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Modeling Reliability for Exponential Distribution Using Maximum Likelihood Method (Case study: A.T.M in Sudan)

Ismaeel Ahmed Mohamed Ahmed

Currant PhD Student, University of Sudan for Science & Technology, Faculty of Science, Department of Statistics

Ahmed Mohamed Abdallah Hamdi

Associate Professor, University of Sudan for Science & Technology, Faculty of Science, Department of Statistics

Afra Hashim Abdelateef

Assistant Professor, University of Sudan for Science & Technology, Faculty of Science, Department of Statistics

Abstract

The exponential distribution is the most commonly used life distribution in applied reliability analysis. The reason for this is its mathematical simplicity and that it leads to realistic lifetime models for certain types of item. The aim of this study is to apply lifetime models on the failure time of the automatic teller machine (ATM) in Sudan and estimate the reliability of the machines, in order to compare between machines, failure's data has been taken from Central Bank of Sudan, which is, type of machine, type of failure, Downtime, Uptime and outage duration in Hrs. during the period of time (1/1/2017-30/6/2017). The comparison between five machines selected randomly out of 28 machines have done. Through the lifetime models estimation (failure distribution, reliability, hazard rate, and mean time to failure (MTTF)), using maximum likelihood estimators in estimate the parameters and reliability of exponential distribution. The comparison is executed through reliability values; from analysis results it is clear that, the failure time of all machines follow exponential distribution with one-parameter, according to the Kolmogorov-Smirnov test and Chi-Squared test for goodness of fit result. The machines no (B5 and B35) have high reliability compered by other machines. When we predict the reliability according to the time we found that the reliability decrease and hazard rate increase and there is relationship between MTTF and reliability.

Keywords: Reliability, Failure rate, Hazard rate, Exponential Distribution, Maximum likelihood, Kolmogorov-Smirnov test, Chi-Squared test.

1. Introduction

Reliability is defined as the ability of the individual device or system or component to perform it is required functions under stated conditions for a specified period of time [1]. Also can be define as the probability of success or the probability that the system will perform it intended function under specified design limits [2]. Reliability that is more specific is the probability that a product is part will operate properly for specified period of time (design life) under the design operating condition without feature. In other words, reliability may be used as measure of the systems success in providing, it is function properly. Reliability is once of quality characteristics that consumer requires form the menu facture of products [2].

The study of reliability appeared in the first decade of the twentieth century, the concentration on this type of study has been crystallized during the world war (II), via studying the military devices reliability. This type has expected recently to include the study of commercial products as a result of extraordinary developments in one hand; and the use of electronic devices and the complex systems in the other. This sort of development has imposed an increasing concern on studying the reasons of breakdowns that may lead to the stoppage of devices and sets in their various kinds [2].

Reliability has gained increasing importance in the last few years in manufacturing organizations, the government and civilian communities. With recent concern about government spending, agencies are trying to buy systems with higher reliability and lower maintenance costs. As consumers, we are mainly concerned with buying products that last longer and are cheaper to maintain, or have higher reliability. There are many reason for wanting high product or component or system reliability such as higher customer satisfaction, increased sales, improved safety, decreased warranty costs, decreased maintenance costs.

The objective of this study is to estimate the reliability and failure models of the automatic teller machines (ATM) in Sudan and compare between machines, through use the models of failure and the suitable statistical methods for analysis the data of faults and maintenance and figure out the percentage of optimal utilization of different machines as well as presenting some proposals and recommendations through which problems, the automatic teller machines suffer can be avoided with the aim of raising its technical and operative efficiency and the improvement of its performance.

2. The Theoretical Side

2.1 The Exponential Distribution

Exponential distribution plays an essential role in reliability engineering because it has constant failure rate. This distribution has been used to model the life time of electronic and electrical components and systems. This distribution is appropriate when a used component that has not failed is as good as anew component – a rather restrictive assumption. Therefore it must be used diplomatically since numerous exist where the restriction of the memory less property may not apply.

Consider an item that is put into operation at time ($t = 0$). The time to failure (T) of the item has probability density function [1]

$$f(t) = \begin{cases} \frac{1}{\theta} e^{-\frac{1}{\theta}t} = \lambda e^{-\lambda t} & \text{for } t > 0, \theta \text{ and } \lambda > 0 \\ 0 & \text{otherwise} \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

This distribution is called exponential distribution with parameter ($\lambda = \frac{1}{\theta}$), and we sometimes write $T \sim \exp(\lambda)$.

The reliability function of the item is

$$R(t) = \Pr(T > t) = \int_t^{\infty} f(u)du = e^{-\frac{1}{\theta}t} \;, \quad \text{for } t > 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The probability density function f(t) and the reliability function R(t) for the exponential distribution are illustrated in Figure (1).

