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Modeling the Effects of Interference in Fertility Rate of Kenya

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Abstract

Many studies have been done on fertility for many years. However, very little has been documented in the existing literature concerning modeling of fertility in the presence of interference, yet interference to fertility is a common phenomenon. In this study fertility data sets for Kenya were modeled both before and after interference. The parameters of the model were estimated by the maximum likelihood estimation method. Using Akaike's Information Criteria, (AIC), it was established that amongst the distributions fitted; Gamma, Weibull and Lognormal, Gamma gave the best fit for the Kenya fertility rate data and interference simply shifts the Gamma distribution parameters. The result of this study would help Governments to understand fully the effect of interference on fertility rate and plan for it. Demographers would also benefit from this study since it can be used to project population growth after an interference.

Keywords: *Fertility, Interference and Kenya.*

1. Introduction

The term fertility in demography context, refers to the actual production of children and not the physical capability to produce them, which is termed as fecundity. Demographers have always measured how quickly a population grows, by determining how frequently people are added to the population by being born. This has been done by measuring fertility rate of a population which can be done in two broad ways namely; the period measures and the cohort measures. The period measures are measures which are based on a cross section of a population in one year. They include the Crude Birth Rate, CBR (the number

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of live births per 1000 women of a population in a given year), the General Fertility Rate, GFR (the number of live births per 1000 women of between ages 15-49 years in a given year) and the Child Woman Ratio, CWR (the number of children under 5 years of age per 1000 women aged 15-49 years in a given year). On the other hand, cohort measures are measures which follow the same people over a period of decades. They include: Age Specific Fertility Rate (ASFR), which refers to the annual number of live births per woman in a particular age group expressed per 1000 women in that age group and also the Total Fertility Rate (TFR), which refers to the average number of children a woman would potentially have, if she were to fast forward through all her child bearing years in a given year, under all the age specific fertility rates for that given year. Barret, Bogue and Anderson [1] describe total fertility rate as a synthetic rate which is neither based on fertility of any real group of women, because this would involve waiting until they complete childbearing, nor based on counting up of the total number of children actually born over her reproductive lifetime, but based on the age specific fertility rate of women in their child bearing years, which in conventional international statistical usage is ages 15 - 49 years. TFR is therefore a measure of fertility of an imaginary woman who passes through her reproductive life and is subjected to all the age specific fertility rates for ages 15-49 years that were recorded for a given population in a given year. This measure represents the number of children that would be born to a hypothetical cohort of 1,000 women who follow a set of a current schedule of age specific fertility rates, assuming that none of the women die before reaching the end of the childbearing period.

According to Onoja and Osayomore [14], TFR is not only a more direct measure of the level of fertility of a population but also, an indicator of the potentiality for population change in a country. A rate of two children per woman is considered to be the replacement rate for a population, leading to stability in terms of total numbers, a rate of above two children would mean that a population is growing in size, while a rate of below two children would mean that a population is declining in size and growing older.

1.1 Fertility In Response to Interference

In this context, the term interference, refers to a situation of large scale strike of unanticipated phenomenon such as high magnitude earthquake, major floods, Wars and Genocides, which leave many people dead and thousands others displaced. Many investigators like Preston [17], Montogomery and Cohen [12], Guarcello [24,5] and Palloni and Rafalimanana [16] have long observed that a strike of an interference in a population may cause big losses of assets, lives and displacements. Households may then have an incentive to increase the number of children ever born, and as a result, a positive fertility response in excess of replacement effects may be experienced. Much has been documented in literature on the effect of interference on fertility. Stein and Susser [25] in 1975, investigated the effect of massive famines on fertility in Neitherland, China and Bangladesh and observed that fertility had reduced during the famine interference and later on rose up sometime after the interference. In 1999, Lindstrom and Berhanu [11] analysed the impacts of conflict on fertility in Ethiopia and documented a sharp temporary decline in fertility during the early years of the violence, which was followed by a high increase in fertility thereafter. Stiegler [26] in 2006 , analysed fertility before and after the 1994 genocide in Rwanda and documented a decreasing trend of TFR before the Genocide and a sharp rise after the Genocide. A paper by the Nairobi chronicle research group [13] in 2008 also reported that the Kenya population had increased rapidly in the months which followed the post electionviolence. Hosseini and Abbasi [9] in 2013 investigated the impact of the 2004 Bam erthquake in Iran and documented that Iran’s fertility had declined in the year 2004 and then rose in the years 2005-2007.

Modeling fertility curves has also attracted the interest of demographers for many years, and a variety of mathematical models have been proposed in order to describe the age specific fertility pattern of populations. The Hadwiger function proposed by Hadwiger in 1940 [6] was one of the earliest models which was fitted to age specific fertility data. But, this model had a problem of overestimating fertility at the oldest fecund ages. Other models have been proposed by various researchers, for fitting age specific fertility curves. For instance, the Brass polynomial [2] proposed in 1960, the modified Gamma and modified Beta distribution functions [7] both proposed in 1978, the cubic splines [7] proposed in 1978and also the quadratic splines [23] proposed in 2003. However, the polynomial and spline models only fit fertility curves when elevated to a suitable degree.

In 1981, