantitative axis, point A lies between the points E and N; which means we have the following elignment of these points on the axis:

This also means that A point coordinate is positive. There may be another alignment of A, E and N points which mans that A point coordinate is negative.
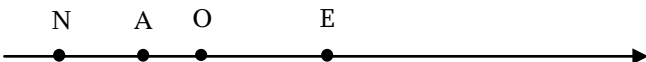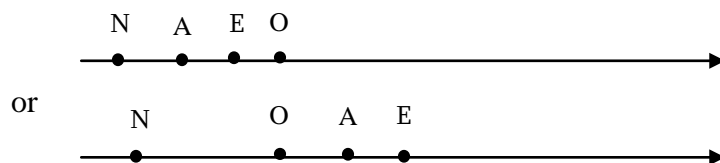

Thus, I condition is not enough to know whether A point coordinate is positive or not.

Let's discuss:

Problem 2. Among A, E and N points on the quantitative axis point A lies between the points E and N, and it is known that N point coordinate is negative. Is A point coordinate positive?

Like in the above analyses, we have two cases of A point location.

N   A   E O

or

N   O   A   E

Respectively, II condition is not enough either to find out A point coordinate is positive or not.

Let's analyze:

Problem 3. Among A, E and N points on the quantitative axis point A lies between the points E and N, and it is known that E point coordinate is positive, and N point coordinate is negative. Is A point coordinate positive?

Let's analyze the quantitative axis;

According to the task E point coordinate is positive, and N point coordinate is negative; it means that E and N points have the following location on the quantitative axis:

N   O   E
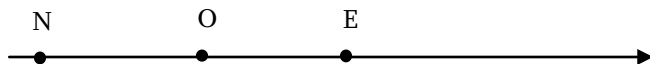
It does not define the location of A point, and A point coordinate marker. Hence, it may be either positive or negative

N   O   A   E

N   A   O   E
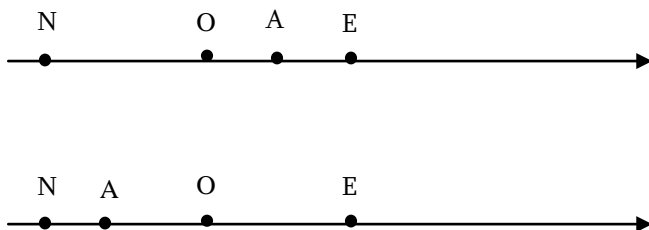
Thus none of the conditions of the problem, taken separately or togeather, is enough to answer the question of the problem. In this case we say that the correct answer is **e) given conditions are not sufficient**.

Assumptions and Theorems – We have analyzed problems about data sufficiency with extra I and II conditions, and have discussed three types of problems:

1) Basic condition + I condition + question;
2) Basic condition + II condition + question;
3) Basic condition + (I + II) condition + question;

Main Result 1) a student perceives the content and structure of the problem, and with argumentative discussion using quantities provided in the problem answers the question stated in the problem.

2) a student has ability to analyze in order to infer what he/she should know to answer the question properly. The provided conditions, are they enough to answer the question of the problem or not.

3) a student is an author as he/she creates new problems and conducts analysis; it helps to build up mathematical skills.

References

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