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Fuzzy Almost p^*-Compact Space

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Abstract

In [1], fuzzy p^*-open set is introduced and studied. Using this concept as a basic tool, in this paper a new type of fuzzy compactness, viz., fuzzy almost p^*-compactness is introduced. Afterwards, this concept is characterized especially by fuzzy net and prefilterbase. In the last section, it is shown that fuzzy almost p^*-compact space is fuzzy almost compact [3] and the converse is true in fuzzy p^*-regular space [1]. Lastly it is proved that fuzzy almost p^*-compactness remains invariant under fuzzy almost p^*-continuous function [1].

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1. Introduction and Preliminaries

From very beginning, many researchers have engaged themselves to introduce and study different types of fuzzy compactness in fuzzy topological space (fts, for short) in the sense of Chang [2] by introducing various types of fuzzy open-like sets. In this paper we also introduce a new type of fuzzy compactness via fuzzy p^*-open set [1]. We characterize this newly defined fuzzy compactness in several ways. This fuzzy compactness implies fuzzy almost compactness but the converse is true only in fuzzy p^*-regular space [1].

Throughout this paper, (X,τ) or simply by X we shall mean an fts. In 1965, L.A. Zadeh introduced fuzzy set [8] A which is a function from a non-empty set X into the closed interval $I=[0,1]$, i.e., $A\in I^X$. The support [8] of a fuzzy set A, denoted by $suppA$ and is defined by $suppA=\{x\in X : A(x)\neq 0\}$. The fuzzy set with the singleton support $\{x\}\subseteq X$ and the value t $(0<t\leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X. The complement [8] of a fuzzy set A in an fts X is denoted by $1_X\setminus A$ and is defined by $(1_X\setminus A)(x)=1-A(x)$, for each $x\in X$. For any two fuzzy sets A,B in X, $A\leq B$ means $A(x)\leq B(x)$, for all $x\in X$ [8] while AqB means A is quasi-coincident (q-coincident, for short) [7] with B, i.e., there exists $x\in X$ such that $A(x)+B(x)>1$. The negation of these two statements will be denoted by $A\not\leq B$ and $A\not{q}B$ respectively. For a fuzzy set A, clA and $intA$ will

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stand for fuzzy closure [2] and fuzzy interior [2] respectively. A fuzzy set A in X is called a fuzzy neighbourhood (nbd, for short) [7] of a fuzzy point x_t if there exists a fuzzy open set G in X such that $x_t \leq G \leq A$. If, in addition, A is fuzzy open, then A is called fuzzy open nbd of x_t. A fuzzy set A is said to be a fuzzy q-nbd of a fuzzy point x_t in an fts X if there is a fuzzy open set U in X such that $x_t qU \leq A$. If, in addition, A is fuzzy open, then A is called a fuzzy open q-nbd [7] of x_t.

A fuzzy set A in an fts (X,τ) is called fuzzy preopen [6] if $A \leq intclA$. The complement of a fuzzy preopen set is called fuzzy preclosed [6]. The union (intersection) of all fuzzy preopen (resp., fuzzy preclosed) sets contained in (resp., containing) a fuzzy set A is called fuzzy preinterior [6] (resp., fuzzy preclosure [6]) of A, denoted by $pintA$ (resp., $pclA$).

2. Some Well-Known Definitions and Results

In this section we recall some definitions and results from [1,2,3,4,5] for ready references.

Definition 2.1 [1]. A fuzzy set A in an fts (X,τ) is called fuzzy p^*-open if $A \leq int(pclA)$. The complement of this set is called fuzzy p^*-closed set.

The union (resp., intersection) of all fuzzy p^*-open (resp., fuzzy p^*-closed) sets contained in (containing) a fuzzy set A is called fuzzy p^*-interior (resp., fuzzy p^*-closure) of A, denoted by p^*intA (resp., p^*clA).

Definition 2.2 [1]. A fuzzy set A in an fts (X,τ) is called fuzzy p^*-nbd of a fuzzy point x_α in X if there exists a fuzzy p^*-open set U in X such that $x_\alpha \leq U \leq A$. If, in addition, A is fuzzy p^*-open, then A is called fuzzy p^*-open nbd of x_α.

Definition 2.3 [1]. A fuzzy set A in an fts (X,τ) is called fuzzy p^*-q-nbd of a fuzzy point x_α in X if there exists a fuzzy p^*-open set U in X such that $x_\alpha qU \leq A$. If, in addition, A is fuzzy p^*-open, then A is called fuzzy p^*-open q-nbd of x_α.

Result 2.4 [1]. Union (resp., intersection) of any two fuzzy p^*-open (resp., fuzzy p^*-closed) sets is also so.

Result 2.5 [1]. $x_\alpha \in p^*clA$ iff every fuzzy p^*-open q-nbd U of x_α, UqA.

Result 2.6 [1]. $p^*cl(p^*clA) = p^*clA$ for any fuzzy set A in an fts (X,τ).

Result 2.7. $p^*cl(A \vee B) = p^*clA \vee p^*clB$, for any two fuzzy sets A, B in X.

Proof. It is clear that

$$p^*clA \vee p^*clB \subseteq p^*cl(A \vee B) \tag{1}$$

Conversely, let $x_\alpha \leq p^*cl(A \vee B)$. Then for any fuzzy p^*-open q-nbd U of x_α,

$Uq(A \vee B) \Rightarrow$ there exists $y \in X$ such that $U(y) + max\{A(y), B(y)\} > 1 \Rightarrow$ either $U(y) + A(y) > 1 \Rightarrow UqA$ or $U(y) + B(y) > 1 \Rightarrow UqB \Rightarrow$ either $x_\alpha \leq p^*clA$ or $x_\alpha \leq p^*clB \Rightarrow$ $x_\alpha \leq p^*clA \vee p^*clB$.

