"Science Stays True Here"
Journal of Mathematics and Statistical Science, 54-75 | Science Signpost Publishing

Second Order Bias Corrected Efficient GMM Estimator

B F Chakalabbi[1], Sagar Matur[2] and Sanmati Neregal[3],

[1,2,3] *Department of Statistics, Karnatak University's Karnatak Arts College, Dharwad - 580001, India*

[1]bfckcd@gmail.com

[2]matursagar@gmail.com

[3]naregalsanmati@gmail.com

Abstract

In this paper, the three conventional GMM estimators First-difference, Level and System GMM estimators with respective efficient initial weight matrices are considered to estimate the autoregressive panel data model. It is observed that the bias of first-difference GMM estimator is heigher and the bias of system GMM estimator is lesser among the above mentioned estimators but as variance ratio increases and as autoregressive parameter approaches to one the bias of all the aforementioned estimators increase. Hence to reduce such bias, second order bias correction method is considered. Through Monte-Carlo simulation it is observed that the considered second order bias correction method works well for first-difference and system GMM estimators, specially when the variance ratio is greater than one.

Mathematics Subject Classification: 62Fxx, 91Bxx.
Keywords: *First-difference GMM estimator, Level GMM estimator, System GMM estimator, Second order bias.*

1. Introduction

The commonly employed technique to estimate the parameters of autoregressive panel data models with unobserved individual specific effects is first-difference GMM estimator proposed by Arellano and Bond(1991), which is based on lagged level variables as an instruments. Arellano and Bover (1995) proposed level GMM estimator, which is based on the level of the model and uses lagged differenced variables as instruments. Blundell and Bond (1998) proposed system GMM estimator, which is proposed by combining both the instruments of first-difference and level GMM estimators. Unfortunately, the system GMM estimator does not work well in terms of bias. Bun and Kiviet(2006) showed that the bias of the system GMM estimator becomes large when the autoregressive parameter is close to unity and when the ratio of the variance of individual effect to that of the idiosyncratic error term departs from unity. Hence to reduce that bias they derived the second order bias of the system GMM estiamtor and examined how the order of magnitude of bias changes when different set of instruments are used.

Kiviet (1995), Hahn and Kuersteiner (2002) and many others have constructed estimators for autoregressive panel data models by modifying the within class estimator. Bun and Carree (2005) estimator overcomes some of the drawbacks of GMM estimators. Their estimator mainly depends on the assumption that the regressors are strictly exogenous with respect to the error term.

Hayakawa(2007) proposed second order bias method and considered $W_d = Z_d' Z_d$ as an initial weight matrix for one-step first-difference GMM estimator, $W_l = Z_l^{nr'} Z_l^{nr}$, where Z_l^{nr} is a non-redundant subset of Z_l and for one-step system GMM estimator, $W_s = Z_s' Z_s$ is considered as an initial weight matrix. Through this he showed that why system GMM estimator gives less bias than the first-difference and level GMM estimators, though it uses more number of instruments.

In this paper with the same approach to check how well the second order bias explains the actual bias for above mentioned estimators, we consider the weight matrix $W_d = N^{-1} \sum_{i=1}^{N} Z_{di}' A_d Z_{di}$ as an efficient initial weight matrix for one-step first-difference GMM estimator, which is efficient when the idiosyncratic errors are homoscedastic and are not serially correlated, where A_d is a $(T-2) \times (T-2)$ matrix with twos in main diagonal, minus ones in first subdiagonals and zeros otherwise (See, Arellano and Bond(1991)). The efficient initial weight matrix for one-step level GMM estimator is given by $W_l = N^{-1} \sum_{i=1}^{N} Z_{li}' A_l Z_{li}$, where A_l is a $(T-2) \times (T-2)$ identity matrix (See, Arellano and Bover(1995)). The initial weight matrix $W_s = \sum_{i=1}^{N} Z_{si}' A_s Z_{si}$ is considered for one-step system GMM estimator, where A_s is a $2(T-2) \times 2(T-2)$ matrix and is given by, $A_s = \begin{bmatrix} A_d & 0 \\ 0 & A_l \end{bmatrix}$ (See, Blundell and Bond(1998)).

The rest of the paper is as follows. Section 2 provides the autoregressive panel data model and its assumptions. Second order bias corrected first-difference, level and system GMM estimators are discussed in Section 3. Section 4 provides Monte Carlo simulation design and discussions. Section 5 presents some concluding remarks. Appendix comprised of some intermediate results and proofs is given in Section 6. Section 7 contains Tables and Figures.

2. The Model and Estimators

We consider first order autoregressive panel data model and is given by

$$\begin{aligned} y_{it} &= \delta y_{it-1} + u_{it}, \\ u_{it} &= \eta_i + v_{it}, \qquad i = 1, 2, \ldots, N; t = 2, 3, \ldots, T. \end{aligned} \tag{2.1}$$

where δ is the parameter of interest with $|\delta| < 1$, N is the number of individuals, T is the time period and we make the following assumptions:

(i) v_{it} are idiosyncratic error terms which are i.i.d across time and individuals with $E(v_{it}) = 0$, $Var(v_{it}) = \sigma_v^2$ and $E(v_{it} v_{is}) = 0$ where $t \neq s$.

(ii) η_i are individual effects which are i.i.d across individuals with $E(\eta_i) = 0$, $Var(\eta_i) = \sigma_\eta^2$. and $E(\eta_i v_{it}) = 0$.

(iii) The initial observations satisfy

$$y_{i1} = \frac{\eta_i}{1-\delta} + w_{i1} \qquad for(i = 1, 2, \ldots, N) \tag{2.2}$$

where, $w_{i1} = \sum_{j=0}^{\infty} \delta^j v_{i1-j}$ and independent of η_i. y_{i1} is not independent of η_i i.e., $E(y_{i1}\eta_i) = \frac{\sigma_\eta^2}{1-\delta}$.

Based on these assumptions, we review three types of GMM estimators viz., first-difference, level and system GMM estimators (For more information See Appendix).

3. Second Order Bias

In this section, we provide analytical forms of the biases of the first-difference, level and system GMM estimators (Here we consider the case for T=4, notations being similar to that of Hayakawa (2007) except for the autoregressive parameter).

We consider the general one-step GMM estimator based on the following moment condition $E[g_i(\delta_0)] = 0$. An efficient one-step GMM estimator is defined as

$$\hat{\delta} = argmin \;\; \hat{g}(\delta)' W^{-1} \hat{g}(\delta) \tag{3.1}$$

where $\hat{g}(\delta) = N^{-1}\sum_{i=1}^{N} g_i(\delta), \hat{W} = N^{-1}\sum_{i=1}^{N} W_i$ and $W_i = W(z_i)$ are symmetric and positive definite matrices which do not depend on parameter δ.

Theorem 1: The Second order bias for the one-step GMM estimator is given by

$$\begin{aligned}
N.Bias(\hat{\delta}) =& \frac{E(G'W^{-1}W_iW^{-1}g_i)}{G'W^{-1}G} - \frac{trace(W^{-1}E(g_iG_i'))}{G'W^{-1}G} + 2\frac{G'W^{-1}E(g_iG_i')W^{-1}G}{(G'W^{-1}G)^2} \\
&- \frac{E(G'W^{-1}g_iG'W_iW^{-1}G)}{(G'W^{-1}G)^2} + \frac{G'W^{-1}\Omega W^{-1}G_\delta}{(G'W^{-1}G)^2} \\
&- 2\frac{G'W^{-1}\Omega W^{-1}E(G_1)W^{-1}G}{(G'W^{-1}G)^2} - \frac{3}{2}\frac{(G'W^{-1}\Omega W^{-1}G)(G'W^{-1}G_\delta)}{(G'W^{-1}G)^3} \\
&+ \frac{trace(W^{-1}E(G_1)W^{-1}\Omega)}{G'W^{-1}G} + \frac{3}{2}\frac{(G'W^{-1}E(G_1)W^{-1}G)(G'W^{-1}\Omega W^{-1}G)}{(G'W^{-1}G)^3} \\
=& \;\; B1 + B2 + B3 + B4 + B5 + B6 + B7 + B8 + B9
\end{aligned} \tag{3.2}$$

where

$$G_i = \frac{\partial g_i(\delta_0)}{\partial \delta} \qquad G = E(G_i) \qquad G_\delta = E\left(\frac{\partial^2 g_i(\delta_0)}{\partial \delta^2}\right) \qquad \Omega = E[g_i(\delta_0)g_i(\delta_0)']$$

Proof is same as Theorem 1 in Hahn, Hausman and Kuersteiner (2001).

The weight matrix considered in this study does not involve the parameter, which implies $E(G_1) = 0$ so sixth, eighth and ninth terms of Theorem 1 equal to zero. Also, the moment restriction is linear in the parameter in interest, which implies $G_\delta = 0$ so fifth and seventh terms of Theorem 1 equal to zero.
Therefore,

$$N.Bias(\hat{\delta}) = \;\; B1 + B2 + B3 + B4 \tag{3.3}$$

Let,

$$
\begin{aligned}
\pi_d =& (E(Z'_{di}A_dZ_{di}))^{-1}E(Z'_{di}\Delta y_{i,-1}) \\
=& \begin{bmatrix} \frac{-\sigma_v^2(2+\delta)}{3(1+\delta)(C+D)} & \frac{-\sigma_v^2[2F+\delta[4(C+D)^2-(C+\delta D)^2]-3(C+D)(C+\delta D)]}{6(1+\delta)(C+D)F} & \frac{-\sigma_v^2[(C+D)-\delta(C+\delta D)]}{2(1+\delta)F} \end{bmatrix}' \\
=& \begin{bmatrix} \pi_{d1} & \pi_{d2} & \pi_{d3} \end{bmatrix}' \\
\pi_l =& (E(Z'_{li}A_lZ_{li}))^{-1}E(Z'_{li}y_{i,-1}) \\
=& \begin{bmatrix} \frac{1}{2} & \frac{\delta+1}{[4-(\delta-1)^2]} & \frac{(2-\delta)(\delta+1)}{[4-(\delta-1)^2]} \end{bmatrix}' \\
=& \begin{bmatrix} \pi_{l1} & \pi_{l2} & \pi_{l3} \end{bmatrix}'
\end{aligned}
$$

where

$$
C = \frac{\sigma_\eta^2}{1-\delta^2}, \quad D = \frac{\sigma_v^2}{(1-\delta)^2} \quad and \quad F = (C+D)^2 - (C+\delta D)^2.
$$

Also,

$$
\begin{aligned}
\phi_d =& E(\Delta y'_{i,-1}Z_{di})(E(Z'_{di}A_dZ_{di}))^{-1}E(Z'_{di}\Delta y_{i,-1}) \\
=& -\frac{\sigma_v^2}{1+\delta}[\pi_{d1} + \delta\pi_{d2} + \pi_{d3}]
\end{aligned}
$$

$$
\begin{aligned}
\phi_l =& E(y'_{i,-1}Z_{li})(E(Z'_{li}A_lZ_{li}))^{-1}E(Z'_{li}y_{i,-1}) \\
=& \frac{\sigma_v^2}{2(1+\delta)} + \frac{2\sigma_v^2}{[4-(\delta-1)^2]}
\end{aligned}
$$

$$
\begin{aligned}
\phi_s =& E(s'_{i,-1}Z_{si})(E(Z'_{si}A_sZ_{si}))^{-1}E(Z'_{si}s_{i,-1}) \\
=& \frac{\sigma_v^2}{(1+\delta)}[1 - \pi_{d1} - \delta\pi_{d2} - \pi_{d3}]
\end{aligned}
$$

Next, the second order biases of one-step first-difference, level and system GMM estimators are provided. The derivations are provided in the appendix.