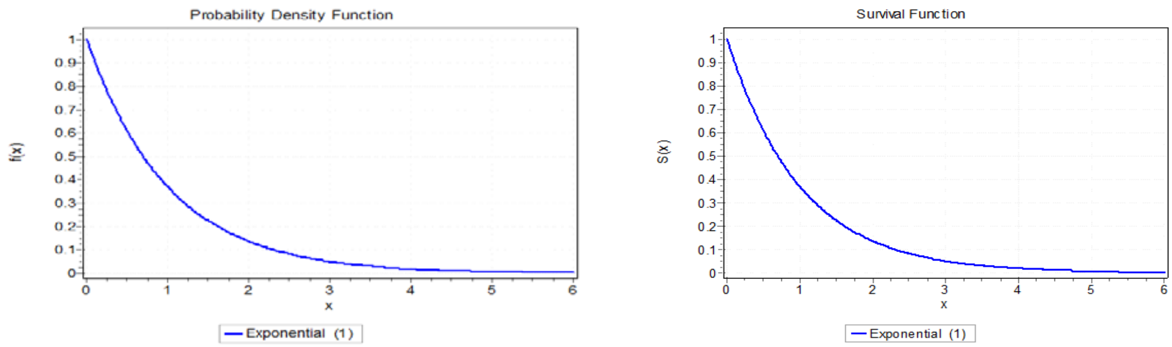


Figure (1) exponential distribution ($\lambda = 1$)
Source: the researcher assume, EasyFit package, 2017

The mean time to failure (MTTF) is

$$MTTF = \int_0^{\infty} R(t)dt = \int_0^{\infty} e^{-\frac{1}{\theta}t}\, dt = \theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

And the variance of T is

$$var(t) = \theta^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

The probability that an item will survive it's mean time to failure is

$$R(MTTF) = R(\theta) = e^{-1} \approx 0.3679$$

The failure rate function is

$$h(t) = \frac{f(t)}{R(t)} = \frac{\frac{1}{\theta}e^{-\frac{1}{\theta}t}}{e^{-\frac{1}{\theta}t}} = \frac{1}{\theta} = \lambda \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Accordingly, the failure rate function of an item with exponential life distribution is constant (i.e., independent of time).

The results (3) and (4) compare well with the use of the concepts in everyday language. If an item on the average has λ=4 failures/year, the MTTF of the item is 1/4 year.

The conditional reliability function:

$$R(x|t) = Pr(T > t + x|T > t) = \frac{Pr(T>t+x)}{Pr(T>t)} = \frac{e^{-\lambda(t+x)}}{e^{-\lambda t}} = e^{-\lambda x} = Pr(T > x) = R(x) \ldots\ldots\ldots(6)$$

The reliability function of an item that has been functioning for t time units is therefore equal to the reliability function of a new item. A new item, and used item (that is still functioning), will therefore have the same probability of surviving a time interval of length. [1]

The mean residual life MRL(t) for the exponential distribution is

$$MRL(t) = \int_0^{\infty} R(x|t)dx = \int_0^{\infty} R(x)dx = MTTF$$

The MRL(t) of an item with exponential life distribution is hence equal to its MTTF irrespective of the age of the item. The item is therefore as good as new machine, as long as it is functioning, and we often say that the exponential distribution has no memory. Therefore, an assumption of exponentially distribution lifetime implies that [5]

- A used item is stochastically as good as new, so there is no reason to replace a functioning time.
- For the estimation of the reliability function, the mean time to failure, and so on, it is sufficient to collect data on the number of observed time in operation and the number of failures. The age of the item is of no interest in this connection.

The exponential distribution is the most commonly used life distribution in applied reliability analysis. The reason for this is its mathematical simplicity and that it leads to realistic lifetime models for certain types of item.

2.2 Maximum likelihood Estimation Method (MLE)

The method of Maximum likelihood Estimation (MLE) is one of the most useful techniques for deriving point estimators. In the exponential distribution with a sample density $f(t_i, \theta)$, assuming that the random variables t_i (i=1,2,3………n) are independent, then the likelihood function $L(t, \theta)$, is the product of the probability density function evaluated at each sample point [2].

In general

$$L(t, \theta) = \prod_{i=1}^{n} f(t_i, \theta)$$

$$\ln L(t, \theta) = \sum_{i=1}^{n} L_n f(t_i, \theta)$$

In the exponential distribution

$$L(t, \theta) = \prod_{i=1}^{n} \frac{1}{\theta} e^{-\frac{1}{\theta} t} = \theta^{-n} e^{-\frac{\sum_{i=1}^{n} t_i}{\theta}}$$

$$\ln L(t, \theta) = -n L_n \theta - \frac{\sum_{i=1}^{n} t_i}{\theta}$$

$$\frac{\partial L_n L}{\partial \theta} = \frac{-n}{\theta} + \frac{\sum t_i}{\theta^2}$$

$$\frac{-n}{\hat{\theta}} + \frac{\sum t_i}{\hat{\theta}^2} = 0$$

$$\hat{\theta}_{MLE} = \frac{\sum_{i=1}^{n} t_i}{n} = \bar{t} \quad \text{……………………………………………..(7)}$$

This unbiased estimation with variance

$$\mathrm{var}(\hat{\theta}_{ML}) = \frac{\theta^2}{n} \quad \text{……………………………………...…….(8)}$$

And mean square error is

$$MSE = Var(\hat{\theta}) + [E(\hat{\theta}) - \theta]^2 = \frac{\theta^2}{n} \quad \text{………………………...…….(9)}$$

Then the reliability function is steamed by

$$\hat{R}(t) = e^{-\left(\frac{t}{\hat{\theta}}\right)} \quad \text{……………………………………………(10)}$$

2.3 Goodness of Fit Techniques

The goodness of fit (GOF) tests measures the compatibility of a random sample with a theoretical probability distribution function. In other words, these tests show how well the distribution you selected fit's to your data.