Definition 2.8 [2, 4]. Let A be a fuzzy set in an fts (X, τ). A collection $\mathcal{U}$ of fuzzy sets in X is called a fuzzy cover of A if $sup\{U(x) : U \in \mathcal{U}\} = 1$, for each $x \in suppA$. If each member of $\mathcal{U}$ is fuzzy open (resp., fuzzy p^*-open), we call $\mathcal{U}$ is fuzzy open (resp., fuzzy p^*-open) cover of A. In particular, if $A = 1_X$, we get the definition of fuzzy cover of X.

Definition 2.9 [2, 3, 4, 5]. A fuzzy cover $\mathcal{U}$ of a fuzzy set A in an fts (X, τ) is said to have a finite (resp., finite proximate) subcover $\mathcal{U}_0$ if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\vee \mathcal{U}_0 \geq A$ (resp., $\vee\{clU : U \in \mathcal{U}_0\} \geq A$). In particular, if $A = 1_X$, we get $\vee \mathcal{U}_0 = 1_X$ (resp., $\vee\{clU : U \in \mathcal{U}_0\} = 1_X$).

Definition 2.10 [3]. An fts (X, τ) is called fuzzy almost compact space if every fuzzy open cover has a finite proximate subcover.

3. Fuzzy Almost p^*-Compactness: Some Characterizations

In this section we first introduce a new type of fuzzy compactness via fuzzy p^*-open and fuzzy regularly p^*-open sets and then characterize it via fuzzy net and prefilterbase.

Definition 3.1. A fuzzy set A in an fts (X, τ) is said to be a fuzzy almost p^*-compact set if every fuzzy p^*-open cover $\mathcal{U}$ of A has a finite p^*-proximate subcover, i.e., there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\vee\{p^*clU : U \in \mathcal{U}_0\} \geq A$. If, in addition, $A = 1_X$, we say that the fts X is fuzzy almost p^*-compact space.

Definition 3.2. Let x_α be a fuzzy point in an fts (X, τ). A prefilterbase $\mathcal{F}$ on X is called

(a) p^*-adhere at x_α, written as $x_\alpha \leq p^*$-ad$\mathcal{F}$, if for each fuzzy p^*-open q-nbd U of x_α and each $F \in \mathcal{F}$, Fqp^*clU, i.e., $x_\alpha \leq p^*clF$, for each $F \in \mathcal{F}$;

(b) p^*-converge to x_α, written as $\mathcal{F} \overrightarrow{p^*} x_\alpha$, if to each fuzzy p^*-open q-nbd U of x_α, there corresponds some $F \in \mathcal{F}$ such that $F \leq p^*clU$.

Definition 3.3. Let x_α be a fuzzy point in an fts (X, τ). A fuzzy net $\{S_n : n \in (D, \geq)\}$ is said to

(a) p^*-adhere at x_α, denoted by $x_\alpha \leq p^*$-ad(S_n), if for each fuzzy p^*-open q-nbd U of x_α and each $n \in D$, there exists $m \in D$ with $m \geq n$ such that $S_m qp^*clU$;

(b) p^*-converge to x_α, denoted by $S_n \overrightarrow{p^*} x_\alpha$, if for each fuzzy p^*-open q-nbd U of x_α, there exists $m \in D$ such that $S_n qp^*clU$, for all $n \geq m (n \in D)$.

Theorem 3.4. For a fuzzy set A in an fts X, the following statements are equivalent:

(a) A is a fuzzy almost p^*-compact set,

(b) for every prefilterbase $\mathcal{B}$ in X, $[\wedge\{p^*clB : B \in \mathcal{B}\}] \wedge A = 0_X \Rightarrow$ there exists a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\wedge\{p^*intB : B \in \mathcal{B}_0\} \not{q} A$,

(c) for any family $\mathcal{F}$ of fuzzy p^*-closed sets in X with $\wedge\{F : F \in \mathcal{F}\} \wedge A = 0_X$, there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\wedge\{p^*intF : F \in \mathcal{F}_0\} \not{q} A$,

(d) every prefilterbase on X, each member of which is q-coincident with A, p^*-adheres at some fuzzy point in A.

Proof (a) $\Rightarrow$ (b). Let $\mathcal{B}$ be a prefilterbase in X such that $[\wedge\{p^*clB : B \in \mathcal{B}\}] \wedge A = 0_X$. Then for any $x \in suppA$, $[\wedge\{p^*clB : B \in \mathcal{B}\}](x) = 0 \Rightarrow 1 - [\wedge\{p^*clB(x) : B \in \mathcal{B}\}] = 1 \Rightarrow \vee[(1_X \setminus p^*clB)(x) : B \in \mathcal{B}] = 1 \Rightarrow sup\{p^*int(1_X \setminus B)(x) : B \in \mathcal{B}\} = 1 \Rightarrow \{p^*int(1_X \setminus B) : B \in \mathcal{B}\}$ is a fuzzy p^*-open cover of A. By (a), there exists a finite p^*-proximate subcover $\{p^*int(1_X \setminus B_1), p^*int(1_X \setminus B_2), ..., p^*int(1_X \setminus B_n)\}$ (say) of it for A. Thus $A \le \bigvee_{i=1}^{n} p^*cl(p^*int(1_X \setminus B_i)) = \bigvee_{i=1}^{n} [1_X \setminus p^*int(p^*clB_i)] = 1_X \setminus \bigwedge_{i=1}^{n} p^*int(p^*clB_i) \Rightarrow \bigwedge_{i=1}^{n} p^*int(p^*clB_i)$ $\le 1_X \setminus A \Rightarrow A \not{q} \bigwedge_{i=1}^{n} p^*int(p^*clB_i) \Rightarrow A \not{q} \bigwedge_{i=1}^{n} p^*intB_i$.

(b) $\Rightarrow$ (a). Let the condition (b) hold, and suppose that there exists a fuzzy p^*-open cover $\mathcal{U}$ of A having no finite p^*-proximate subcover for A. Then for every finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$, there exists $x \in suppA$ such that $sup\{p^*clU(x) : U \in \mathcal{U}_0\} < A(x)$, i.e., $1 - sup\{(p^*clU)(x) : U \in \mathcal{U}_0\} > 1 - A(x) \ge 0 \Rightarrow inf\{(1_X \setminus p^*clU)(x) : U \in \mathcal{U}_0\} > 0$. Thus $\{\bigwedge_{U \in \mathcal{U}_0} (1_X \setminus p^*clU) : \mathcal{U}_0$ is a finite subcollection of $\mathcal{U}\}$ ($= \mathcal{B}$, say) is a prefilterbase in X. If there exists a finite subcollection $\{U_1, U_2, ..., U_n\}$ (say) of $\mathcal{U}$ such that $\bigwedge_{i=1}^{n} p^*int(1_X \setminus p^*clU_i) \not{q} A$, then $A \le 1_X \setminus \bigwedge_{i=1}^{n} p^*int(1_X \setminus p^*clU_i) = \bigvee_{i=1}^{n} [1_X \setminus p^*int(1_X \setminus p^*clU_i)] = \bigvee_{i=1}^{n} p^*cl(p^*clU_i) = \bigvee_{i=1}^{n} p^*clU_i$ (by Result 2.6). Thus $\mathcal{U}$ has a finite p^*-proximate subcover for A, contradicts our hypothesis. Hence for every finite subcollection $\{\bigwedge_{U \in \mathcal{U}_1} (1_X \setminus p^*clU), ..., \bigwedge_{U \in \mathcal{U}_k} (1_X \setminus p^*clU)\}$ of $\mathcal{B}$, where $\mathcal{U}_1, ..., \mathcal{U}_k$ are finite subsets of $\mathcal{U}$, we have $[\bigwedge_{U \in \mathcal{U}_1 \vee ... \vee \mathcal{U}_k} p^*int(1_X \setminus p^*clU)] qA$. By (b), $[\bigwedge_{U \in \mathcal{U}} p^*cl(1_X \setminus p^*clU)] \wedge A \ne 0_X$. Then there exists $x \in suppA$, such that

$\inf_{U\in\mathcal{U}}[p^*cl(1_X \setminus p^*clU)](x) > 0 \Rightarrow 1-\inf_{U\in\mathcal{U}}[p^*cl(1_X \setminus p^*clU)](x) < 1 \Rightarrow \sup_{U\in\mathcal{U}}[1_X \setminus p^*cl(1_X \setminus p^*clU)](x)$ $< 1 \Rightarrow \sup_{U\in\mathcal{U}} U(x) \le \sup_{U\in\mathcal{U}} p^*int(p^*clU)(x) < 1$ which contradicts that $\mathcal{U}$ is a fuzzy p^*-open cover of A.

(a) $\Rightarrow$ (c). Let $\mathcal{F}$ be a family of fuzzy p^*-closed sets in X such that $\wedge\{F : F\in\mathcal{F}\} \wedge A = 0_X$. Then for each $x \in suppA$ and for each positive integer n, there exists some $F_n \in \mathcal{F}$ such that $F_n(x) < 1/n \Rightarrow 1 - F_n(x) > 1 - 1/n \Rightarrow \sup_{F\in F}[(1_X \setminus F)(x)] = 1$ and so $\{1_X \setminus F : F \in \mathcal{F}\}$ is a fuzzy p^*-open cover of A. By (a), there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $A \le \bigvee_{F\in\mathcal{F}_0} p^*cl(1_X \setminus F) \Rightarrow 1_X \setminus A \ge 1_X \setminus \bigvee_{F\in\mathcal{F}_0} p^*cl(1_X \setminus F) = \bigwedge_{F\in\mathcal{F}_0} (1_X \setminus p^*cl(1_X \setminus F)) = \bigwedge_{F\in\mathcal{F}_0} p^*intF$. Hence $A\not{q}(\bigwedge_{F\in\mathcal{F}_0} p^*intF)$, where $\mathcal{F}_0$ is a finite subcollection of $\mathcal{F}$.

(c) $\Rightarrow$ (b). Let $\mathcal{B}$ be a prefilterbase in X such that $[\wedge\{p^*clB : B\in\mathcal{B}\}] \wedge A = 0_X$. Then the family $\mathcal{F} = \{p^*clB : B\in\mathcal{B}\}$ is a family of fuzzy p^*-closed sets in X with $(\wedge\mathcal{F}) \wedge A = 0_X$. By (c), there is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $[\wedge\{p^*int(p^*clB) : B\in\mathcal{B}_0\}]\not{q}A \Rightarrow (\bigwedge_{B\in\mathcal{B}_0} p^*intB)\not{q}A$.

(a) $\Rightarrow$ (d). Let $\mathcal{F}$ be a prefilterbase in X, each member of which is q-coincident with A. If possible, let $\mathcal{F}$ do not p^*-adhere at any fuzzy point in A. Then for each $x \in suppA$, there exists $n_x \in \mathcal{N}$ such that $x_{1/n} \le A$. Then there are a fuzzy p^*-open set $U^x_{n_x}$ and a member $F^x_{n_x}$ of $\mathcal{F}$ such that $x_{1/n}qU^x_{n_x}$ and $p^*clU^x_{n_x}\not{q}F^x_{n_x}$. Thus $U^x_{n_x}(x) > 1 - 1/n$ so that $sup\{U^x_n(x) : n\in\mathcal{N}, n \ge n_x\} = 1$. Thus $\{U^x_n : n\in\mathcal{N}, n\ge n_x, x\in suppA\}$ forms a fuzzy p^*-open cover of A. By (a), there exist finitely many points $x_1, x_2, ..., x_k \in suppA$ and $n_1, n_2, ..., n_k \in \mathcal{N}$ such that $A \le \bigvee_{i=1}^{k} p^*clU^{x_i}_{n_{x_i}}$. Choose $F\in\mathcal{F}$ such that $F \le \bigwedge_{i=1}^{k} F^{x_i}_{n_i}$. Then $F\not{q}[\bigvee_{i=1}^{k} p^*clU^{x_i}_{n_{x_i}}]$, i.e., $F\not{q}A$, a contradiction.