Theorem 2: The second order bias of one-step first-difference GMM estimator is given by (using 6.4.1 - 6.4.4)

$$
\begin{aligned}
N.Bias(\hat{\delta}_{dif}) =& \; B_{1dif} + B_{2dif} + B_{3dif} + B_{4dif} \\
=& -\frac{\sigma_v^2}{6\phi_d}(7+2\delta) + \frac{2\sigma_v^2}{\phi_d^2}\left[(\pi_{d1}^2 + \pi_{d2}^2 + \pi_{d3}^2)(C+D) + \pi_{d1}\pi_{d2}(\delta-2)(C+D)\right. \\
& \left. + 2\pi_{d2}\pi_{d3}(C+\delta D) - \pi_{d1}\pi_{d3}[(2-\delta)C - \delta(2\delta-3)D]\right] \\
& - \frac{2\sigma_v^2}{\phi_d^2}\left[(2\pi_{d1}\pi_{d2}\pi_{d3} - \pi_{d1}^2\pi_{d3})(C+D) + 2\pi_{d1}\pi_{d3}^2(C+\delta D)\right]
\end{aligned}
$$

Theorem 3: The second order bias of one-step level GMM estimator is given by (using 6.5.1 - 6.5.4)

$$\begin{aligned}
N.Bias(\hat{\delta}_{lev}) &= B_{1lev} + B_{2lev} + B_{3lev} + B_{4lev} \\
&= \frac{3\sigma_\eta^2}{\phi_l(1-\delta)} + \frac{-2}{\phi_l^2}\left[\frac{2\sigma_\eta^2\sigma_v^2}{(1-\delta^2)}\left\{\pi_{l1}^2 + \pi_{l2}^2 + \pi_{l3}^2 + 2\pi_{l1}\pi_{l2} - (1-\delta)\pi_{l1}\pi_{l3} - (1-\delta)\pi_{l2}\pi_{l3}\right\}\right. \\
&\left. + \frac{\sigma_v^4}{1+\delta}\left\{(2\delta-1)\pi_{l1}\pi_{l3} + 2\pi_{l1}\pi_{l2}\right\}\right] + \frac{2\sigma_v^4}{\phi_l^2(1+\delta)}\left[2\pi_{l1}\pi_{l2}\pi_{l3} + (\delta-1)\pi_{l1}\pi_{l3}^2\right]
\end{aligned}$$

Theorem 4: The second order bias of one-step system GMM estimator is given by (using 6.6.1 - 6.6.4)

$$\begin{aligned}
N.Bias(\hat{\delta}_{sys}) &= B_{1sys} + B_{2sys} + B_{3sys} + B_{4sys} \\
&= \frac{1}{\phi_s}\left[\frac{2\sigma_\eta^2}{1-\delta} - \frac{\sigma_v^2(7+2\delta)}{6}\right] + \frac{\phi_d^2}{\phi_s^2}B_{3dif} - \frac{1}{\phi_s^2}\left[\frac{2\pi_{d1}\sigma_\eta^2\sigma_v^2}{1-\delta} - \frac{\pi_{d2}\delta\sigma_\eta^2\sigma_v^2}{(1-\delta)^2} + \frac{\sigma_v^4(2\delta-1)}{2(1+\delta)}\right. \\
&+ \frac{\sigma_\eta^2\sigma_v^2}{1-\delta} - \frac{\pi_{d1}\sigma_v^4[2\delta-1+\delta(2\delta^2-4\delta+1)]}{1-\delta^2} - \frac{\pi_{d2}\sigma_v^4(2\delta^2-2\delta+1)}{1-\delta^2} - \frac{\pi_{d3}\delta\sigma_\eta^2\sigma_v^2}{(1-\delta)^2} \\
&\left. + \frac{\pi_{d3}\sigma_v^4(2-3\delta)}{1-\delta^2}\right] + \frac{\phi_d^2}{\phi_s^2}B_{4dif} + \frac{1}{\phi_s^2}\left[\frac{(\delta-1)\sigma_v^4}{4(\delta+1)} + \frac{\pi_{d1}\sigma_v^4(\delta^2-2\delta+1)}{2(1+\delta)} - \frac{2\pi_{d1}^2\sigma_\eta^2\sigma_v^2}{1-\delta}\right. \\
&+ \frac{\pi_{d2}\delta\sigma_v^4}{2(1+\delta)} - \frac{2\pi_{d2}^2\sigma_\eta^2\sigma_v^2}{1-\delta} + \frac{\pi_{d1}\pi_{d3}\sigma_\eta^2\sigma_v^2}{1-\delta} + 2\frac{\pi_{d1}\pi_{d2}\sigma_\eta^2\sigma_v^2}{1-\delta} - 2\frac{\pi_{d2}\pi_{d3}\sigma_\eta^2\sigma_v^2}{1-\delta} - 2\frac{\pi_{d3}^2\sigma_\eta^2\sigma_v^2}{1+\delta} \\
&\left. + \frac{\pi_{d3}\sigma_v^4}{2(1+\delta)}\right]
\end{aligned}$$

Theorem 5: The second order bias corrected one-step first-difference, level and system GMM estimators are respectively given by

$$\begin{aligned}
\hat{\hat{\delta}}_{dif} &= \hat{\delta}_{dif} - Bias(\hat{\delta}_{dif}) \\
\hat{\hat{\delta}}_{lev} &= \hat{\delta}_{lev} - Bias(\hat{\delta}_{lev}) \\
\hat{\hat{\delta}}_{sys} &= \hat{\delta}_{sys} - Bias(\hat{\delta}_{sys})
\end{aligned}$$

where

$$\begin{aligned}
Bias(\hat{\delta}_{dif}) &= \frac{B_{1dif} + B_{2dif} + B_{3dif} + B_{4dif}}{N} \\
Bias(\hat{\delta}_{lev}) &= \frac{B_{1lev} + B_{2lev} + B_{3lev} + B_{4lev}}{N} \\
Bias(\hat{\delta}_{sys}) &= \frac{B_{1sys} + B_{2sys} + B_{3sys} + B_{4sys}}{N}
\end{aligned}$$

4. Simulation Design

To investigate the finite sample perferomance of the above mentioned estimators we carry out the Monte Carlo simulation. The data generating process for the autoregressive panel data model is as follows

$$y_{it} = \delta y_{it-1} + \eta_i + v_{it}$$
$$y_{i1} = \frac{\eta_i}{1-\delta} + w_{i1}$$
$$w_{i1} \sim N\left(0, \frac{\sigma_v^2}{1-\delta^2}\right);\ \ \eta i \sim N\left(0, \sigma_\eta^2\right);\ \ v_{it} \sim N(0, \sigma_v^2)$$

for the number of individuals $N = 50$, time period $T = 4$, variance of idiosyncratic error term $\sigma_v^2 = 1$, values of parameter $\delta = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$ and variance of individual effects $\sigma_\eta^2 = \left\{\frac{1}{4}, 1, 4\right\}$ and the results are based on 10000 replicatons.

4.1 Discussions

Table 1 presents Mean, Bias and Root Mean Square Errors (RMSEs) of one-step first-difference, level and system GMM estimators. Figures 1 and 2 show the comparison between first-difference, level and system GMM estimators in terms of bias and RMSE repectively. As δ approaches unity and as the variance ratio $(vr = \sigma_\eta^2/\sigma_v^2)$ increases, the bias of all the above estimators increase. When $vr \leq 1$ and for $\delta \leq 0.5$ the bias of level and system GMM estimators are almost same. For different values of δ and vr the bias of system estimator is less than the bias of other two estimators. From figure 2 it is observed that for different values of δ and for different values of vr, the RMSE of system GMM estimator is less than the first-difference and level GMM estimators.

As of Hayakawa (2007), it is observed that how well the second order bias explains the actual bias by comparing relative biases of actual bias and second order bias. Table 2, 3 and 4 describe the relative biases of actual and second order biases of first-difference, level and system GMM estimators for different values of δ and vr. Relative bias is given by $Bias(\hat{\delta})/\delta \times 100$. From Table 2 it is observed that, for different values of vr the relative biases of actual bias and second order bias of first-difference GMM estimator are very close to each other when $\delta \leq 0.8$ (Figure 3). Where as in case of level GMM estimator (Table 3), when $vr = 1/4$ the relative biases of actual bias and second order bias are close when $\delta \leq 0.5$. As vr increases, the actual bias does not explain second order bias well for any values of δ (Figure 4). From Table 4 it is observed that for $vr \leq 1$, the relative biases of actual bias and second order bias of system GMM estimator are close when $\delta \leq 0.5$. For $vr = 4$, the relative biases of actual bias and second order bias of system GMM estimator are close for all values of δ (Figure 5).

Table 5 describes the comparison between first-difference GMM estimator's actual mean, bias and RMSE and the second order bias corrected first-difference GMM estimator's mean, bias and RMSE. Similarly, Table 6 compares system GMM estimator's actual mean, bias and RMSE with that of the second order bias corrected system GMM estimator. Second order bias corrected estimator gives less bias and less RMSE compared to actual bias and RMSE.

5. Conclusion

In this paper, the three conventional GMM estimators first-difference, level and system GMM estimators with respective efficient weight matrices are considered to estimate the autoregressive panel data model. It is observed that first-difference GMM estimator gives more bias than the other two estimators for different values of δ and vr, whereas, system GMM estimator has least bias and RMSE among the three. But for the case $vr > 1$ and δ approaches unity, bias and RMSE of system GMM estimator increase. So this study performs second order bias correction to reduce the actual bias and RMSE of first-difference and system GMM estimators.

References

[1] M. Arellano and S. Bond, Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations. Review of Economic Studies 58 (1991) 277-298.

[2] M. Arellano and O. Bover, Another Look at the Instrumental Variable Estimation of Error-Components Models. Journal of Econometrics 68 (1995) 29-51.

[3] R. Blundell and S. Bond, Initial Conditions and Moment Restrictions in Dynamic Panel Data Models. Journal of Econometrics 87 (1998) 115-143.

[4] R. Blundell, S. Bond and F. Windmeijer, Estimation in dynamic panel data models: improving on the performance of the standard GMM estimator. IFS Working Papers No. W00/12 (2000). http://dx.doi.org/10.1920/wp.ifs.2000.0012.

[5] M. Bun and M. Carree, Bias-Corrected Estimation in Dynamic Panel Data Models. Journal of Business and Economic Statistics 23 (2005) 200-210.

[6] M. Bun and J. Kiviet, The Effects of Dynamic Feedbacks on LS and MM Estimator Accuracy in Panel Data Models. Journal of Econometrics 132 (2006) 409-444.

[7] J. Hahn, J. Hausman and G. Kuersteiner, Bias corrected instrumental variables estimation for dynamic panel models with fixed effects. MIT Department of Economics Working Paper 01-24 (2001).

[8] J. Hahn and G. Kuersteiner, Asymptotically unbiased inference for a dynamic panel model with fixed effects when both n and T are large. Econometrica 70 (2002) 1639-1657.

[9] K. Hayakawa, Small sample bias properties of the system GMM estimator in dynamic panel data models. Economics Letters 95 (2007) 32-38.

[10] J. Kiviet, On Bias, Inconsistency, and Efficiency of Various Estimators in Dynamic Panel Data Models. Journal of Econometrics 68 (1995) 53-78.

6. Appendix

Here, we review the conventional one-step first-difference, level and system GMM estimators with efficient initial weight matrices (for more information see, Blundell, Bond and Windmeijer (2000)).

6.1 First-difference GMM estimator

Based on $(T-1)(T-2)/2$ moment conditions, Arellano and Bond(1991) proposed one-step first-difference GMM estimator given by

$$E(Z_{di}^{'}\Delta u_i) = 0 \tag{6.1.1}$$

where Z_{di} is a $(T-2)\times(T-1)(T-2)/2$ instrumental matrix and Δu_i is a $(T-2)\times 1$ vector.

$$Z_{di} = \begin{bmatrix} y_{i1} & 0 & 0 & \dots & 0 & \dots & 0 \\ 0 & y_{i1} & y_{i2} & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & y_{i1} & \dots & y_{iT-2} \end{bmatrix}; \quad \Delta u_i = \begin{bmatrix} \Delta u_{i3} \\ \Delta u_{i4} \\ \vdots \\ \Delta u_{iT} \end{bmatrix}$$

Based on the moment conditions (6.1.1) the one-step first-difference GMM estimator is obtained and is given by,

$$\hat{\delta}_{dif} = (\Delta y_{-1}' Z_d W_d^{-1} Z_d' \Delta y_{-1})^{-1} (\Delta y_{-1}' Z_d W_d^{-1} Z_d' \Delta y) \tag{6.1.2}$$

where $\Delta y_{i,-1}'$ is the $1\times(T-2)$ vector $(\Delta y_{i2}, \Delta y_{i3}, \dots, \Delta y_{iT-1})$, $\Delta y_i'$ is the $1\times(T-2)$ vector $(\Delta y_{i3}, \Delta y_{i4}, \dots, \Delta y_{iT})$, Δy_{-1} and Δy are stacked across individuals. Z_d is a $N(T-2)\times(T-1)(T-2)/2$ matrix $(Z_{d1}, Z_{d2}, \dots, Z_{dN})$ and $W_d = N^{-1}\sum_{i=1}^{N} Z_{di}' A_d Z_{di}$, where A_d is a $(T-2)\times(T-2)$ matrix with twos in main daigonal, minus ones in first sub diagonals and zero otherwise.