The general procedure consists of defining a test statistic which is some function of the data measuring the distance between the hypothesis and the data, and then calculating the probability of obtaining data which have a still larger value of this test statistic than the value observed, assuming the hypothesis is true. This probability is called the confidence level.

Small probabilities (say, less than one percent) indicate a poor fit. Especially high probabilities (close to one) correspond to a fit which is too good to happen very often, and may indicate a mistake in the way the test was applied.

2.3.1 Kolmogorov-Smirnov Test

This test is used to decide if a sample comes from a hypothesized continuous distribution. It is based on the empirical cumulative distribution function (ECDF). Assume that we have a random sample $x_1, \dots , x_n$ from some continuous distribution with CDF F(x). The empirical CDF is denoted by [4]:

$$\boldsymbol{F_n(x) = \frac{1}{n}.[Number\ of\ observations \leq x]}.$$

The Kolmogorov-Smirnov statistic (D) is based on the largest vertical difference between $F(x)$ and $\boldsymbol{F_n(x)}$. It is defined as

$$D_n = \sup_x |F_n(x) - F(x)| \quad \text{...(11)}$$

H_0: The data follow the specified distribution.
H_1: The data do not follow the specified distribution.

The hypothesis regarding the distributional form is rejected at the chosen significance level (alpha) if the test statistic, D, is greater than the critical value obtained from a table.

2.3.2 Chi-Squared Test

The Chi-Squared test is used to determine if a sample comes from a population with a specific distribution.

The following statistic [2]:

$$\chi^2 = \sum_{i=1}^{k} \left(\frac{x_i - \mu}{\sigma_i}\right) \quad \text{.. (12)}$$

Has Chi-Squared χ^2 distribution with k degree of freedom. The steps of Chi-Squared test are as follows:

Divide the sample data into the mutually exclusive cells such the range of random variable is covered.

Determine the frequency (f_i), of sample observation in each cell.

Determine the theoretical frequency, F_i for each cell.(are a number density function between cell boundaries X_n total sample size).

From the statistic

$$S = \frac{\sum_{i=1}^{k}(f_i - F_i)}{F_i} \quad \ldots\ldots (13)$$

H_0: The data follow the specified distribution.

H_1: The data do not follow the specified distribution.

Form the χ^2 table; choose value of χ^2 with the desired significance level (α) and degree of freedom (k-1-r), where r is number of population parameters estimated.

Reject the hypothesis that the sample distribution is the same as theoretical distribution if $S > \chi^2_{1-\alpha, k-1-r}$.

3. The Application Side

In this side we will describe the data and test the distribution of the data and estimate the reliability of the machines.

3.1 Description of Study's Data

We applied this study on automatic teller machines (ATM) in Sudan.

The data on this study have been collocated for five deferent banks in Sudan selected randomly out of 28 banks.

There are three types of faults in ATM, out of journals, out of serves and out of cash; in this study we applied the data of out of journal.

Table (1) Descriptive Statistics of out of journal time for each machines

Machine code	N	Mean		Std. Deviation
	Statistic	Statistic	Std. Error	Statistic
B3	51	53.5543	6.50364	46.44527
B5	25	70.7788	12.79589	63.97943
B13	123	49.9072	4.43891	49.22987
B22	51	48.0298	6.65307	47.51243
B35	44	61.5161	8.97677	59.54515

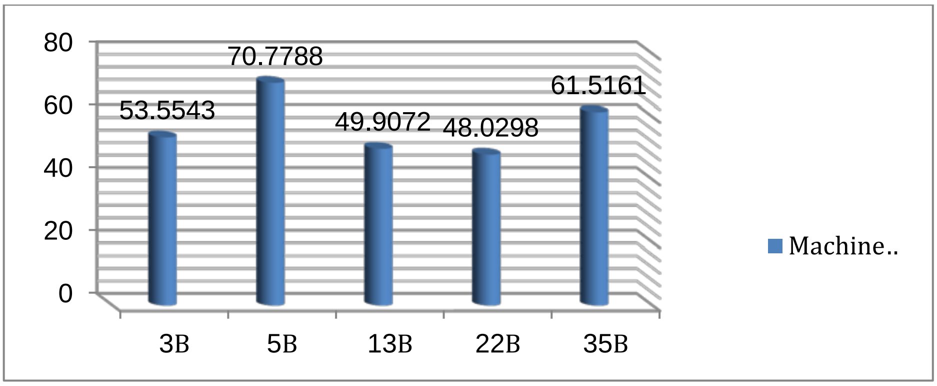


Figure (2) Descriptive Statistics of out of journal time for each machine

From table (1) and figure (2), it is shows that according to the mean values for the five machines, the machine (B5) have the highest mean failure time (out of journals) with mean the time of failure value (70.7788) hours, followed by machine (B35) with the mean time of failure value (61.5161) hours, followed by machine (B3) with the mean time of failure value (53.5543) hours, and the machine (B22) have the less mean failure time (out of journals) with the mean time of failure value (48.0298) hours.