(d) $\Rightarrow$ (a). If possible, let there exist a fuzzy p^*-open cover $\mathcal{U}$ of A such that for every finite subset $\mathcal{U}_0$ of $\mathcal{U}$, $\vee\{p^*clU : U\in\mathcal{U}_0\} \not\ge A$. Then $\mathcal{F} = \{1_X \setminus \bigvee_{U\in\mathcal{U}_0} p^*clU : \mathcal{U}_0$ is a finite subset of $\mathcal{U}\}$ is a prefilterbase on X such that FqA, for each $F\in\mathcal{F}$. By (d), $\mathcal{F}$ p^*-adheres at some fuzzy point $x_\alpha \le A$. As $\mathcal{U}$ is a fuzzy cover of A, $\sup_{U\in\mathcal{U}} U(x) = 1 \Rightarrow$ there exists $U_0 \in \mathcal{U}$ such that $U_0(x) > 1-\alpha \Rightarrow x_\alpha q U_0$. As $x_\alpha \le p^* - \text{ad}\mathcal{F}$ and $1_X \setminus p^*clU_0 \in \mathcal{F}$, we have $p^*clU_0 q(1_X \setminus p^*clU_0)$, a contradiction.

Theorem 3.5. For a fuzzy set A in an fts X, the following implications hold:

(a) every fuzzy net in A p^*-adheres at some fuzzy point in A,

$\Leftrightarrow$ (b) every fuzzy net in A has a p^*-convergent fuzzy subnet,

$\Leftrightarrow$ (c) every prefilterbase in A p^*-adheres at some fuzzy point in A,

$\Rightarrow$ (d) for every family $\{B_\alpha : \alpha \in \Lambda\}$ of non-null fuzzy sets with $[\bigwedge_{\alpha\in\Lambda} p^*clB_\alpha] \wedge A = 0_X$, there is a finite subset Λ_0 of Λ such that $(\bigwedge_{\alpha\in\Lambda_0} B_\alpha) \wedge A = 0_X$,

$\Rightarrow$ (e) A is fuzzy almost p^*-compact.

Proof (a) $\Rightarrow$ (b). Let a fuzzy net $\{S_n : n \in (D,\geq)\}$ in A where $(D,\geq)$ is a directed set, p^*-adhere at a fuzzy point $x_\alpha \leq A$. Let Q_{x_α} denote the set of the fuzzy p^*-closures of all fuzzy p^*-open q-nbds of x_α. For any $B \in Q_{x_\alpha}$, we can choose some $n \in D$ such that $S_n qB$. Let E denote the set of all ordered pairs (n,B) with the property that $n \in D$, $B \in Q_{x_\alpha}$ and $S_n qB$. Then $(E,\gg)$ is a directed set where $(m,C) \gg (n,B)$ iff $m \geq n$ in D and $C \leq B$. Then $T:(E,\gg) \to (X,\tau)$ given by $T(n,B) = S_n$, is a fuzzy subnet of $\{S_n : n \in (D,\geq)\}$. Let V be any fuzzy p^*-open q-nbd of x_α. Then there is $n \in D$ such that that $(n, p^*clV) \in E$ and hence $S_n qp^*clV$. Now, for any $(m,U) \gg (n, p^*clV)$, $T(m,U) = S_m qU \leq p^*clV \Rightarrow T(m,U)qp^*clV$. Hence $T \overrightarrow{p^*} x_\alpha$.

(b) $\Rightarrow$ (a). If a fuzzy net $\{S_n : n \in (D,\geq)\}$ does not p^*-adhere at a fuzzy point x_α, then there is a fuzzy p^*-open q-nbd U of x_α and an $n \in D$ such that $S_m \not{q} p^*clU$, for all $m \geq n$. Then obviously no fuzzy subnet of the fuzzy net can p^*-converge to x_α.

(a) $\Rightarrow$ (c). Let $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a prefilterbase in A. For each $\alpha \in \Lambda$, choose a fuzzy point $x_{F_\alpha} \leq F_\alpha$ and construct the fuzzy net $S = \{x_{F_\alpha} : F_\alpha \in \mathcal{F}\}$ in A with $(\mathcal{F},\gg)$ as domain, where for two members $F_\alpha, F_\beta \in \mathcal{F}$, $F_\alpha \gg F_\beta$ iff $F_\alpha \leq F_\beta$. By (a), the fuzzy net S p^*-adheres at some fuzzy point x_t $(0 < t \leq 1) \leq A$. Then for any fuzzy p^*-open q-nbd U of x_t and any $F_\alpha \in \mathcal{F}$, there exists $F_\beta \in \mathcal{F}$ such that $F_\beta \gg F_\alpha$ and $x_{F_\beta} qp^*clU$. Then $F_\beta qp^*clU$ and hence $F_\alpha qp^*clU$. Thus $\mathcal{F}$ p^*-adheres at x_t.

(c) $\Rightarrow$ (a). Let $\{S_n : n \in (D,\geq)\}$ be a fuzzy net in A. Consider the prefilterbase $\mathcal{F} = \{T_n : n \in D\}$ generated by the net, where $T_n = \{S_m : m \in D, m \geq n\}$. By (c), there exists a fuzzy point $a_\alpha \leq A$ such that $\mathcal{F}$ p^*-adheres at a_α. Then for each fuzzy p^*-open q-nbd U of a_α and each $F \in \mathcal{F}$, Fqp^*clU, i.e., p^*clUqT_n, for all $n \in D$. Hence the given fuzzy net p^*-adheres at a_α.

(c) $\Rightarrow$ (d). Let $\mathcal{B} = \{B_\alpha : \alpha \in \Lambda\}$ be a family of fuzzy sets in X such that for every finite subset

Λ_0 of Λ, $(\bigwedge_{\alpha\in\Lambda_0} B_\alpha)\wedge A\neq 0_X$. Then $\mathcal{F}=\{(\bigwedge_{\alpha\in\Lambda_0} B_\alpha)\wedge A:\Lambda_0$ is a finite subset of $\Lambda\}$ is a prefilterbase in A. By (c), $\mathcal{F}$ p^*-adheres at some fuzzy point $a_t\leq A$ $(0<t\leq 1)$. Then for each $\alpha\in\Lambda$ and each fuzzy p^*-open q-nbd U of a_t, $B_\alpha q p^* clU$, i.e., $a_t\leq p^*clB_\alpha$, for each $\alpha\in\Lambda$. Consequently, $(\bigwedge_{\alpha\in\Lambda} p^*clB_\alpha)\wedge A\neq 0_X$.