6.2 Level GMM estimator

Arellano and Bover (1995) proposed level GMM estimator which is based on the moment conditions

$$E(Z_{li}' u_i) = 0 \tag{6.2.1}$$

where Z_{li} is a $(T-2)\times(T-1)(T-2)/2$ instrumental matrix and Δu_i is a $(T-2)\times 1$ vector.

$$Z_{li} = \begin{bmatrix} \Delta y_{i2} & 0 & 0 & \dots & 0 & \dots & 0 \\ 0 & \Delta y_{i2} & \Delta y_{i3} & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & \Delta y_{i2} & \dots & \Delta y_{iT-1} \end{bmatrix}; \quad u_i = \begin{bmatrix} u_{i3} \\ u_{i4} \\ \vdots \\ u_{iT} \end{bmatrix}$$

Based on the moment conditions (6.2.1) the one-step level GMM estimator is obtained and is given by,

$$\hat{\delta}_{lev} = (y_{-1}' Z_l W_l^{-1} Z_l' y_{-1})^{-1} (y_{-1}' Z_l W_l^{-1} Z_l' y) \tag{6.2.2}$$

where $y_{i,-1}'$ is the $1\times(T-2)$ vector $(y_{i2}, y_{i3}, \dots, y_{iT-1})$, y_i' is the $1\times(T-2)$ vector $(y_{i3}, y_{i4}, \dots, y_{iT})$, y_{-1} and y are stacked across individuals. Z_l is a $N(T-2)\times(T-1)(T-2)/2$ matrix $(Z_{l1}, Z_{l2}, \dots, Z_{lN})$ and $W_l = N^{-1}\sum_{i=1}^{N} Z_{li}' A_l Z_{li}$, where A_l is the $(T-2)\times(T-2)$ identity matrix.

6.3 System GMM estimator

Blundell and Bond(1998) proposed system GMM estimator in which the moment conditions of first difference and level GMM estimators are jointly used. The moment condition used in constructing one-step system GMM estimator is given by

$$E(Z'_{si}u'_{si}) = 0 \tag{6.3.1}$$

where,

$$Z_{si} = \begin{bmatrix} Z_{di} & 0 \\ 0 & Z_{li}^{nr} \end{bmatrix}; \quad u_{si} = \begin{bmatrix} \Delta u_i \\ u_i \end{bmatrix}$$

with Z_{li}^{nr} is a non-redundant subset of Z_{li} (See Blundell et al (2000)).

Based on the moment conditions (6.3.1) the one-step system GMM estimator is given by,

$$\hat{\delta}_{sys} = (s'_{-1}Z_s W_s^{-1} Z'_s s_{-1})^{-1}(s'_{-1}Z_s W_s^{-1} Z'_{sl} s) \tag{6.3.2}$$

where $s'_{i,-1}$ is the $1 \times 2(T-2)$ vector $(\Delta y_{i2}, \ldots, \Delta y_{iT-1}, y_{i3}, \ldots, y_{iT-1})$, s'_i is the $1 \times 2(T-2)$ vector $(\Delta y_{i3}, \ldots, \Delta y_{iT}, y_{i3}, \ldots, y_{iT})$, s_{-1} and s are stacked across individuals. Z_s is a $2N(T-2) \times ((T-2) + (T-1)(T-2)/2)$ matrix $(Z_{s1}, Z_{s2}, \ldots, Z_{sN})$ and $W_s = N^{-1}\sum_{i=1}^{N} Z'_{si}A_s Z_{si}$, where A_s is a $2(T-2) \times 2(T-2)$ matrix and is given by

$$A_s = \begin{bmatrix} A_d & 0 \\ 0 & A_l \end{bmatrix}$$

6.4 Proof of Theorem 2

$$\begin{aligned}
B_{1dif} = & -\frac{E[E(\Delta y'_{i,-1}Z_{di})(E(Z'_{di}A_d Z_{di}))^{-1}Z'_{di}A_d Z_{di}(E(Z'_{di}A_d Z_{di}))^{-1}Z'_{di}\Delta u_i]}{E(\Delta y'_{i,-1}Z_{di})(E(Z'_{di}A_d Z_{di}))^{-1}E(Z'_{di}\Delta y_{i,-1})} \\
= & -\frac{1}{\phi_d}\left[\frac{\pi_{d1}}{(C+D)}E(y_{i1}^3\Delta u_{i3}) + \left(\frac{\pi_{d2}(C+D)}{F} - \frac{\pi_{d1}(C+\delta D)^2}{2(C+D)F}\right)E(y_{i1}^3\Delta u_{i4})\right. \\
& + \left(\frac{\pi_{d1}(C+\delta D)}{F} - 2\frac{\pi_{d2}(C+\delta D)}{F} + \frac{\pi_{d3}(C+D)}{F}\right)E(y_{i1}^2 y_{i2}\Delta u_{i4}) \\
& + \left(-\frac{\pi_{d1}(C+D)}{2F} + \frac{\pi_{d2}(C+D)}{F} - 2\frac{\pi_{d3}(C+\delta D)}{F}\right)E(y_{i1}y_{i2}^2\Delta u_{i4}) \\
& \left. + \frac{\pi_{d3}(C+D)}{F}E(y_{i2}^3\Delta u_{i4})\right]
\end{aligned}$$

Since

$$E(y_{i1}^3\Delta u_{i3}) = E(y_{i1}^3\Delta u_{i4}) = E(y_{i1}^2 y_{i2}\Delta u_{i4}) = E(y_{i1}y_{i2}^2\Delta u_{i4}) = E(y_{i2}^3\Delta u_{i4}) = 0.$$

Therefore B_{1dif} reduces to

$$B_{1dif} = 0. \tag{6.4.1}$$

Next

$$B_{2dif} = \frac{trace[E(Z'_{di}A_dZ_{di})^{-1}E(Z'_{di}\Delta u_i\Delta y'_{i,-1}Z_{di})]}{E(\Delta y'_{i,-1}Z_{di})E(Z'_{di}A_dZ_{di})^{-1}E(Z'_{di}\Delta y_{i,-1})}$$

$$= \frac{1}{\phi_d}\Bigg[\frac{2E(y_{i1}^2\Delta y_{i2}\Delta u_{i3}) + E(y_{i1}^2\Delta y_{i2}\Delta u_{i4}) + E(y_{i1}^2\Delta y_{i3}\Delta u_{i3})}{3(C+D)}$$

$$+\frac{[4(C+D)^2-(C+\delta D)^2]E(y_{i1}^2\Delta y_{i3}\Delta u_{i4})}{6(C+D)F}$$

$$+\frac{(C+D)E(y_{i2}^2\Delta y_{i3}\Delta u_{i4}) - 2(C+\delta D)E(y_{i1}y_{i2}\Delta y_{i3}\Delta u_{i4})}{2F}\Bigg]$$

Since

$$E(y_{i1}^2\Delta y_{i2}\Delta u_{i3}) = E(y_{i1}^2\Delta y_{i3}\Delta u_{i4}) = E(y_{i2}^2\Delta y_{i3}\Delta u_{i4}) = -\sigma_v^2(C+D);$$
$$E(y_{i1}^2\Delta y_{i2}\Delta u_{i4}) = 0;\;\; E(y_{i1}^2\Delta y_{i3}\Delta u_{i3}) = \sigma_v^2(2-\delta)(C+D);$$
$$E(y_{i1}y_{i2}\Delta y_{i3}\Delta u_{i4}) = -\sigma_v^2(C+\delta D)$$

Therefore B_{2dif} reduces to

$$B_{2dif} = -\frac{\sigma_v^2}{6\phi_d}(7+2\delta) \tag{6.4.2}$$

$$B_{3dif} = -2\frac{E(\Delta y'_{i,-1}Z_{di})E(Z'_{di}A_dZ_{di})^{-1}E(Z'_{di}\Delta u_i\Delta y'_{i,-1}Z_{di})E(Z'_{di}A_dZ_{di})^{-1}E(Z'_{di}\Delta y_{i,-1})}{[E(\Delta y'_{i,-1}Z_{di})E(Z'_{di}A_dZ_{di})^{-1}E(Z'_{di}\Delta y_{i,-1})]^2}$$

$$= -\frac{2}{\phi_d^2}\left[\pi_{d1}^2E(y_{i1}^2\Delta y_{i2}\Delta u_{i3}) + \pi_{d2}^2E(y_{i1}^2\Delta y_{i3}\Delta u_{i4}) + \pi_{d3}^2E(y_{i2}^2\Delta y_{i3}\Delta u_{i4})\right.$$
$$+\pi_{d1}\pi_{d2}E(y_{i1}^2\Delta y_{i2}\Delta u_{i4}) + \pi_{d1}\pi_{d2}E(y_{i1}^2\Delta y_{i3}\Delta u_{i3}) + \pi_{d1}\pi_{d3}E(y_{i1}y_{i2}\Delta y_{i2}\Delta u_{i4})$$
$$\left.+\pi_{d1}\pi_{d3}E(y_{i1}y_{i2}\Delta y_{i3}\Delta u_{i3}) + 2\pi_{d2}\pi_{d3}E(y_{i1}y_{i2}\Delta y_{i3}\Delta u_{i4})\right]$$

Since

$$E(y_{i1}^2\Delta y_{i2}\Delta u_{i3}) = E(y_{i1}^2\Delta y_{i3}\Delta u_{i4}) = E(y_{i2}^2\Delta y_{i3}\Delta u_{i4}) = -\sigma_v^2(C+D);$$
$$E(y_{i1}^2\Delta y_{i2}\Delta u_{i4}) = 0;\;\; E(y_{i1}^2\Delta y_{i3}\Delta u_{i3}) = \sigma_v^2(2-\delta)(C+D);$$
$$E(y_{i1}y_{i2}\Delta y_{i3}\Delta u_{i4}) = -\sigma_v^2(C+\delta D); E(y_{i1}y_{i2}\Delta y_{i2}\Delta u_{i4}) = 0;$$
$$E(y_{i1}y_{i2}\Delta y_{i3}\Delta u_{i3}) = \sigma_v^2[(2-\delta)C - \delta(2\delta-3)D].$$

Therefore B_{3dif} reduces to

$$B_{3dif} = \frac{2\sigma_v^2}{\phi_d^2}\left[(\pi_{d1}^2+\pi_{d2}^2+\pi_{d3}^2)(C+D) + \pi_{d1}\pi_{d2}(\delta-2)(C+D) + 2\pi_{d2}\pi_{d3}(C+\delta D)\right.$$
$$\left.-\pi_{d1}\pi_{d3}[(2-\delta)C - \delta(2\delta-3)D]\right] \tag{6.4.3}$$

$$B_{4dif} = \frac{E\left[E(\Delta y'_{i,-1}Z_{di})E(Z'_{di}A_dZ_{di})^{-1}Z'_{di}\Delta u_iE(\Delta y'_{i,-1}Z_{di})\right.}{[E(\Delta y'_{i,-1}Z_{di})E(Z'_{di}A_dZ_{di})^{-1}E(Z'_{di}\Delta y_{i,-1})]^2}$$
$$\frac{\left.\times E(Z'_{di}A_dZ_{di})^{-1}Z'_{di}A_dZ_{di}E(Z'_{di}A_dZ_{di})^{-1}E(Z'_{di}\Delta y_{i,-1})\right]}{[E(\Delta y'_{i,-1}Z_{di})E(Z'_{i}A_dZ_{di})^{-1}E(Z'_{di}\Delta y_{i,-1})]^2}$$

$$= \frac{1}{\phi_d^2}\left[(2\pi_{d1}^3 - 2\pi_{d1}^2\pi_{d2} + 2\pi_{d1}\pi_{d2}^2)E(y_{i1}^3\Delta u_{i3}) + (2\pi_{d1}^2\pi_{d2} - 2\pi_{d1}\pi_{d2}^2 + 2\pi_{d2}^3)E(y_{i1}^3\Delta u_{i4})\right.$$
$$+2\pi_{d3}^3E(y_{i2}^3\Delta u_{i4}) + (4\pi_{d1}\pi_{d2}\pi_{d3} - 2\pi_{d1}^2\pi_{d3})E(y_{i1}^2y_{i2}\Delta u_{i3})$$
$$+(4\pi_{d2}^2\pi_{d3} - 4\pi_{d1}\pi_{d2}\pi_{d3} + 2\pi_{d1}^2\pi_{d3} + 2\pi_{d2}^2\pi_{d3})E(y_{i1}^2y_{i2}\Delta u_{i4})$$
$$\left.+(4\pi_{d2}pi_{d3}^2 - 2\pi_{d1}\pi_{d3}^2)E(y_{i1}y_{i2}^2\Delta u_{i4}) + 2\pi_{d1}\pi_{d3}^2E(y_{i1}y_{i2}^2\Delta u_{i3})\right]$$