3.2 Goodness of Fit – Details

To test the failure time data follow exponential distribution we used Kolmogorov-Smirnov test and Chi-Squared test as the following:

H_0: The failure time data follow exponential distribution

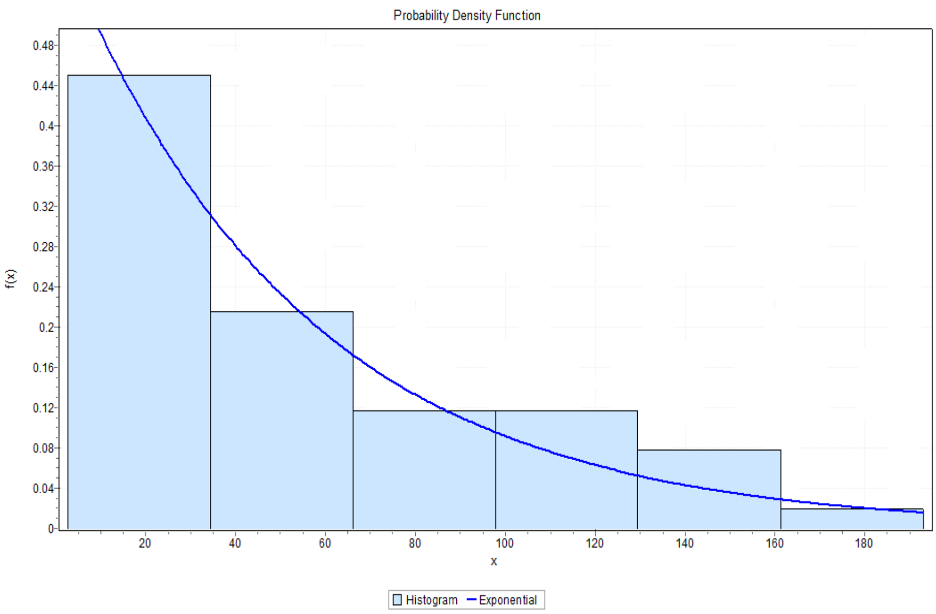
H_1: The failure time data not follow exponential distribution

3.2.1 Goodness of Fit – Details for machine (B3)

Table (2) Kolmogorov-Smirnov test and Chi-Squared test for machine (B3)

Kolmogorov-Smirnov							
Sample Size	51	α	0.2	0.1	0.05	0.02	0.01
Statistic	0.06238	Critical Value	0.14697	0.16796	0.18659	0.20864	0.22386
P-Value	0.98161	RejectH_0	No	No	No	No	No
Chi-Squared							
Deg. of freedom	5	α	0.2	0.1	0.05	0.02	0.01
Statistic	0.53138	Critical Value	7.2893	9.2364	11.07	13.388	15.086
P-Value	0.99093	RejectH_0	No	No	No	No	No

Source: the researcher from applied study, EasyFit package, 2017



Source: the researcher from applied study, EasyFit package, 2017

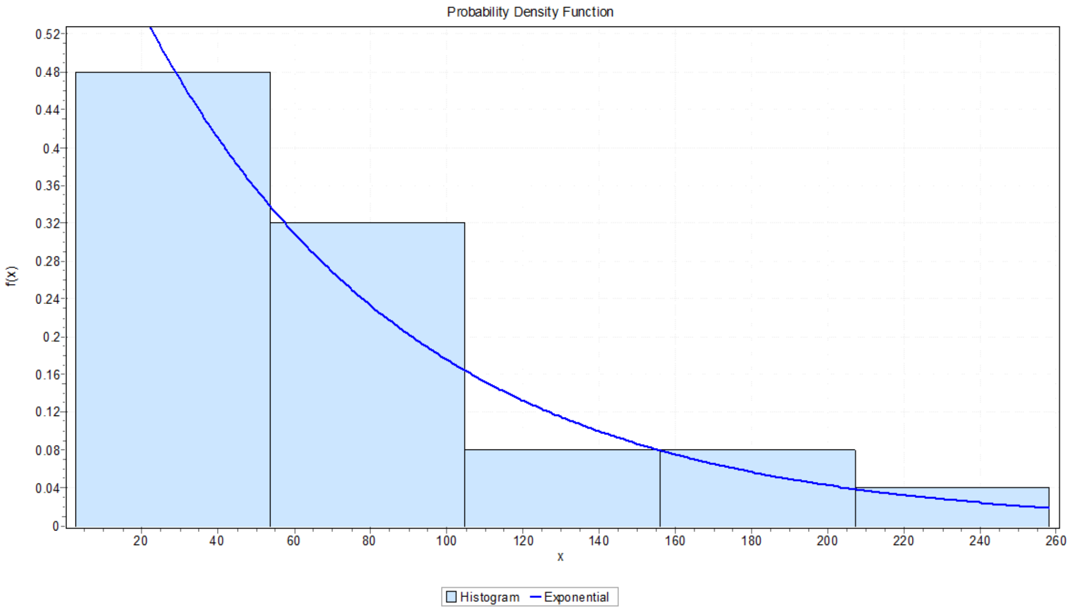
Figure (3) density function of Exponential vs. time for machine (B3)

From above table, it is obvious that the P-Value of Kolmogorov-Smirnov test and Chi-Squared test are (0.98161) and (0.99093) are greater than all significant levels (0.2,0.1,0.05,0.02 and 0.01) that mean the failure time data of machine (B3) follow exponential distribution with parameter ($\lambda = 0.01867$) and the probability density function is illustrated in figure (3).