(d) $\Rightarrow$ (e). Let $\mathcal{U}=\{U_\alpha:\alpha\in\Lambda\}$ be a fuzzy p^*-open cover of a fuzzy set A. Then by (d), $A\wedge[\bigwedge_{\alpha\in\Lambda}(1_X\setminus p^*\text{int}(p^*clU_\alpha))]\leq A\wedge[\bigwedge_{\alpha\in\Lambda}(1_X\setminus U_\alpha)]=0_X$. If for some $\alpha\in\Lambda$, $1_X\setminus p^*clU_\alpha=0_X$, then we are done. If $1_X\setminus p^*clU_\alpha$ $(=B_\alpha$, say) $\neq 0_X$, then for each $\alpha\in\Lambda$, $\mathcal{B}=\{B_\alpha:\alpha\in\Lambda\}$ is a family of non-null fuzzy sets. We show that $\bigwedge_{\alpha\in\Lambda} p^*clB_\alpha\leq\bigwedge_{\alpha\in\Lambda}(1_X\setminus U_\alpha)$. In fact, let x_t $(0<t\leq 1)$ be a fuzzy point such that $x_t\leq p^*clB_\alpha=p^*cl(1_X\setminus p^*clU_\alpha)$. If $x_t qU_\alpha$, then $p^*clU_\alpha q(1_X\setminus p^*clU_\alpha)$, which is absurd. Hence $x_t \not{q} U_\alpha\Rightarrow x_t\leq 1_X\setminus U_\alpha$. Then $[\bigwedge_{\alpha\in\Lambda} p^*clB_\alpha]\wedge A\leq A\wedge[\bigwedge_{\alpha\in\Lambda}(1_X\setminus p^*\text{int}(p^*clU_\alpha))]\leq A\wedge[\bigwedge_{\alpha\in\Lambda}(1_X\setminus U_\alpha)]=0_X$. By (d), there exists a finite subset Λ_0 of Λ such that $[\bigwedge_{\alpha\in\Lambda_0} B_\alpha]\wedge A=0_X$, i.e., $A\leq 1_X\setminus\bigwedge_{\alpha\in\Lambda_0} B_\alpha=\bigvee_{\alpha\in\Lambda_0}(1_X\setminus B_\alpha)$ $=\bigvee_{\alpha\in\Lambda_0} p^*clU_\alpha$ and (e) follows.

Definition 3.6. A fuzzy set A in an fts (X,τ) is said to be fuzzy regularly p^*-open if $A=p^*int(p^*clA)$. The complement of such a set is called fuzzy regularly p^*-closed.

Definition 3.7. A fuzzy point x_α in X is said to be a fuzzy p^*-cluster point of a prefilterbase $\mathcal{B}$ if $x_\alpha\leq p^*clB$, for all $B\in\mathcal{B}$. If, in addition, $x_\alpha\leq A$, for a fuzzy set A, then $\mathcal{B}$ is said to have a fuzzy p^*-cluster point in A.

Theorem 3.8. A fuzzy set A in an fts (X,τ) is fuzzy almost p^*-compact iff for each prefilterbase $\mathcal{F}$ in X which is such that for each set of finitely many members $F_1,F_2,\ldots,F_n$ from $\mathcal{F}$ and for any fuzzy regular p^*-closed set C containing A, one has $(F_1\wedge\ldots\wedge F_n)qC$, $\mathcal{F}$ has a fuzzy p^*-cluster point in A.

Proof. Let A be fuzzy almost p^*-compact set and suppose $\mathcal{F}$ be a prefilterbase in X such that $[\wedge\{p^*clF:F\in\mathcal{F}\}]\wedge A=0_X\ldots(1)$. Let $x\in suppA$. Consider any $n\in\mathcal{N}$ such that $1/n<A(x)$, i.e., $x_{1/n}\leq A$. By (1), $x_{1/n}\not\leq p^*clF_x^n$, for some $F_x^n\in\mathcal{F}$. Then there exists a fuzzy p^*-open q-nbd U_x^n of $x_{1/n}$ such that $U_x^n \not{q} F_x^n$. Now $U_x^n(x)>1-1/n$ $\Rightarrow sup\{U_x^n(x):1/n<A(x),n\in N\}=1\Rightarrow\mathcal{U}=\{U_x^n:x\in suppA,n\in N\}$ forms a fuzzy p^*-open cover

of A such that for U_x^n, there exists $F_x^n \in \mathcal{F}$ with $U_x^n \not{q} F_x^n$. Since A is fuzzy almost p^*-compact, there exist finitely many members $U_{x_1}^{n_1},,,,,U_{x_k}^{n_k}$ of $\mathcal{U}$ such that $A \le \bigvee_{i=1}^{k} p^*clU_{x_i}^{n_i} = p^*cl(\bigvee_{i=1}^{k} U_{x_i}^{n_i})$ (=U, say). Now $F_{x_1}^{n_1},...,F_{x_k}^{n_k} \in \mathcal{F}$ such that $U_{x_i}^{n_i} \not{q} F_{x_i}^{n_i}$ for $i = 1,2,...,k$. Now U is a fuzzy regularly p^*-closed set containing A such that $U \not{q} (F_{x_1}^{n_1} \wedge .. \wedge F_{x_k}^{n_k})$.

Conversely, let $\mathcal{B}$ be a prefilterbase in X having no fuzzy p^*-cluster point in A. Then by hypothesis, there is a fuzzy regularly p^*-closed set C containing A such that for some finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$, $(\wedge \mathcal{B}_0) \not{q} C$. Then $(\wedge \mathcal{B}_0) \not{q} A$. By Theorem 3.4 (b) $\Rightarrow$ (a), A is fuzzy almost p^*-compact set.

From Theorem 3.4, Theorem 3.5 and Theorem 3.8, we have the characterizations of fuzzy almost p^*-compact space as follows.