Since

$$E(y_{i1}^3\Delta u_{i3}) = E(y_{i1}^3\Delta u_{i4}) = E(y_{i2}^3\Delta u_{i4}) = E(y_{i1}^2y_{i2}\Delta u_{i4}) = E(y_{i1}y_{i2}^2\Delta u_{i4}) = 0;$$
$$E(y_{i1}^2y_{i2}\Delta u_{i3}) = -\sigma_v^2(C+D); E(y_{i1}y_{i2}^2\Delta u_{i3}) = -2\sigma_v^2(C+\delta D)$$

Therefore B_{4dif} reduces to

$$B_{4dif} = \frac{-2\sigma_v^2}{\phi_d^2}\left[(2\pi_{d1}\pi_{d2}\pi_{d3} - \pi_{d1}^2\pi_{d3})(C+D) + 2\pi_{d1}\pi_{d3}^2(C+\delta D)\right] \quad (6.4.4)$$

6.5 Proof of Theorem 3

The proof of the Theorem 2 is the same as Theorem 1. The second order bias of one-step level GMM estimator is given by

$$B_{1lev} = -\frac{E[E(y'_{i,-1}Z_{li})E(Z'_{li}A_lZ_{li})^{-1}Z'_{li}A_lZ_{li}E(Z'_{li}A_lZ_{li})^{-1}Z'_{li}u_i]}{E(y'_{i,-1}Z_{li})E(Z'_{li}A_lZ_{li})^{-1}E(Z'_{li}y_{i,-1})}$$
$$B_{1lev} = \frac{-1}{\sigma_v^2[4-(\delta-1)^2]\phi_l}\left[2(1+\delta)\pi_{l2}\left\{E(\Delta y_{i2}^3u_{i4}) + E(\Delta y_{i2}\Delta y_{i3}^2u_{i4})\right\}\right.$$
$$+2(1+\delta)\pi_{l3}\left\{E(\Delta y_{i3}^3u_{i4}) + E(\Delta y_{i2}^2\Delta y_{i3}u_{i4})\right\} + 2\pi_{l3}(1-\delta^2)E(\Delta y_{i2}\Delta y_{i3}^2u_{i4})$$
$$\left.+2\pi_{l2}(1-\delta^2)E(\Delta y_{i2}^2\Delta y_{i3}u_{i4})\right] - \frac{\pi_{l1}(1+\delta)}{2\sigma_v^2\phi_l}E(\Delta y_{i2}^3u_{i3})$$

Since

$$E(\Delta y_{i2}^3u_{i4}) = E(\Delta y_{i3}^3u_{i4}) = E(\Delta y_{i2}\Delta y_{i3}^2u_{i4}) = E(\Delta y_{i2}^2\Delta y_{i3}u_{i4}) = E(\Delta y_{i2}^3u_{i3}) = 0$$

Therefore

$$B_{1lev} = 0. \quad (6.5.1)$$

$$B_{2lev} = \frac{trace[E(Z'_{li}A_lZ_{li})^{-1}E(Z'_{li}u_iy'_{i,-1}Z_{li})]}{E(y'_{i,-1}Z_{li})E(Z'_{li}A_lZ_{li})^{-1}E(Z'_{li}y_{i,-1})}$$
$$= \frac{1}{\phi_l}\left[\frac{(1+\delta)}{2\sigma_v^2}E(y_{i2}\Delta y_{i2}^2u_{i3}) + \frac{2(1+\delta)}{\sigma_v^2[4-(\delta-1)^2]}\left\{E(y_{i3}\Delta y_{i2}^2u_{i4}) + E(y_{i3}\Delta y_{i3}^2u_{i4})\right\}\right.$$
$$\left.+\frac{2(1-\delta^2)}{\sigma_v^2[4-(\delta-1)^2]}E(y_{i3}\Delta y_{i2}\Delta y_{i3}u_{i4})\right]$$

Since

$$E(y_{i2}\Delta y_{i2}^2u_{i3}) = E(y_{i3}\Delta y_{i2}^2u_{i4}) = E(y_{i3}\Delta y_{i3}^2u_{i4}) = \frac{2\sigma_\eta^2\sigma_v^2}{(1-\delta^2)};$$
$$E(y_{i3}\Delta y_{i2}\Delta y_{i3}u_{i4}) = \frac{-\sigma_\eta^2\sigma_v^2}{1+\delta}$$

Therefore

$$B_{2lev} = \frac{3\sigma_\eta^2}{\phi_l(1-\delta)} \tag{6.5.2}$$

$$\begin{aligned}
B_{3lev} =& -2\frac{E(\Delta y'_{i,-1}Z_{li})E(Z'_{li}A_lZ_{li})^{-1}E(Z'_{li}u_iy'_{i,-1}Z_{li})E(Z'_{li}A_lZ_{li})^{-1}E(Z'_{li}y_{i,-1})}{[E(y'_{i,-1}Z_{li})E(Z'_{li}A_lZ_{li})^{-1}E(Z'_{li}y_{i,-1})]^2} \\
=& \frac{-2}{\phi_l^2}\left[\pi_{l1}^2E(y_{i2}\Delta y_{i2}^2u_{i3}) + \pi_{l2}^2E(y_{i3}\Delta y_{i2}^2u_{i4}) + \pi_{l3}^2E(y_{i3}\Delta y_{i3}^2u_{i4})\right. \\
&+\pi_{l1}\pi_{l2}E(y_{i2}\Delta y_{i2}^2u_{i4}) + \pi_{l1}\pi_{l2}E(y_{i3}\Delta y_{i2}^2u_{i3}) + \pi_{l1}\pi_{l3}E(y_{i2}\Delta y_{i2}\Delta y_{i3}u_{i4}) \\
&\left.+\pi_{l1}\pi_{l3}E(y_{i3}\Delta y_{i2}\Delta y_{i3}u_{i3}) + 2\pi_{l2}\pi_{l3}E(y_{i3}\Delta y_{i2}\Delta y_{i3}u_{i4})\right]
\end{aligned}$$

Since

$$\begin{aligned}
&E(y_{i2}\Delta y_{i2}^2u_{i3}) = E(y_{i2}\Delta y_{i2}^2u_{i4}) = E(y_{i3}\Delta y_{i2}^2u_{i4}) = E(y_{i3}\Delta y_{i3}^2u_{i4}) = \frac{2\sigma_\eta^2\sigma_v^2}{1-\delta^2} \\
&E(y_{i3}\Delta y_{i2}\Delta y_{i3}u_{i4}) = E(y_{i2}\Delta y_{i2}\Delta y_{i3}u_{i4}) = \frac{-\sigma_\eta^2\sigma_v^2}{1+\delta} \\
&E(y_{i3}\Delta y_{i2}^2u_{i3}) = \frac{2\sigma_\eta^2\sigma_v^2}{1-\delta^2} + \frac{2\sigma_v^4}{1+\delta}; \quad E(y_{i3}\Delta y_{i2}\Delta y_{i3}u_{i3}) = \frac{-\sigma_\eta^2\sigma_v^2}{1+\delta} + \frac{(2\delta-1)\sigma_v^4}{1+\delta}
\end{aligned}$$

Therefore

$$\begin{aligned}
B_{3lev} =& \frac{-2}{\phi_l^2}\left[\frac{2\sigma_\eta^2\sigma_v^2}{(1-\delta^2)}\left\{\pi_{l1}^2 + \pi_{l2}^2 + \pi_{l3}^2 + 2\pi_{l1}\pi_{l2} - (1-\delta)\pi_{l1}\pi_{l3} - (1-\delta)\pi_{l2}\pi_{l3}\right\}\right. \\
&\left.+\frac{\sigma_v^4}{1+\delta}\left\{(2\delta-1)\pi_{l1}\pi_{l3} + 2\pi_{l1}\pi_{l2}\right\}\right] \qquad (6.5.3)
\end{aligned}$$

$$\begin{aligned}
B_{4lev} =& \frac{E\left[E(y'_{i,-1}Z_{li})E(Z'_{li}A_lZ_{li})^{-1}Z'_{li}u_iE(y'_{i,-1}Z_{li})\right.}{[E(y'_{i,-1}Z_{li})E(Z'_{li}A_lZ_{li})^{-1}E(Z'_{li}y_{i,-1})]^2} \\
&\frac{\left.\times E(Z'_{li}A_lZ_{li})^{-1}Z'_{li}A_lZ_{li}E(Z'_{li}A_lZ_{li})^{-1}E(Z'_{li}y_{i,-1})\right]}{[E(y'_{i,-1}Z_{li})E(Z'_{li}A_lZ_{li})^{-1}E(Z'_{li}y_{i,-1})]^2} \\
=& \frac{1}{\phi_l^2}\left[\pi_{l1}^3E(\Delta y_{i2}^3u_{i3}) + \pi_{l2}^3E(\Delta y_{i2}^3u_{i4}) + \pi_{l3}^3E(\Delta y_{i3}^3u_{i4}) + \pi_{l1}^2\pi_{l2}E(\Delta y_{i2}^3u_{i4})\right. \\
&+\pi_{l1}^2\pi_{l3}E(\Delta y_{i2}^2\Delta y_{i3}u_{i4}) + \pi_{l1}\pi_{l2}^2E(\Delta y_{i2}^3u_{i3}) + 2\pi_{l2}^2\pi_{l3}E(\Delta y_{i2}^2\Delta y_{i3}u_{i4}) \\
&\left.+2\pi_{l1}\pi_{l2}\pi_{l3}E(\Delta y_{i2}^2\Delta y_{i3}u_{i3}) + 3\pi_{l2}\pi_{l3}^2E(\Delta y_{i2}\Delta y_{i3}^2u_{i4}) + \pi_{l1}\pi_{l3}^2E(\Delta y_{i2}\Delta y_{i3}^2u_{i3})\right]
\end{aligned}$$

Since

$$\begin{aligned}
&E(\Delta y_{i2}^3u_{i3}) = E(\Delta y_{i2}^3u_{i4}) = E(\Delta y_{i3}^3u_{i4}) = E(\Delta y_{i2}^2\Delta y_{i3}u_{i4}) = E(\Delta y_{i2}\Delta y_{i3}^2u_{i4}) = 0 \\
&E(\Delta y_{i2}^2\Delta y_{i3}u_{i3}) = \frac{2\sigma_v^4}{(1+\delta)} \\
&E(\Delta y_{i2}\Delta y_{i3}^2u_{i3}) = \frac{2\sigma_v^4(\delta-1)}{(1+\delta)}
\end{aligned}$$

Therefore

$$B_{4lev} = \frac{2\sigma_v^4}{\phi_l^2(1+\delta)}\left[2\pi_{l1}\pi_{l2}\pi_{l3} + (\delta-1)\pi_{l1}\pi_{l3}^2\right] \tag{6.5.4}$$

6.6 Proof of Theorem 4

The proof of the second order bias of one-step system GMM estimator is given by

$$\begin{aligned}
B_{1sys} =& -\frac{E[E(s'_{i,-1}Z_{si})E(Z'_{si}A_sZ_{si})^{-1}Z'_{si}A_sZ_{si}E(Z'_{si}A_sZ_{si})^{-1}Z'_{si}u_{si}]}{E(s'_{i,-1}Z_{si})E(Z'_{si}A_sZ_{si})^{-1}E(Z'_{si}s_{i,-1})} \\
=& \frac{1}{\phi_s}\left[\frac{\pi_{d1}}{C+D}E(y_{i1}^3\Delta u_{i3}) - \frac{\pi_{d1}(C+\delta D)^2}{2(C+D)F}E(y_{i1}^3\Delta u_{i4}) + \frac{\pi_{d2}(C+D)}{F}E(y_{i1}^3\Delta u_{i4})\right. \\
&+\frac{\pi_{d1}(C+\delta D)}{F}E(y_{i1}^2y_{i2}\Delta u_{i4}) - 2\frac{\pi_{d2}(C+\delta D)}{F}E(y_{i1}^2y_{i2}\Delta u_{i4}) + \frac{\pi_{d3}(C+D)}{F}E(y_{i1}^2y_{i2}\Delta u_{i4}) \\
&-\frac{\pi_{d1}(C+D)}{2F}E(y_{i1}y_{i2}^2\Delta u_{i4}) + \frac{\pi_{d2}(C+D)}{F}E(y_{i1}y_{i2}^2\Delta u_{i4}) - 2\frac{\pi_{d3}(C+\delta D)}{F}E(y_{i1}y_{i2}^2\Delta u_{i4}) \\
&\left.+\frac{\pi_{d3}(C+D)}{F}E(y_{i2}^3\Delta u_{i4}) + \frac{1+\delta}{4\sigma_v^2}E(\Delta y_{i2}^3u_{i3}) + \frac{1+\delta}{4\sigma_v^2}E(\Delta y_{i3}^3u_{i4})\right]
\end{aligned}$$