3.2.2 Goodness of Fit – Details for machine (B5)

Table (3) Kolmogorov-Smirnov test and Chi-Squared test for machine (B5)

Kolmogorov-Smirnov							
Sample Size	25	α	0.2	0.1	0.05	0.02	0.01
Statistic	0.09653	Critical Value	0.2079	0.23768	0.26404	0.29516	0.31657
P-Value	0.95646	RejectH0	No	No	No	No	No
Chi-Squared							
Deg. of freedom	2	α	0.2	0.1	0.05	0.02	0.01
Statistic	0.48266	Critical Value	3.2189	4.6052	5.9915	7.824	9.2103
P-Value	0.78558	RejectH0	No	No	No	No	No



Source: the researcher from applied study, EasyFit package, 2017

Figure (4) density function of Exponential Vs. time for machine (B5)

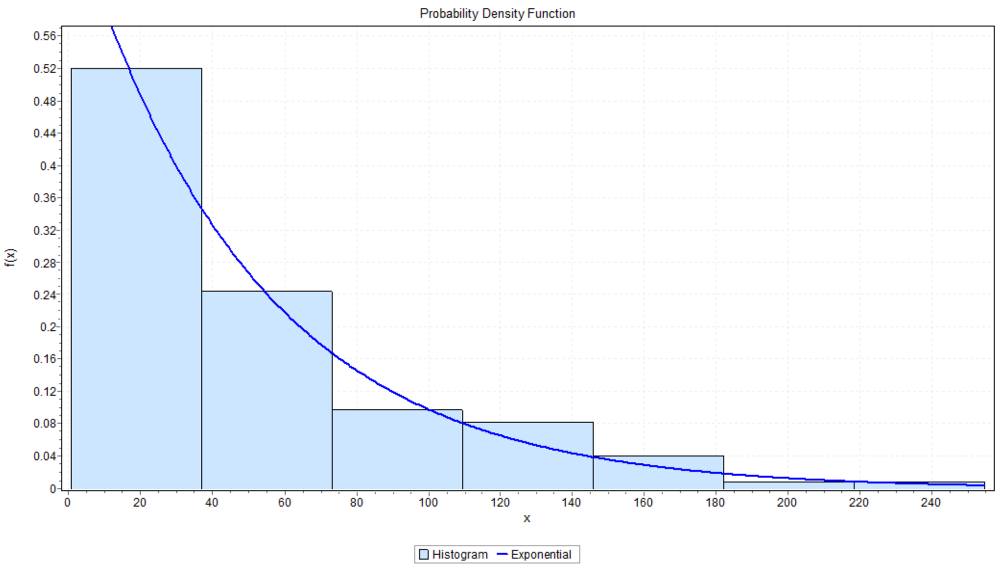
From above table, it is obvious that the P-Value of Kolmogorov-Smirnov test and Chi-Squared test are (0.95646) and (0.78558) are greater than all significant levels (0.2,0.1,0.05,0.02 and 0.01) that mean the failure time data of machine (B5) follow exponential distribution with parameter ($\lambda = 0.0141$) and the probability density function is illustrated in figure (4).

3.2.3 Goodness of Fit – Details for machine (B13)

Table (4) Kolmogorov-Smirnov test and Chi-Squared test for machine (B13)

Kolmogorov-Smirnov							
Sample Size	123	α	0.2	0.1	0.05	0.02	0.01
Statistic	0.06647	Critical Value	0.09675	0.11027	0.12245	0.13687	0.14688
P-Value	0.62448	RejectH_0	No	No	No	No	No
Chi-Squared							
Deg. of freedom	6	α	0.2	0.1	0.05	0.02	0.01
Statistic	1.1613	Critical Value	8.5581	10.645	12.592	15.033	16.812
P-Value	0.97875	RejectH_0	No	No	No	No	No

Source: the researcher from applied study, EasyFit package, 2017



Source: the researcher from applied study, EasyFit package, 2017

Figure (5) density function of Exponential Vs. time for machine (B13)

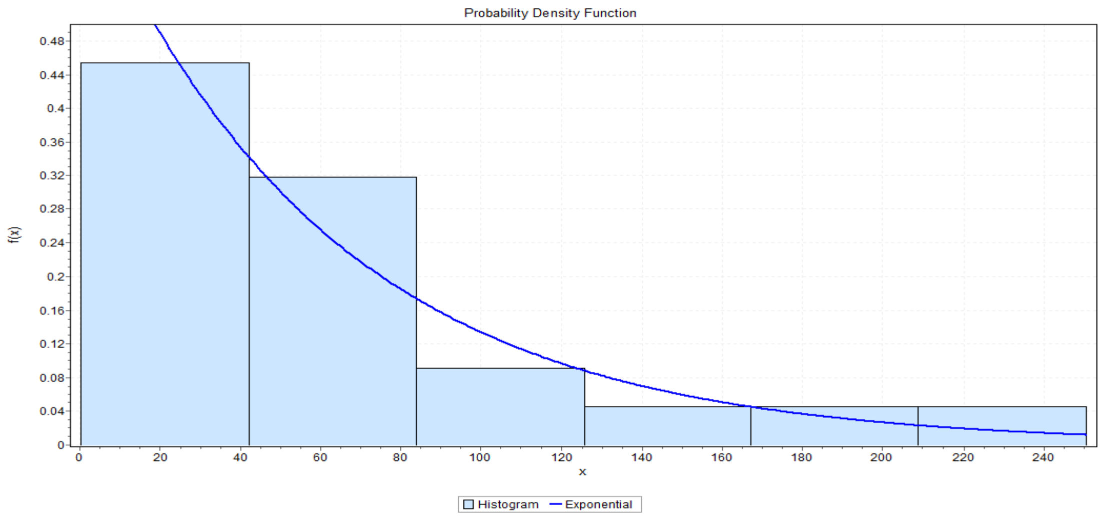
From above table, it is obvious that the P-Value of Kolmogorov-Smirnov test and Chi-Squared test are (0.62448) and (0.97875) are greater than all significant levels (0.2,0.1,0.05,0.02 and 0.01) that mean the failure time data of machine (B13) follow exponential distribution with parameter(λ=0.02004), and the probability density function is illustrated in figure (5).