Theorem 3.9. For an fts X, the following statements are equivalent :

(a) X is fuzzy almost p^*-compact,

(b) every fuzzy net in X p^*-adheres at some fuzzy point in X,

(c) every fuzzy net in X has a p^*-convergent fuzzy subnet,

(d) every prefilterbase in X p^*-adheres at some fuzzy point in X,

(e) for every family $\{B_\alpha : \alpha \in \Lambda\}$ of non-null fuzzy sets with $\bigwedge_{\alpha \in \Lambda} p^*clB_\alpha = 0_X$, there is a finite subset Λ_0 of Λ such that $(\bigwedge_{\alpha \in \Lambda_0} B_\alpha) = 0_X$,

(f) for every prefilterbase $\mathcal{B}$ in X with $\wedge\{p^*clB : B \in \mathcal{B}\} = 0_X$, there is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\wedge\{p^*intB : B \in \mathcal{B}_0\} = 0_X$,

(g) for any family $\mathcal{F}$ of fuzzy p^*-closed sets in X with $\wedge \mathcal{F} = 0_X$, there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\wedge\{p^*intF : F \in \mathcal{F}_0\} = 0_X$.

Theorem 3.10. An fts X is fuzzy almost p^*-compact iff for any collection $\{F_\alpha : \alpha \in \Lambda\}$ of fuzzy p^*-open sets in X having finite intersection property $\wedge\{p^*clF_\alpha : \alpha \in \Lambda\} \neq 0_X$.

Proof. Let X be fuzzy almost p^*-compact space and $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a collection of fuzzy p^*-open sets in X with finite intersection property. Suppose $\wedge\{p^*clF_\alpha : \alpha \in \Lambda\} = 0_X$. Then $\{1_X \setminus p^*clF_\alpha : \alpha \in \Lambda\}$ is a fuzzy p^*-open cover of X. By hypothesis, there exists a finite subset Λ_0 of Λ such that $1_X = \vee\{p^*cl(1_X \setminus p^*clF_\alpha) : \alpha \in \Lambda_0\} = \vee\{1_X \setminus p^*int(p^*clF_\alpha) : \alpha \in \Lambda_0\}$ $\le \vee\{1_X \setminus F_\alpha : \alpha \in \Lambda_0\} = 1_X \setminus \bigwedge_{\alpha \in \Lambda_0} F_\alpha \Rightarrow \bigwedge_{\alpha \in \Lambda_0} F_\alpha = 0_X$ which contradicts the fact that $\mathcal{F}$ has finite intersection property.

Conversely, suppose that X is not fuzzy almost p^*-compact. Then there is a fuzzy p^*-open cover $\mathcal{F}=\{F_\alpha:\alpha\in\Lambda\}$ of X such that for every finite subset Λ_0 of Λ, $\vee\{p^*clF_\alpha:\in\Lambda_0\}\neq 1_X$. Then $1_X\setminus\vee\{p^*clF_\alpha:\alpha\in\Lambda_0\}\neq 0_X \Rightarrow \bigwedge_{\alpha\in\Lambda_0}(1_X\setminus p^*clF_\alpha)\neq 0_X$, for every finite subset Λ_0 of Λ. Thus $\{1_X\setminus p^*clF_\alpha:\alpha\in\Lambda\}$ is a collection of fuzzy p^*-open sets with finite intersection property. By hypothesis, $\bigwedge_{\alpha\in\Lambda} p^*cl(1_X\setminus p^*clF)\neq 0_X$, i.e., $1_X\setminus\bigvee_{\alpha\in\Lambda} p^*int(p^*clF_\alpha)\neq 0_X\Rightarrow\bigvee_{\alpha\in\Lambda} p^*int(p^*clF_\alpha)\neq 1_X$. Hence $\bigvee_{\alpha\in\Lambda} F_\alpha\neq 1_X$, a contradiction as $\mathcal{F}$ is a fuzzy p^*-open cover of X.

Definition 3.11. Let $\{S_n:n\in(D,\geq)\}$ be a fuzzy net of fuzzy p^*-open sets in X, i.e., for each member n of a directed set $(D,\geq)$, S_n is a fuzzy p^*-open set in X. A fuzzy point x_α in X is said to be a fuzzy p^*-cluster point of the fuzzy net if for every $n\in D$ and every fuzzy p^*-open q-nbd V of x_α, there exists $m\in D$ with $m\geq n$ such that S_mqV.

Theorem 3.12. An fts X is fuzzy almost p^*-compact iff every fuzzy net of fuzzy p^*-open sets in X has a fuzzy p^*-cluster point in X.

Proof. Let $\mathcal{U}=\{S_n:n\in(D,\geq)\}$ be a fuzzy net of fuzzy p^*-open sets in a fuzzy almost p^*-compact space X. For each $n\in D$, let us put $F_n=p^*cl[\vee\{S_m:m\in D$ and $m\geq n\}]$. Then $\mathcal{F}=\{F_n:n\in D\}$ is a family of fuzzy p^*-closed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\wedge\{p^*intF:F\in\mathcal{F}_0\}\neq 0_X$. By Theorem 3.9 (a)$\Rightarrow$ (g), $\bigwedge_{n\in D}F_n\neq 0_X$. Let $x_\alpha\in\bigwedge_{n\in D}F_n$. Then $x_\alpha\in F_n$, for all $n\in D$. Thus for any fuzzy p^*-open q-nbd A of x_α and any $n\in D$, $Aq[\vee\{S_m:m\geq n\}]$ and so there exists some $m\in D$ with $m\geq n$ and $AqS_m\Rightarrow x_\alpha$ is a fuzzy p^*-cluster point of $\mathcal{U}$.