Since all the above expectations are equal to zero.

$$B_{1sys} = 0. \tag{6.6.1}$$

$$\begin{aligned}
B_{2sys} =& \frac{trace[E(Z'_{si}A_sZ_{si})^{-1}E(Z'_{si}u_{si}s'_{i,-1}Z_{si})]}{E(s'_{i,-1}Z_{si})E(Z'_{si}A_sZ_{si})^{-1}E(Z'_{si}s_{i,-1})} \\
=&\frac{1}{\phi_s}\left[\frac{2E(y_{i1}^2\Delta y_{i2}\Delta u_{i3}) + E(y_{i1}^2\Delta y_{i2}\Delta u_{i4}) + E(y_{i1}^2\Delta y_{i3}\Delta u_{i3})}{3(C+D)} - \frac{2(C+\delta D)E(y_{i1}y_{i2}\Delta y_{i3}\Delta u_{i4})}{2F}\right. \\
&+\frac{[4(C+D)^2-(C+\delta D)^2]E(y_{i1}^2\Delta y_{i3}\Delta u_{i4})}{6(C+D)F} + \frac{(1+\delta)[E(y_{i2}\Delta y_{i2}^2u_{i3}) + E(y_{i3}\Delta y_{i3}^2u_{i4})]}{2\sigma_v^2} \\
&\left.+\frac{(C+D)E(y_{i2}^2\Delta y_{i3}\Delta u_{i4})}{2F}\right]
\end{aligned}$$

Since

$$\begin{aligned}
&E(y_{i1}^2\Delta y_{i2}\Delta u_{i3}) = E(y_{i1}^2\Delta y_{i3}\Delta u_{i4}) = E(y_{i2}^2\Delta y_{i3}\Delta u_{i4}) = -\sigma_v^2(C+D); \\
&E(y_{i1}^2\Delta y_{i2}\Delta u_{i4}) = 0; \quad E(y_{i1}^2\Delta y_{i3}\Delta u_{i3}) = \sigma_v^2(2-\delta)(C+D) \\
&E(y_{i1}y_{i2}\Delta y_{i3}\Delta u_{i4}) = -\sigma_v^2(C+\delta D) \\
&E(y_{i2}\Delta y_{i2}^2u_{i3}) = E(y_{i3}\Delta y_{i3}^2u_{i4}) = \frac{2\sigma_\eta^2\sigma_v^2}{(1-\delta^2)}
\end{aligned}$$

Therefore

$$B_{2sys} = \frac{1}{\phi_s}\left[\frac{2\sigma_\eta^2}{1-\delta} - \frac{\sigma_v^2(7+2\delta)}{6}\right] \tag{6.6.2}$$

$$B_{3sys} = -2\frac{E(s'_{i,-1}Z_{si})E(Z'_{si}A_sZ_{si})^{-1}E(Z'_{si}u_{si}s'_{i,-1}Z_{si})E(Z'_{si}A_sZ_{si})^{-1}E(Z'_{si}s_{i,-1})}{[E(s'_{i,-1}Z_{si})E(Z'_{si}A_sZ_{si})^{-1}E(Z'_{si}s_{i,-1})]^2}$$

$$
\begin{aligned}
= \quad & \frac{-2}{\phi_s^2}\left[\pi_{d1}^2 E(y_{i1}^2\Delta y_{i2}\Delta u_{i3}) + \pi_{d2}^2 E(y_{i1}^2\Delta y_{i3}\Delta u_{i4}) + \pi_{d3}^2 E(y_{i2}^2\Delta y_{i3}\Delta u_{i4})\right. \\
& +\pi_{d1}\pi_{d2}\left\{E(y_{i1}^2\Delta y_{i2}\Delta u_{i4}) + E(y_{i1}^2\Delta y_{i3}\Delta u_{i3})\right\} + 2\pi_{d2}\pi_{d3}E(y_{i1}y_{i2}\Delta y_{i3}\Delta u_{i4}) \\
& +\pi_{d1}\pi_{d3}\left\{E(y_{i1}y_{i2}\Delta y_{i2}\Delta u_{i4}) + E(y_{i1}y_{i2}\Delta y_{i3}\Delta u_{i3})\right\} \\
& +\frac{\pi_{d1}}{2}\left\{E(y_{i1}y_{i2}\Delta y_{i2}\Delta u_{i3}) + E(y_{i1}y_{i3}\Delta y_{i3}\Delta u_{i3}) + E(y_{i1}\Delta y_{i2}^2 u_{i3}) + E(y_{i1}\Delta y_{i2}\Delta y_{i3}u_{i4})\right\} \\
& +\frac{\pi_{d2}}{2}\left\{E(y_{i1}y_{i2}\Delta y_{i2}\Delta u_{i4}) + E(y_{i1}y_{i3}\Delta y_{i3}\Delta u_{i4}) + E(y_{i1}\Delta y_{i3}^2 u_{i4}) + E(y_{i1}\Delta y_{i2}\Delta y_{i3}u_{i3})\right\} \\
& +\frac{\pi_{d3}}{2}\left\{E(y_{i2}^2\Delta y_{i2}\Delta u_{i4}) + E(y_{i2}y_{i3}\Delta y_{i3}\Delta u_{i4}) + E(y_{i2}\Delta y_{i3}^2 u_{i4}) + E(y_{i2}\Delta y_{i2}\Delta y_{i3}u_{i3})\right\} \\
& \left.+\frac{1}{4}\left\{E(y_{i2}\Delta y_{i2}^2 u_{i3}) + E(y_{i2}\Delta y_{i2}\Delta y_{i3}u_{i4}) + E(y_{i3}\Delta y_{i3}^2 u_{i4}) + E(y_{i3}\Delta y_{i2}\Delta y_{i3}u_{i3})\right\}\right]
\end{aligned}
$$

Since

$$
\begin{aligned}
& E(y_{i1}^2\Delta y_{i2}\Delta u_{i3}) = E(y_{i1}^2\Delta y_{i3}\Delta u_{i4}) = E(y_{i2}^2\Delta y_{i3}\Delta u_{i4}) = -\sigma_v^2(C+D); \\
& E(y_{i1}^2\Delta y_{i2}\Delta u_{i4}) = E(y_{i1}y_{i2}\Delta y_{i2}\Delta u_{i4}) = E(y_{i2}^2\Delta y_{i2}\Delta u_{i4}) = 0; \\
& E(y_{i1}\Delta y_{i2}^2 u_{i3}) = E(y_{i1}\Delta y_{i3}^2 u_{i4}) = E(y_{i2}\Delta y_{i3}^2 u_{i4}) = E(y_{i2}\Delta y_{i2}^2 u_{i3}) = E(y_{i3}\Delta y_{i3}^2 u_{i4}) = \frac{2\sigma_\eta^2\sigma_v^2}{(1-\delta^2)} \\
& E(y_{i1}\Delta y_{i2}\Delta y_{i3}u_{i4}) = E(y_{i2}\Delta y_{i2}\Delta y_{i3}u_{i4}) = \frac{-\sigma_\eta^2\sigma_v^2}{(1+\delta)};\;\; E(y_{i1}y_{i2}\Delta y_{i3}\Delta u_{i4}) = -\sigma_v^2(C+\delta D) \\
& E(y_{i1}\Delta y_{i2}\Delta y_{i3}u_{i3}) = -\frac{(\sigma_v^4+\sigma_\eta^2\sigma_v^2)}{1+\delta};\;\; E(y_{i2}\Delta y_{i2}\Delta y_{i3}u_{i3}) = \frac{\sigma_v^4-\sigma_\eta^2\sigma_v^2}{1+\delta} \\
& E(y_{i1}^2\Delta y_{i3}\Delta u_{i3}) = \sigma_v^2(2-\delta)(C+D);\;\; E(y_{i1}y_{i2}\Delta y_{i3}\Delta u_{i3}) = \frac{(2-\delta)\sigma_\eta^2\sigma_v^2}{(1-\delta)^2} + \frac{(3\delta-2\delta^2)\sigma_v^4}{(1-\delta^2)} \\
& E(y_{i1}y_{i2}\Delta y_{i2}\Delta u_{i3}) = E(y_{i2}y_{i3}\Delta y_{i3}\Delta u_{i4}) = -\frac{\sigma_\eta^2\sigma_v^2}{(1-\delta)^2} - \frac{(2\delta-1)\sigma_v^4}{(1-\delta^2)} \\
& E(y_{i1}y_{i3}\Delta y_{i3}\Delta u_{i4}) = -\frac{\sigma_\eta^2\sigma_v^2}{(1-\delta)^2} - \frac{\delta(2\delta-1)\sigma_v^4}{(1-\delta^2)} \\
& E(y_{i3}\Delta y_{i2}\Delta y_{i3}\Delta u_{i3}) = \frac{(2\delta-1)\sigma_v^4-\sigma_\eta^2\sigma_v^2}{(1+\delta)} \\
& E(y_{i1}y_{i3}\Delta y_{i3}\Delta u_{i3}) = \frac{(2-\delta)\sigma_\eta^2\sigma_v^2}{(1-\delta)^2} - \frac{\delta(2\delta^2-4\delta+1)\sigma_v^4}{(1-\delta^2)}
\end{aligned}
$$

Therefore

$$
\begin{aligned}
B_{3sys} = \; & \frac{\phi_d^2}{\phi_s^2}B_{3dif} - \frac{1}{\phi_s^2}\left[\frac{\sigma_v^4(2\delta-1)}{2(1+\delta)} + \frac{\sigma_\eta^2\sigma_v^2}{1-\delta} + \frac{2\pi_{d1}\sigma_\eta^2\sigma_v^2}{1-\delta} - \frac{\pi_{d1}\sigma_v^4[2\delta-1+\delta(2\delta^2-4\delta+1)]}{1-\delta^2}\right. \\
& \left.-\frac{\pi_{d2}\delta\sigma_\eta^2\sigma_v^2}{(1-\delta)^2} - \frac{\pi_{d2}\sigma_v^4(2\delta^2-2\delta+1)}{1-\delta^2} + \frac{\pi_{d3}\sigma_v^4(2-3\delta)}{1-\delta^2} - \frac{\pi_{d3}\delta\sigma_\eta^2\sigma_v^2}{(1-\delta)^2}\right] \qquad (6.6.3)
\end{aligned}
$$