3.2.4 Goodness of Fit – Details for machine (B22)

Table (5) Kolmogorov-Smirnov test and Chi-Squared test for machine (B22)

Kolmogorov-Smirnov							
Sample Size	51	α	0.2	0.1	0.05	0.02	0.01
Statistic	0.07683	Critical Value	0.14697	0.16796	0.18659	0.20864	0.22386
P-Value	0.9015	RejectH0	No	No	No	No	No
Chi-Squared							
Deg. of freedom	5	α	0.2	0.1	0.05	0.02	0.01
Statistic	1.6685	Critical Value	7.2893	9.2364	11.07	13.388	15.086
P-Value	0.89285	RejectH0	No	No	No	No	No

Source: the researcher from applied study, EasyFit package, 2017



Source: the researcher from applied study, EasyFit package, 2017

Figure (6) density function of Exponential Vs. time for machine (B22)

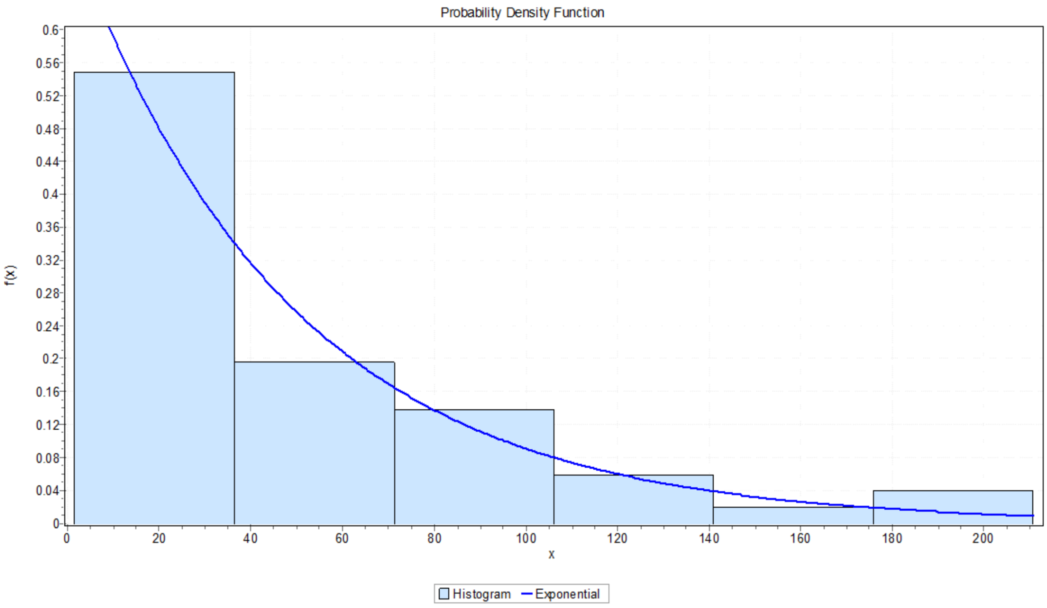
From above table, it is obvious that the P-Value of Kolmogorov-Smirnov test and Chi-Squared test are (0.9015) and (0.89285) are greater than all significant levels (0.2, 0.1, 0.05, 0.02 and 0.01) that mean the failure time data of machine (B22) follow exponential distribution with parameter (λ= 0.02082), and the probability density function is illustrated in figure (6).

3.2.5 Goodness of Fit – Details for machine (B35)

Table (6) Kolmogorov-Smirnov test and Chi-Squared test for machine (B35)

Kolmogorov-Smirnov							
Sample Size	44	α	0.2	0.1	0.05	0.02	0.01
Statistic	0.0725	Critical Value	0.15796	0.18053	0.20056	0.22426	0.2406
P-Value	0.96189	RejectH0	No	No	No	No	No
Chi-Squared							
Deg. of freedom	4	α	0.2	0.1	0.05	0.02	0.01
Statistic	3.6612	Critical Value	5.9886	7.7794	9.4877	11.668	13.277
P-Value	0.45379	RejectH0	No	No	No	No	No

Source: the researcher from applied study, EasyFit package, 2017



Source: the researcher from applied study, EasyFit package, 2017

Figure (7) density function of Exponential Vs. time for machine (B35)

From above table, it is obvious that the P-Value of Kolmogorov-Smirnov test and Chi-Squared test are (0.96189) and (0.45379) are greater than all significant levels (0.2,0.1,0.05,0.02 and 0.01) that mean the failure time data of machine (B35) follow exponential distribution with parameter (λ=0.01626), and the probability density function is illustrated in figure (7).