Conversely, let $\mathcal{F}$ be a collection of fuzzy p^*-closed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\wedge\{p^*intF:F\in\mathcal{F}_0\}\neq 0_X$. Let $\mathcal{F}^*$ denote the family of all finite intersections of members of F directed by the relation '$\gg$' such that for $F_1,F_2\in\mathcal{F}^*$, $F_1\gg F_2$ iff $F_1\leq F_2$. Let $F^*=p^*intF$, for each $F\in\mathcal{F}^*$. Then $F^*\neq 0_X$. Consider the fuzzy net $\mathcal{U}=\{F^*:F\in(\mathcal{F}^*,\gg)\}$ of non-null fuzzy p^*-open sets of X. By hypothesis, $\mathcal{U}$ has a fuzzy p^*-cluster point, say x_α. We claim that $x_\alpha\in\wedge\mathcal{F}$. In fact, let $F\in\mathcal{F}$ be arbitrary and A be any fuzzy p^*-open q-nbd of x_α. Since $F\in\mathcal{F}^*$ and x_α is a fuzzy p^*-cluster point of $\mathcal{U}$, there exists $G\in\mathcal{F}^*$ such that $G\gg F$ (i.e., $G\leq F$) and $G^*qA\Rightarrow GqA\Rightarrow FqA\Rightarrow x_\alpha\in p^*clF=F$, for each $F\in\mathcal{F}\Rightarrow x_\alpha\in\wedge\mathcal{F}\Rightarrow\wedge\mathcal{F}\neq 0_X$. By Theorem 3.9 (g) $\Rightarrow$ (a), X is fuzzy almost p^*-compact.

Definition 3.13. A fuzzy cover $\mathcal{U}$ by fuzzy p^*-closed sets of an fts (X,τ) will be called a fuzzy p^*-cover of X if for each fuzzy point x_α $(0<\alpha<1)$ in X, there exists $U\in\mathcal{U}$ such that U is a fuzzy p^*-open nbd of x_α.

Theorem 3.14. An fts (X,τ) is fuzzy almost p^*-compact iff every fuzzy p^*-cover of X has a finite subcover.

Proof. Let X be fuzzy almost p^*-compact and $\mathcal{U}$ be any fuzzy p^*-cover of X. Then for each $n\in N$ with $n>1$, there exist $U_x^n\in\mathcal{U}$ and a fuzzy p^*-open set V_x^n in X such that $x_{1-1/n}\le V_x^n\le U_x^n$. Then $V_x^n(x)\ge 1-1/n \;\Rightarrow sup\{V_x^n(x):n\in N\}=1\Rightarrow \mathcal{V}=\{V_x^n : x\in X, n\in N, n>1\}$ is a fuzzy p^*-open cover of X. As X is fuzzy almost p^*-compact, there exist finitely many points $x_1,x_2,...,x_m\in X$ and $n_1,n_2,...,n_m\in N\setminus\{1\}$ such that $1_X=\bigvee_{k=1}^{m} p^*clV_{x_k}^{n_k}\le\bigvee_{k=1}^{m} p^*clU_{x_k}^{n_k}=\bigvee_{k=1}^{m} U_{x_k}^{n_k}$.

Conversely, let $\mathcal{U}$ be fuzzy p^*-open cover of X. For any fuzzy point x_α $(0<\alpha<1)$ in X, as $\sup\limits_{U\in\mathcal{U}} U(x)=1$, there exists $U_{x_\alpha}\in\mathcal{U}$ such that $U_{x_\alpha}(x)\ge\alpha$ $(0<\alpha<1)$. Then $\mathcal{V}=\{p^*clU : U\in\mathcal{U}\}$ is a fuzzy p^*-cover of X and the rest is clear.

The following theorem gives a necessary condition for an fts to be fuzzy almost p^*-compact.

Theorem 3.15. If an fts X is fuzzy almost p^*-compact, then every prefilterbase on X with at most one p^*-adherent point is p^*-convergent.

Proof. Let $\mathcal{F}$ be a prefilterbase with at most one p^*-adherent point in a fuzzy almost p^*-compact fts X. Then by Theorem 3.9, $\mathcal{F}$ has at least one p^*-adherent point in X. Let x_α be the unique p^*-adherent point of $\mathcal{F}$ and if possible, let $\mathcal{F}$ do not p^*-converge to x_α. Then for some fuzzy p^*-open q-nbd U of x_α and for each $F\in\mathcal{F}$, $F\not\le p^*clU$, so that $F\wedge\{1_X\setminus p^*clU\}\ne 0_X$. Then $\mathcal{G}=\{F\wedge(1_X\setminus p^*clU):F\in\mathcal{F}\}$ is a prefilterbase in X and hence has a p^*-adherent point y_t (say) in X. Now $p^*clU\not q G$, for all $G\in\mathcal{G}$ so that $x_\alpha\ne y_t$. Again, for each fuzzy p^*-open q-nbd V of y_t and each $F\in\mathcal{F}$, $p^*clVq(F\wedge(1_X\setminus p^*clU))\Rightarrow p^*clVqF\Rightarrow y_t$ is a fuzzy p^*-adherent point of $\mathcal{F}$, where $x_\alpha\ne y_t$. This contradicts the fact that x_α is the only fuzzy p^*-adherent point of $\mathcal{F}$.

Some results on fuzzy almost p^*-compactness of an fts are given by the following theorem.

Theorem 3.16. Let (X,τ) be an fts and $A\in I^X$. Then the following statements are true :

(a) If A is fuzzy almost p^*-compact, then so is p^*clA,

(b) Union of two fuzzy almost p^*-compact sets is also so,

(c) If X is fuzzy almost p^*-compact, then every fuzzy regularly p^*-closed set A in X is fuzzy almost p^*-compact set.

Proof (a). Let $\mathcal{U}$ be a fuzzy p^*-open cover of p^*clA. Then $\mathcal{U}$ is also a fuzzy p^*-open cover of A. As A is fuzzy almost p^*-compact, there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $A \le \vee\{p^*clU : U \in \mathcal{U}_0\} = p^*cl[\vee\{U : U \in \mathcal{U}_0\}] \Rightarrow p^*clA \le p^*cl\{p^*cl[\vee\{U : U \in \mathcal{U}_0\}]\} = p^*cl[\vee\{U : U \in \mathcal{U}_0\}] = \vee\{p^*clU : U \in \mathcal{U}_0\}$. Hence the result.

(b). Obvious.