$$B_{4sys} = \frac{E\left[E(s'_{i,-1}Z_{si})E(Z'_{si}A_sZ_{si})^{-1}Z'_{si}u_{si}E(s'_{i,-1}Z_{si})\right.}{[E(s'_{i,-1}Z_{si})E(Z'_{si}A_sZ_{si})^{-1}E(Z'_{si}s_{i,-1})]^2}$$
$$\frac{\times E(Z'_{si}A_sZ_{si})^{-1}Z'_{si}A_sZ_{si}E(Z'_{si}A_sZ_{si})^{-1}E(Z'_{si}s_{i,-1})]}{[E(s'_{i,-1}Z_{si})E(Z'_{si}A_sZ_{si})^{-1}E(Z'_{si}s_{i,-1})]^2}$$
$$\begin{aligned}
= \frac{1}{\phi_s^2}\Big[\Big\{ & 2\pi_{d1}^3E(y_{i1}^3\Delta u_{i3}) + 2\pi_{d1}^2\pi_{d2}E(y_{i1}^3\Delta u_{i4}) + 2\pi_{d1}^2\pi_{d3}E(y_{i1}^2y_{i2}\Delta u_{i4}) + \pi_{d1}^2E(y_{i1}^2\Delta y_{i2}u_{i3}) \\
& +\pi_{d1}^2E(y_{i1}^2\Delta y_{i3}u_{i4}) - \pi_{d1}^2\pi_{d2}E(y_{i1}^3\Delta u_{i3}) - \pi_{d1}\pi_{d2}^2E(y_{i1}^3\Delta u_{i4}) - 2\pi_{d1}\pi_{d2}\pi_{d3}E(y_{i1}^2y_{i2}\Delta u_{i4}) \\
& -\pi_{d1}^2\pi_{d3}E(y_{i1}^2y_{i2}\Delta u_{i3}) - \pi_{d1}\pi_{d3}^2E(y_{i1}y_{i2}^2\Delta u_{i4}) - \frac{\pi_{d1}\pi_{d2}}{2}[E(y_{i1}^2\Delta y_{i2}u_{i3}) + E(y_{i1}^2\Delta y_{i3}u_{i4})] \\
& -\frac{\pi_{d1}\pi_{d3}}{2}[E(y_{i1}y_{i2}\Delta y_{i2}u_{i3}) + E(y_{i1}y_{i2}\Delta y_{i3}u_{i4})]\Big\} + \Big\{-\pi_{d1}^2\pi_{d2}E(y_{i1}^3\Delta u_{i3}) \\
& -\pi_{d1}\pi_{d2}^2E(y_{i1}^3\Delta u_{i4}) - \pi_{d1}\pi_{d2}\pi_{d3}E(y_{i1}^2y_{i2}\Delta u_{i4}) + 2\pi_{d1}\pi_{d2}^2E(y_{i1}^3\Delta u_{i3}) + 2\pi_{d2}^3E(y_{i1}^3\Delta u_{i4}) \\
& +2\pi_{d2}^2\pi_{d3}E(y_{i1}^2y_{i2}\Delta u_{i4}) + \pi_{d2}^2[E(y_{i1}^2\Delta y_{i2}u_{i3}) + E(y_{i1}^2\Delta y_{i3}u_{i4})] + 2\pi_{d1}\pi_{d2}\pi_{d3}E(y_{i1}^2y_{i2}\Delta u_{i3}) \\
& +2\pi_{d2}^2\pi_{d3}E(y_{i1}^2y_{i2}\Delta u_{i4}) + 2\pi_{d2}\pi_{d3}^2E(y_{i1}y_{i2}^2\Delta u_{i4}) + \pi_{d2}\pi_{d3}[E(y_{i1}y_{i2}\Delta y_{i2}u_{i3}) \\
& +E(y_{i1}y_{i2}\Delta y_{i3}u_{i4})] - \frac{\pi_{d1}\pi_{d2}}{2}[E(y_{i1}^2\Delta y_{i2}u_{i3}) + E(y_{i1}^2\Delta y_{i3}u_{i4})]\Big\} + \Big\{-\pi_{d1}^2\pi_{d3}E(y_{i1}^2y_{i2}\Delta u_{i3}) \\
& -\pi_{d1}\pi_{d3}^2E(y_{i1}y_{i2}^2\Delta u_{i4}) - \pi_{d1}\pi_{d2}\pi_{d3}E(y_{i1}^2y_{i2}\Delta u_{i4}) + 2\pi_{d1}\pi_{d2}\pi_{d3}E(y_{i1}^2y_{i2}\Delta u_{i3}) \\
& +2\pi_{d2}^2\pi_{d3}E(y_{i1}^2y_{i2}\Delta u_{i4}) + 2\pi_{d2}\pi_{d3}^2E(y_{i1}y_{i2}^2\Delta u_{i4}) + \pi_{d2}\pi_{d3}[E(y_{i1}y_{i2}\Delta y_{i2}u_{i3}) \\
& +E(y_{i1}y_{i2}\Delta y_{i3}u_{i4})] + 2\pi_{d1}\pi_{d3}^2E(y_{i1}y_{i2}^2\Delta u_{i3}) + 2\pi_{d2}\pi_{d3}^2E(y_{i1}y_{i2}^2\Delta u_{i4}) + 2\pi_{d3}^3E(y_{i2}^3\Delta u_{i4}) \\
& +\pi_{d3}^2[E(y_{i2}^2\Delta y_{i2}u_{i3}) + E(y_{i2}^2\Delta y_{i3}u_{i4})] - \frac{\pi_{d1}\pi_{d3}}{2}[E(y_{i1}y_{i2}\Delta y_{i2}u_{i3}) + E(y_{i1}y_{i2}\Delta y_{i3}u_{i4})]\Big\} \\
& +\left\{\frac{\pi_{d1}E(y_{i1}\Delta y_{i2}^2\Delta u_{i3})}{4} + \frac{\pi_{d2}E(y_{i1}\Delta y_{i2}^2\Delta u_{i4})}{4} + \frac{\pi_{d3}E(y_{i1}\Delta y_{i2}^2\Delta u_{i4})}{4}\right. \\
& \left.+\frac{E(\Delta y_{i2}^3u_{i3}) + E(\Delta y_{i2}^2\Delta y_{i3}u_{i4})}{8}\right\} + \left\{\frac{\pi_{d1}E(y_{i1}\Delta y_{i3}^2\Delta u_{i3})}{4} + \frac{\pi_{d2}E(y_{i1}\Delta y_{i3}^2\Delta u_{i4})}{4}\right. \\
& \left.+\frac{\pi_{d3}E(y_{i2}\Delta y_{i3}^2\Delta u_{i4})}{4} + \frac{E(\Delta y_{i3}^3u_{i4}) + E(\Delta y_{i2}\Delta y_{i3}^2u_{i3})}{8}\right\}\Big]
\end{aligned}$$

Since

$$E(y_{i1}^2\Delta y_{i2}u_{i3}) = \frac{-2\sigma_\eta^2\sigma_v^2}{(1-\delta^2)};\ \ E(y_{i1}^2\Delta y_{i3}u_{i4}) = \frac{-2\delta\sigma_\eta^2\sigma_v^2}{(1-\delta^2)};\ \ E(y_{i1}^2y_{i2}\Delta u_{i3}) = -\sigma_v^2(C+D)$$
$$E(y_{i1}y_{i2}\Delta y_{i3}u_{i4}) = \frac{-\sigma_\eta^2\sigma_v^2}{1-\delta};\ \ E(y_{i1}y_{i2}^2\Delta u_{i3}) = -2\sigma_v^2(C+\delta D);\ \ E(y_{i2}^2\Delta y_{i2}u_{i3}) = \frac{2\delta\sigma_\eta^2\sigma_v^2}{1-\delta^2}$$
$$E(y_{i2}^2\Delta y_{i3}u_{i4}) = \frac{-2\sigma_\eta^2\sigma_v^2}{1-\delta^2};\ \ E(y_{i1}\Delta y_{i2}^2\Delta u_{i3}) = E(y_{i2}\Delta y_{i3}^2\Delta u_{i4}) = \frac{2\sigma_v^4}{1+\delta}$$
$$E(y_{i1}\Delta y_{i3}^2\Delta u_{i3}) = \frac{2\delta(\delta-2)\sigma_v^4}{1+\delta};\ \ E(y_{i1}\Delta y_{i3}^2\Delta u_{i4}) = \frac{2\delta\sigma_v^4}{1+\delta};\ \ E(\Delta y_{i2}\Delta y_{i3}^2u_{i3}) = \frac{2(\delta-1)\sigma_v^4}{1+\delta}$$

and all other expectaions are equals to zero.

Therefore

$$\begin{aligned}
B_{4sys} = \frac{\phi_d^2}{\phi_s^2}B_{4dif} + \frac{1}{\phi_s^2}\Bigg[& \frac{(\delta-1)\sigma_v^4}{4(\delta+1)} + \frac{\pi_{d1}\sigma_v^4(\delta^2-2\delta+1)}{2(1+\delta)} - \frac{2\pi_{d1}^2\sigma_\eta^2\sigma_v^2}{1-\delta} + \frac{\pi_{d2}\delta\sigma_v^4}{2(1+\delta)} - \frac{2\pi_{d2}^2\sigma_\eta^2\sigma_v^2}{1-\delta} \\
& +\frac{\pi_{d1}\pi_{d3}\sigma_\eta^2\sigma_v^2}{1-\delta} + 2\frac{\pi_{d1}\pi_{d2}\sigma_\eta^2\sigma_v^2}{1-\delta} - 2\frac{\pi_{d2}\pi_{d3}\sigma_\eta^2\sigma_v^2}{1-\delta} - 2\frac{\pi_{d3}^2\sigma_\eta^2\sigma_v^2}{1+\delta} + \frac{\pi_{d3}\sigma_v^4}{2(1+\delta)}\Bigg] \qquad (6.6.4)
\end{aligned}$$

7. Tables and Figures

Table 1: Comparison of biases and RMSEs of one-step first-difference, level and system GMM estimators

			δ								
vr			0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1/4	Diff	Mean	0.0838	0.1779	0.2735	0.3624	0.4491	0.5270	0.5782	0.5259	0.3036
		Bias	-0.0162	-0.0221	-0.0265	-0.0376	-0.0509	-0.0730	-0.1218	-0.2741	-0.5964
		RMSE	0.1586	0.1763	0.1982	0.2285	0.2688	0.3402	0.4818	0.7470	1.1096
	Lev	Mean	0.1010	0.2021	0.3025	0.3966	0.4972	0.5978	0.6982	0.8194	0.9400
		Bias	0.0010	0.0021	0.0025	-0.0034	-0.0028	-0.0022	-0.0018	0.0194	0.0400
		RMSE	0.1518	0.1575	0.1672	0.1768	0.1845	0.1930	0.2217	0.2559	0.2248
	Sys	Mean	0.0978	0.1971	0.2970	0.3917	0.4888	0.5874	0.6798	0.7835	0.9056
		Bias	-0.0022	-0.0029	-0.0030	-0.0083	-0.0112	-0.0126	-0.0202	-0.0165	0.0056
		RMSE	0.1355	0.1425	0.1521	0.1607	0.1691	0.1770	0.1855	0.1916	0.1797
1	Diff	Mean	0.0777	0.1689	0.2637	0.3336	0.4122	0.4314	0.4170	0.3179	0.1596
		Bias	-0.0223	-0.0311	-0.0363	-0.0664	-0.0878	-0.1686	-0.2830	-0.4821	-0.7404
		RMSE	0.1997	0.2227	0.2617	0.3105	0.3992	0.5468	0.7315	0.9089	1.1447
	Lev	Mean	0.1174	0.2207	0.3226	0.4212	0.5335	0.6452	0.7736	0.8999	0.9785
		Bias	0.0174	0.0207	0.0226	0.0212	0.0335	0.0452	0.0736	0.0999	0.0785
		RMSE	0.1931	0.2039	0.2130	0.2317	0.2444	0.2889	0.2856	0.2549	0.1590
	Sys	Mean	0.1143	0.2180	0.3210	0.4160	0.5225	0.6198	0.7316	0.8565	0.9634
		Bias	0.0143	0.0180	0.0210	0.0160	0.0225	0.0198	0.0316	0.0565	0.0634
		RMSE	0.1595	0.1684	0.1788	0.1857	0.1967	0.2009	0.2055	0.1978	0.1525
4	Diff	Mean	0.0730	0.1563	0.2342	0.3036	0.3359	0.3275	0.2760	0.1690	0.1146
		Bias	-0.0270	-0.0437	-0.0658	-0.0964	-0.1641	-0.2725	-0.4240	-0.6310	-0.7854
		RMSE	0.2548	0.3037	0.3626	0.5163	0.5548	0.7799	0.8891	1.1049	1.1571
	Lev	Mean	0.1917	0.3133	0.4290	0.5655	0.6832	0.8079	0.9023	0.9598	0.9930
		Bias	0.0917	0.1133	0.1290	0.1655	0.1832	0.2079	0.2023	0.1598	0.0930
		RMSE	0.2895	0.3168	0.3407	0.3751	0.3814	0.3634	0.3264	0.2365	0.1203
	Sys	Mean	0.1726	0.2831	0.3897	0.5101	0.6211	0.7466	0.8590	0.9405	0.9893
		Bias	0.0726	0.0831	0.0897	0.1101	0.1211	0.1466	0.1590	0.1405	0.0893
		RMSE	0.2205	0.2363	0.2491	0.2564	0.2657	0.2672	0.2573	0.2141	0.1249

Table 2: Comparison of relative biases of one-step First-difference GMM estimator(in %).