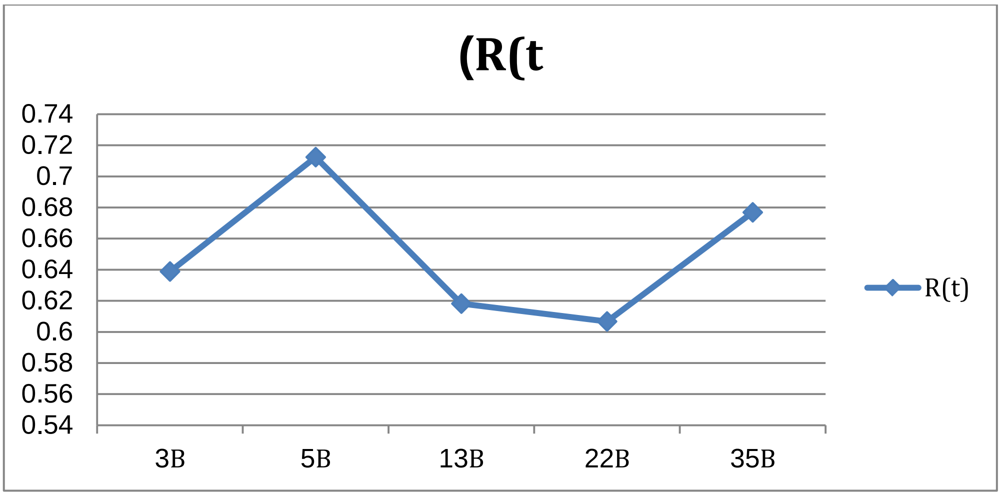
3.3 Reliability Model of Machines

The Reliability model of machines have been conducted from all machines for a period of time (24 hours) and the following measure have been calculated:

Table (7) Result of Reliability model test of machines at time (t=24) hours

Machine code	Measure		
	f(t)	R(t)	h(t)
B3	0.01193	0.63885	0.01867
B5	0.01007	0.7124	0.01413
B13	0.01239	0.61819	0.02004
B22	0.01263	0.60672	0.02082
B35	0.01101	0.67689	0.01626

Source: the researcher from applied study, EasyFit package, 2017



Source: the researcher from applied study, Excel package, 2017

Figure (8) Result of Reliability model test of machines at time (t=24)

From above table and figure, it is obvious that according to the reliability values for the five machines, the machine (B5) have the highest reliability R(t=24=0.7124) with probability fault (f(t=24=0.01007) and hazard rate h(t=24= 0.01413), this mean that the probability of work (24) hours without fault for machine (B5) is (0.7124) followed by machine (B35) with R(t=24=0.67689) and probability fault (f(t=24=0.01101) and hazard rate h(t=24= 0.01626), this mean that the probability of work (24) hours without fault for machine (B35) is (0.67689), followed by machine (B3) with

R(t=24=0.63885) and probability fault f(t=24=0.01193) and hazard rate h(t=24= 0.01867), this mean that the probability of work (24) hours without fault for machine (B3) is (0.63885)

Table (8) Result of Reliability model test of machines

Code	Masseur	Time/hour						
		0	24	48	72	96	120	144
B3	R(t)	1	0.63885	0.40813	0.26074	0.16657	0.10642	0.06798
	h(t)	0.01867	0.01867	0.01867	0.01867	0.01867	0.01867	0.01867
	Cum.h(t)	0.00	0.44808	0.89616	1.3442	1.7923	2.2404	2.6885
B5	R(t)	1.00	0.7124	0.50824	0.36233	0.25831	0.18415	0.13128
	h(t)	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141	0.0141
	Cum.h(t)	0.00	0.3384	0.6768	1.0152	1.3536	1.692	2.0304
B13	R(t)	1.00	0.61819	0.38216	0.23625	0.14605	0.09028	0.05581
	h(t)	0.02004	0.02004	0.02004	0.02004	0.02004	0.02004	0.02004
	Cum.h(t)	0.00	0.48096	0.96192	1.4429	1.9238	2.4048	2.8858
B22	R(t)	1.00	0.60672	0.36811	0.22334	0.13551	0.08222	0.04988
	h(t)	0.02082	0.02082	0.02082	0.02082	0.02082	0.02082	0.02082
	Cum.h(t)	0.00	0.49968	0.99936	1.499	1.9987	2.4984	2.9981
B35	R(t)	1.00	0.67689	0.45819	0.31014	0.20993	0.1421	0.09619
	h(t)	0.01626	0.01626	0.01626	0.01626	0.01626	0.01626	0.01626
	Cum.h(t)	0.00	0.39024	0.78048	1.1707	1.561	1.9512	2.3414

Source: the researcher from applied study, EasyFit package, 2017

From above table (8), it is obvious that the reliability of machines decreases whenever the working time of the machines increases.

Fore machine (B3), at time (t=0) hour the reliability is (100%), at time (t=24) hours the reliability is about (63%), at time (t=48) hours the reliability is about (41%), at time (t=72) hours the reliability is about (26%), at time (t=96) hours the reliability is about (17%), at time (t=120) hours the reliability is about (11%)and at time (t=144) hours the reliability is about(7%).