(c). Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy p^*-open cover of a fuzzy regularly p^*-closed set A in X. Then for each $x \notin suppA$, $A(x) = 0 \Rightarrow (1_X \setminus A)(x) = 1 \Rightarrow \mathcal{U} \vee \{(1_X \setminus A)\}$ is a fuzzy p^*-open cover of X. Since X is fuzzy almost p^*-compact, there are finitely many members $U_1, U_2, ..., U_n$ in $\mathcal{U}$ such that $1_X = (p^*clU_1 \vee ... \vee p^*clU_n) \vee p^*cl(1_X \setminus A)$. We claim that $p^*intA \le p^*clU_1 \vee ... \vee p^*clU_n$. If not, there exists a fuzzy point $x_\beta \le p^*intA$, but $x_\beta \not\le (p^*clU_1 \vee ... \vee p^*clU_n)$, i.e., $\beta > max\{(p^*clU_1)(x), ..., (p^*clU_n)(x)\}$. As $1_X = (p^*clU_1 \vee ... \vee p^*clU_n) \vee p^*cl(1_X \setminus A)$, $[p^*cl(1_X \setminus A)](x) = 1 \Rightarrow 1 - p^*intA(x) = 1 \Rightarrow p^*intA(x) = 0 \Rightarrow x_\beta \notin p^*intA$, a contradiction. Hence $A = p^*cl(p^*intA) \le p^*cl(p^*clU_1 \vee .. \vee p^*clU_n) = p^*clU_1 \vee ... \vee p^*clU_n$ (by Result 2.6 and Result 2.7) $\Rightarrow A$ is fuzzy almost p^*-compact.

4. Mutual Relation

In this section we first establish the mutual relation between fuzzy almost compact space [3] and fuzzy almost p^*-compact space. It is also established that fuzzy almost p^*-compactness remains invariant under fuzzy almost p^*-continuous function [1].

Since for any fuzzy set A in an fts X, $p^*clA \le clA$ (as every fuzzy closed set is fuzzy p^*-closed [1]), we can state the following theorem easily.

Theorem 4.1. Every fuzzy almost p^*-compact space is fuzzy almost compact.

To get the converse we have to recall the following definition and theorem for ready references.

Definition 4.2 [1]. An fts (X, τ) is said to be fuzzy p^*-regular if for each fuzzy p^*-closed set F in X and each fuzzy point x_α in X with $x_\alpha q(1_X \setminus F)$, there exists a fuzzy open set U in X and a fuzzy p^*-open set V in X such that $x_\alpha qU$, $F \le V$ and $U \not{q} V$.

Theorem 4.3 [1]. An fts (X, τ) is fuzzy p^*-regular iff every fuzzy p^*-closed set is fuzzy closed.

Theorem 4.4. A fuzzy p^*-regular, fuzzy almost compact space X is fuzzy almost p^*-compact.

Proof. Let $\mathcal{U}$ be a fuzzy p^*-open cover of a fuzzy p^*-regular, fuzzy almost compact space X.

Then by Theorem 4.3, $\mathcal{U}$ is a fuzzy open cover of X. As X is fuzzy almost compact, there is a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\vee\{clU : U \in U_0\} = \vee\{p^*clU : U \in \mathcal{U}_0\}$ (by Theorem 4.3) $= 1_X \Rightarrow X$ is fuzzy almost p^*-compact.

Next we recall the following definition and theorem for ready references.

Definition 4.5 [1]. A function $f : X \to Y$ is said to be fuzzy almost p^*-continuous if the inverse image of every fuzzy p^*-open set in Y is fuzzy p^*-open in X.

Theorem 4.6 [1]. For a function $f : X \to Y$, the following statements are equivalent :

(i) f is fuzzy almost p^*-continuous,

(ii) $f(p^*clA) \leq p^*cl(f(A))$, for all $A \in I^X$,

(iii) for each fuzzy point x_α in X and each fuzzy p^*-open q-nbd V of $f(x_\alpha)$, there exists a fuzzy p^*-open set U in X with $x_\alpha \leq U$, $f(U) \leq V$.

Theorem 4.7. Fuzzy almost p^*-continuous image of a fuzzy almost p^*-compact space is fuzzy almost p^*-compact.

Proof. Let $f : X \to Y$ be fuzzy almost p^*-continuous surjection from a fuzzy almost p^*-compact space X to an fts Y, and let $\mathcal{V}$ be a fuzzy p^*-open cover of Y. Let $x \in X$ and $f(x) = y$. Since $sup\{V(y) : V \in \mathcal{V}\} = 1$, for each $n \in \mathcal{N}$, there exists some $V_x^n \in \mathcal{V}$ with $V_x^n(y) > 1 - 1/n$ and so $y_{1/n} q V_x^n$. By fuzzy almost p^*-continuity of f, by Theorem 4.6 (i)$\Rightarrow$ (iii), $f(U_x^n) \leq V_x^n$,for some fuzzy p^*-open set U_x^n in X q-coincident with $x_{1/n}$. Since $U_x^n(x) > 1 - 1/n$, $sup\{U_x^n(x) : n \in \mathcal{N}\} = 1$. Then $\mathcal{U} = \{U_x^n : n \in \mathcal{N}, x \in X\}$ is a fuzzy p^*-open cover of X. By fuzzy almost p^*-compactness of X, $\bigvee_{i=1}^{k} p^*clU_{x_i}^{n_i} = 1_X$, for some finite subcollection $\{U_{x_1}^{n_1}, \ldots, U_{x_k}^{n_k}\}$ of $\mathcal{U}$. Then $1_Y = f(\bigvee_{i=1}^{k} p^*clU_{x_i}^{n_i}) = \bigvee_{i=1}^{k} f(p^*clU_{x_i}^{n_i}) \leq \bigvee_{i=1}^{k} p^*cl(f(U_{x_i}^{n_i}))$ (by Theorem 4.6 (i) $\Rightarrow$ (ii)) $\leq \bigvee_{i=1}^{k} p^*clV_{x_i}^{n_i} \Rightarrow Y$ is fuzzy almost p^*-compact space.

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