vr	1/4		1		4	
δ	Actual Bias	Second Order Bias	Actual Bias	Second Order Bias	Actual Bias	Second Order Bias
0.1	-16.234	-15.206	-22.311	-21.115	-27.039	-27.618
0.2	-11.063	-9.935	-15.543	-14.629	-21.837	-20.220
0.3	-8.817	-8.595	-12.110	-13.507	-21.929	-19.793
0.4	-9.394	-8.425	-16.606	-14.238	-24.093	-22.193
0.5	-10.177	-9.030	-17.563	-16.550	-32.819	-27.542
0.6	-12.174	-10.604	-28.101	-21.295	-45.411	-37.992
0.7	-17.400	-14.112	-40.432	-31.442	-60.575	-60.422
0.8	-34.259	-23.437	-60.262	-58.831	-78.879	-122.476
0.9	-66.265	-67.576	-82.264	-194.900	-87.263	-442.658

Table 3: Comparison of relative biases of one-step Level GMM estimator(in %).

vr	1/4		1		4	
δ	Actual Bias	Second Order Bias	Actual Bias	Second Order Bias	Actual Bias	Second Order Bias
0.1	0.984	2.719	17.353	15.539	91.747	66.818
0.2	1.036	0.443	10.368	6.656	56.651	31.507
0.3	0.844	-0.401	7.534	3.464	43.014	18.924
0.4	-0.838	-0.889	5.300	1.660	41.377	11.855
0.5	-0.553	-1.243	6.692	0.355	36.647	6.746
0.6	-0.371	-1.550	7.525	-0.800	34.656	2.200
0.7	-0.251	-1.878	10.519	-2.075	28.898	-2.863
0.8	2.429	-2.354	12.493	-3.976	19.980	-10.467
0.9	4.441	-3.541	8.721	-8.759	10.337	-29.633

Table 4: Comparison of relative biases of one-step System GMM estimator(in %).

vr	1/4		1		4	
δ	Actual Bias	Second Order Bias	Actual Bias	Second Order Bias	Actual Bias	Second Order Bias
0.1	-2.175	-0.874	14.277	16.492	72.583	84.432
0.2	-1.450	-1.266	8.979	8.875	41.544	47.610
0.3	-0.984	-1.559	6.998	6.127	29.908	34.935
0.4	-2.087	-1.910	4.006	4.431	27.524	28.027
0.5	-2.243	-2.406	4.498	2.954	24.230	23.168
0.6	-2.094	-3.156	3.296	1.357	24.439	19.102
0.7	-2.892	-4.311	4.512	-0.545	22.719	15.299
0.8	-2.058	-6.051	7.068	-2.819	17.561	11.535
0.9	0.623	-8.443	7.040	-5.390	9.927	7.752

Table 5: First-difference GMM estimator

	vr	1/4		1		4	
δ		Actual	Bias Corrected	Actual	Bias Corrected	Actual	Bias Corrected
0.1	Mean	0.08377	0.09897	0.07769	0.09880	0.07296	0.10058
	Bias	-0.01623	-0.00103	-0.02231	-0.00120	-0.02704	0.00058
	RMSE	0.15863	0.15780	0.19968	0.19844	0.25481	0.25337
0.2	Mean	0.17787	0.19775	0.16891	0.19817	0.15633	0.19677
	Bias	-0.02213	-0.00225	-0.03109	-0.00183	-0.04367	-0.00323
	RMSE	0.17627	0.17490	0.22273	0.22056	0.30374	0.30060
0.3	Mean	0.27355	0.29933	0.26367	0.30419	0.23421	0.29359
	Bias	-0.02645	-0.00067	-0.03633	0.00419	-0.06579	-0.00641
	RMSE	0.19816	0.19639	0.26167	0.25917	0.36256	0.35659
0.4	Mean	0.36243	0.39613	0.33358	0.39053	0.30363	0.39240
	Bias	-0.03757	-0.00387	-0.06642	-0.00947	-0.09637	-0.00760
	RMSE	0.22854	0.22546	0.31048	0.30344	0.51632	0.50731
0.5	Mean	0.44911	0.49426	0.41218	0.49493	0.33590	0.47362
	Bias	-0.05089	-0.00574	-0.08782	-0.00507	-0.16410	-0.02638
	RMSE	0.26881	0.26401	0.39924	0.38950	0.55478	0.53062
0.6	Mean	0.52696	0.59058	0.43140	0.55917	0.32753	0.55548
	Bias	-0.07304	-0.00942	-0.16860	-0.04083	-0.27247	-0.04452
	RMSE	0.34018	0.33237	0.54681	0.52177	0.77993	0.73214
0.7	Mean	0.57820	0.67698	0.41698	0.63707	0.27597	0.69893
	Bias	-0.12180	-0.02302	-0.28302	-0.06293	-0.42403	-0.00107
	RMSE	0.48180	0.46672	0.73146	0.67741	0.88914	0.78152
0.8	Mean	0.52593	0.71342	0.31790	0.78855	0.16897	1.14878
	Bias	-0.27407	-0.08658	-0.48210	-0.01145	-0.63103	0.34878
	RMSE	0.74702	0.70030	0.90887	0.77056	1.10489	0.97171
0.9	Mean	0.30362	0.91180	0.15962	1.91372	0.11463	4.09856
	Bias	-0.59638	0.01180	-0.74038	1.01372	-0.78537	3.19856
	RMSE	1.10956	0.93573	1.14471	1.33784	1.15706	3.30949

Table 6: System GMM estimator

	vr	1/4		1		4	
δ		Actual	Bias Corrected	Actual	Bias Corrected	Actual	Bias Corrected
	Mean	0.09783	0.09870	0.11428	0.09778	0.17258	0.08815
0.1	Bias	-0.00217	-0.00130	0.01428	-0.00222	0.07258	-0.01185
	RMSE	0.13553	0.13552	0.15953	0.15890	0.22048	0.20853
	Mean	0.19710	0.19963	0.21796	0.20021	0.28309	0.18787
0.2	Bias	-0.00290	-0.00037	0.01796	0.00021	0.08309	-0.01213
	RMSE	0.14246	0.14243	0.16838	0.16742	0.23633	0.22157
	Mean	0.29705	0.30172	0.32099	0.30261	0.38972	0.28492
0.3	Bias	-0.00295	0.00172	0.02099	0.00261	0.08972	-0.01508
	RMSE	0.15206	0.15204	0.17884	0.17762	0.24909	0.23286
	Mean	0.39165	0.39929	0.41602	0.39830	0.51010	0.39799
0.4	Bias	-0.00835	-0.00071	0.01602	-0.00170	0.11010	-0.00201
	RMSE	0.16073	0.16051	0.18570	0.18502	0.25644	0.23161
	Mean	0.48878	0.50081	0.52249	0.50772	0.62115	0.50531
0.5	Bias	-0.01122	0.00081	0.02249	0.00772	0.12115	0.00531
	RMSE	0.16912	0.16875	0.19672	0.19558	0.26570	0.23653
	Mean	0.58744	0.60637	0.61978	0.61164	0.74663	0.63202
0.6	Bias	-0.01256	0.00637	0.01978	0.01164	0.14663	0.03202
	RMSE	0.17696	0.17663	0.20089	0.20025	0.26720	0.22565
	Mean	0.67975	0.70993	0.73158	0.73540	0.85904	0.75194
0.7	Bias	-0.02025	0.00993	0.03158	0.03540	0.15904	0.05194
	RMSE	0.18550	0.18466	0.20545	0.20607	0.25726	0.20878
	Mean	0.78353	0.83195	0.85654	0.87909	0.94049	0.84821
0.8	Bias	-0.01647	0.03195	0.05654	0.07909	0.14049	0.04821
	RMSE	0.19156	0.19351	0.19781	0.20540	0.21406	0.16855
	Mean	0.90561	0.98160	0.96336	1.01187	0.98934	0.91957
0.9	Bias	0.00561	0.08160	0.06336	0.11187	0.08934	0.01957
	RMSE	0.17974	0.19732	0.15245	0.17816	0.12487	0.08941

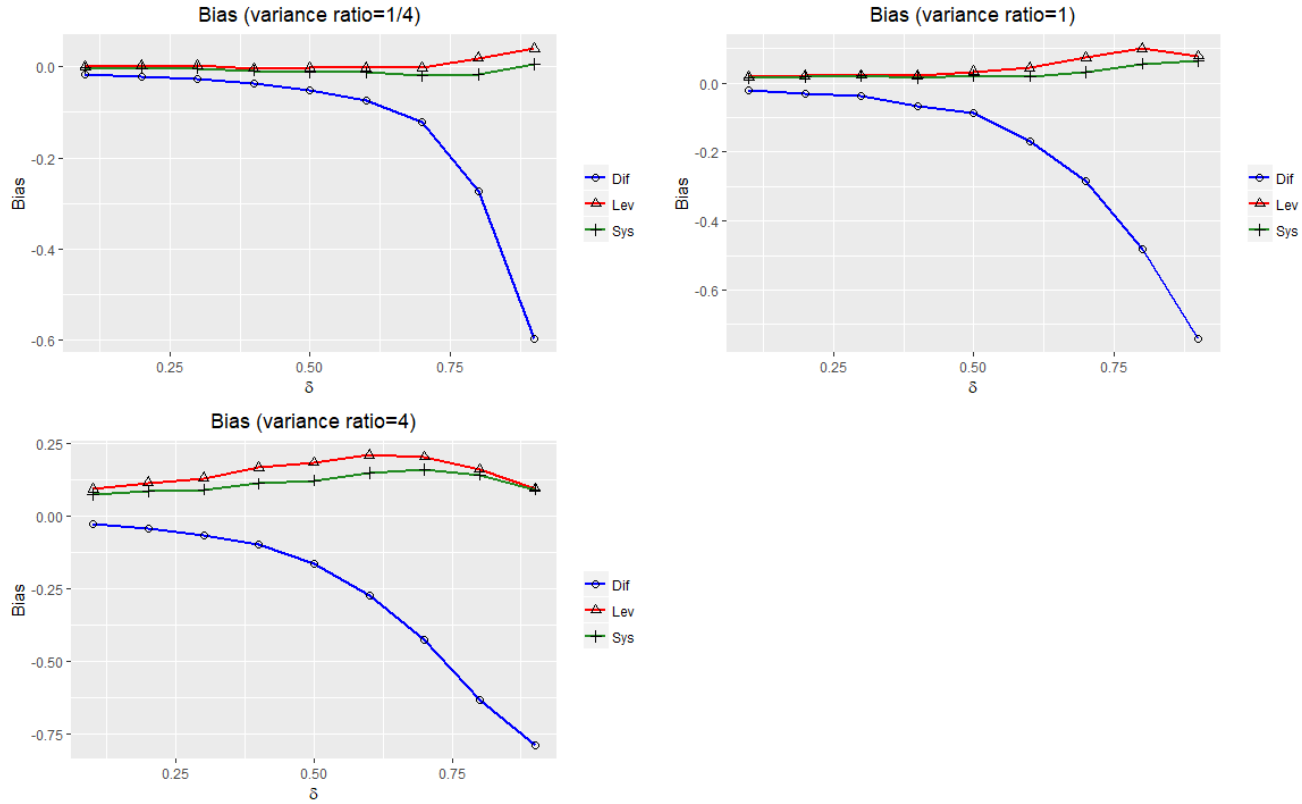

Figure 1: Comparison of biases of First-difference(Dif), Level(Lev) and System(Sys) GMM estimators.