Fore machine (B5), at time (t=0) hour the reliability is (100%), at time (t=24) hours the reliability is about (71%), at time (t=48) hours the reliability is about (51%), at time (t=72) hours the reliability is about (36%), at time (t=96) hours the reliability is about (26%), at time (t=120) hours the reliability is about (18%) and at time (t=144) hours the reliability is about (13%).

Fore machine (B13), at time (t=0) hour the reliability is (100%), at time (t=24) hours the reliability is about (62%), at time (t=48) hours the reliability is about (38%), at time (t=72) hours the reliability is about

(24%), at time (t=96) hours the reliability is about (15%), at time (t=120) hours the reliability is about (9%) and at time (t=144) hours the reliability is about (6%).

Fore machine (B22), at time (t=0) hour the reliability is (100%), at time (t=24) hours the reliability is about (61%), at time (t=48) hours the reliability is about (37%), at time (t=72) hours the reliability is about (22%), at time (t=96) hours the reliability is about (14%), at time (t=120) hours the reliability is about (8%) and at time (t=144) hours the reliability is about (5%).

Fore machine (B35), at time (t=0) hour the reliability is (100%), at time (t=24) hours the reliability is about (68%), at time (t=48) hours the reliability is about (46%), at time (t=72) hours the reliability is about (31%), at time (t=96) hours the reliability is about (21%), at time (t=120) hours the reliability is about (14%) and at time (t=144) hours the reliability is about (10%).

4. Conclusions

The main findings of this study are:

1. Time of failures follows exponential distribution with one parameter for all sample machines.
2. When operation time of machines increase the performance decreased or the machine got fault.
3. The hazard rate of machines is constant according to the time.
4. The machines with high reliability have a low faults probability and hazard rate.
5. The machines with low reliability have a high fault probability and hazard rate.
6. Whenever mean time between failures for machines increase that indicate the machine has high reliability.
7. All machines have high reliability but the machine (B5 and B35) have the highest reliability.

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Appendix

The technical fault data for the five machines collocated from the Sudan central Bank, during the period (1/1/2017-30/6/2017).

B35	B22	B13	B5	B3
65.97	5.23	10.96	54.93	11.10
128.05	83.27	0.59	63.44	86.68
45.94	28.94	34.30	24.24	60.46
14.27	150.27	11.17	116.59	16.03
250.71	15.88	60.33	12.17	24.87
168.86	117.50	30.75	37.11	51.79
17.26	6.11	18.80	57.24	33.74
0.24	92.54	2.15	178.40	2.58
29.75	68.75	12.27	53.18	29.38
10.98	45.57	68.69	28.00	67.27
36.91	25.19	83.87	79.69	63.31
2.55	50.05	8.24	203.47	2.72
51.77	4.01	97.99	10.39	40.23
177.25	10.99	9.08	15.58	100.32
2.70	1.35	113.94	92.97	43.51
62.52	28.04	106.12	2.70	120.07
40.67	5.59	17.80	26.80	14.48
2.64	32.18	20.14	88.00	78.81
99.05	70.99	31.71	91.95	28.96
10.34	106.75	12.63	21.60	5.54
136.38	27.21	49.83	20.51	131.65
72.39	12.07	41.49	258.14	35.51
4.66	15.22	11.79	64.11	53.28
10.85	46.78	75.20	48.35	78.00
90.80	23.06	55.82	119.91	69.44
49.87	2.19	19.50		99.53
38.77	210.93	60.77		140.60
19.02	186.48	137.75		21.89
76.26	83.57	103.54		40.48
11.39	13.53	2.70		102.55
98.15	25.00	5.44		12.02
49.09	23.34	168.42		34.98
244.67	26.01	2.34		8.34

32.84	18.14	9.57	118.52
81.24	46.88	159.16	98.44
67.96	9.62	9.81	9.28
9.28	93.49	45.67	193.00
69.66	19.10	14.01	4.90
54.17	77.83	23.77	148.36
52.27	86.91	51.17	17.61
25.51	11.92	30.88	17.73
93.48	37.46	131.10	23.46
67.03	12.39	10.67	5.08
32.54	24.90	40.86	66.36
	19.62	16.26	57.37
	39.07	4.89	13.46
	128.43	112.39	14.27
	46.35	139.58	10.96
	45.34	18.36	50.62
	81.82	21.50	24.43
	5.66	29.24	147.30
		7.23	
		34.14	
		61.84	
		133.52	
		86.69	
		90.27	
		41.00	
		3.82	
		13.66	
		57.46	
		73.06	
		4.45	
		5.15	
		10.32	
		6.12	
		12.88	
		149.61	
		10.05	
		17.37	
		40.40	
		5.59	
		4.30	
		27.28	
		169.07	
		42.57	
		72.43	
		17.48	
		48.80	
		20.74	

174.88
50.93
62.90
5.69
107.31
45.74
39.51
49.10
110.38
17.78
14.01
72.86
84.73
86.65
20.36
20.67
1.82
39.03
24.10
112.34
31.30
11.25
37.80
114.68
2.81
47.47
52.77
254.70
32.92
62.71
1.89
36.89
116.33
6.40
28.36
14.62
63.04
76.41
19.49
2.18
71.26
205.42
84.78

Source: Sudan central Bank, 2017

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