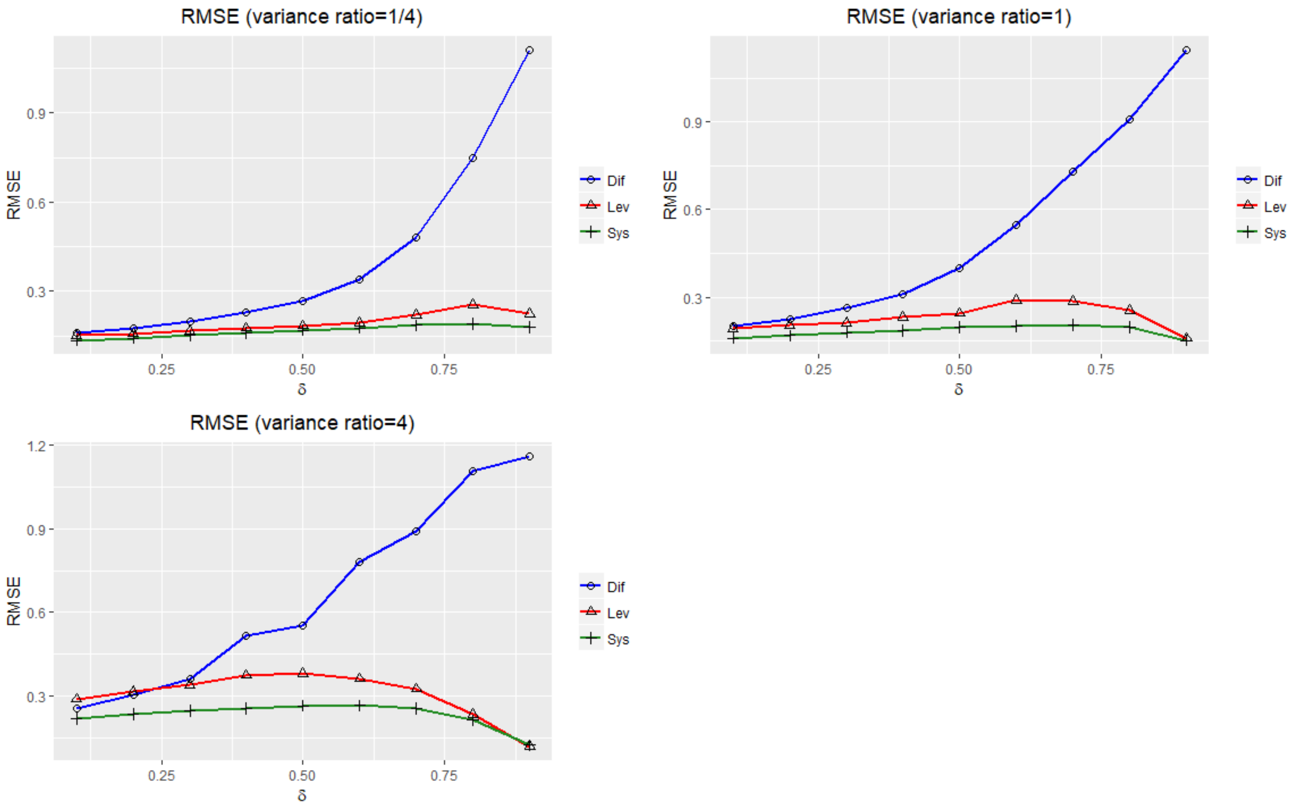

Figure 2: Comparison of RMSEs of First-difference(Dif), Level(Lev) and System(Sys) GMM estimators.

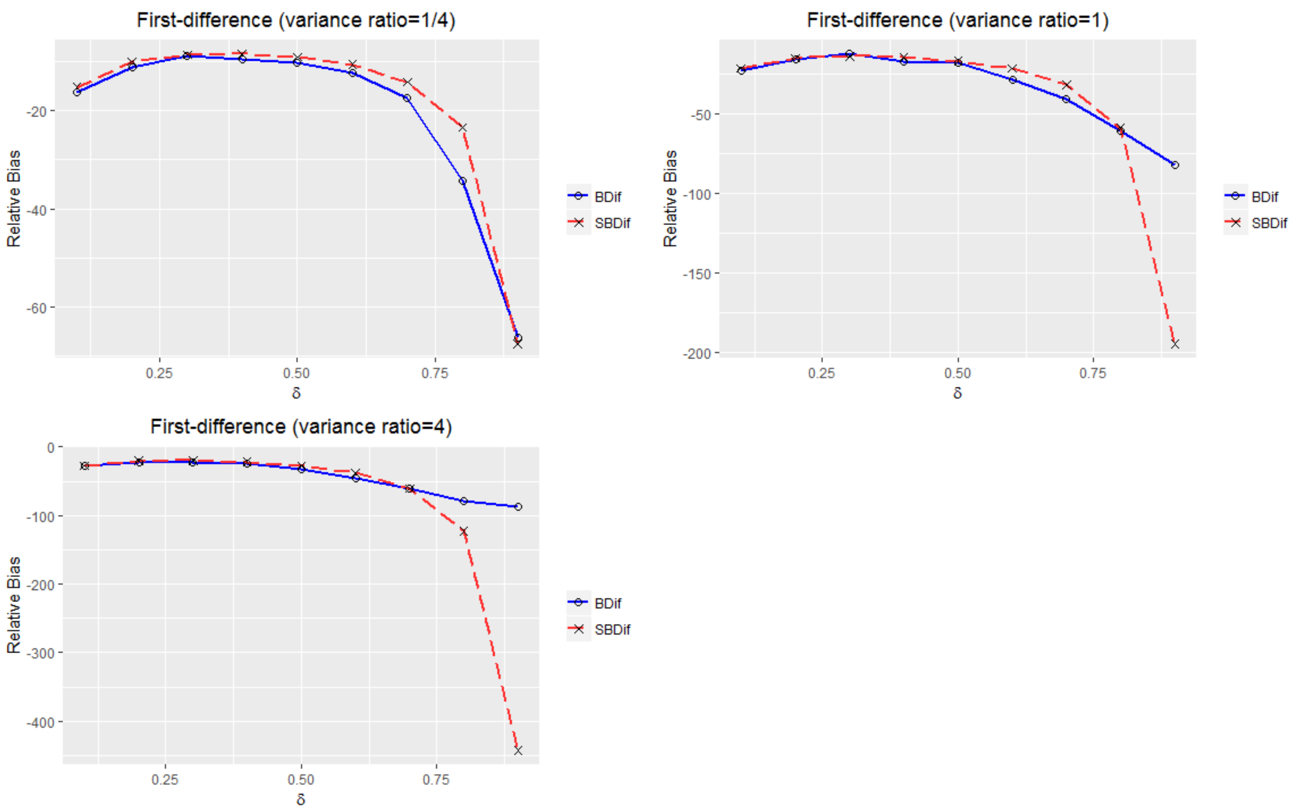

Figure 3: Comparison of relative biases of actual bias and second order bias of first-difference GMM estimator.

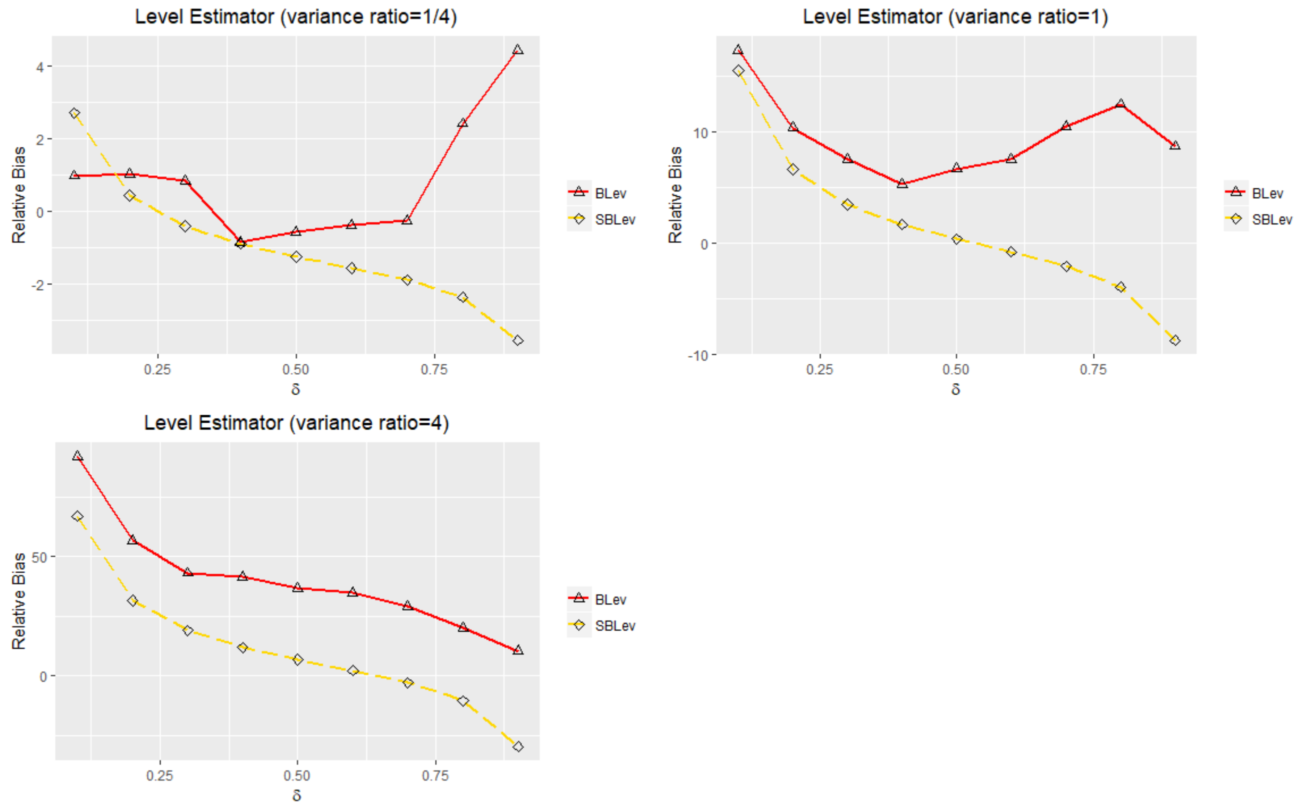

Figure 4: Comparison of relative biases of actual bias and second order bias of level GMM estimator.

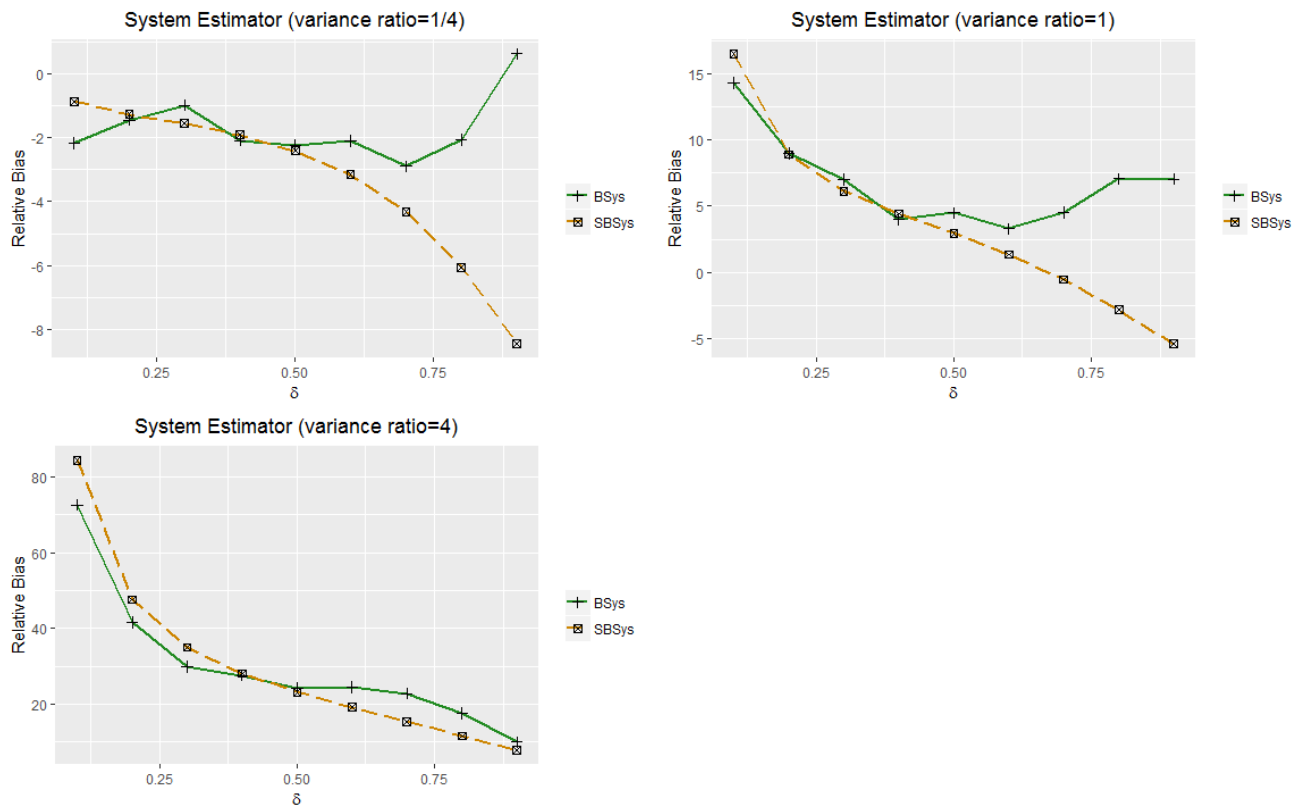

Figure 5: Comparison of relative biases of actual bias and second order bias of system GMM estimator.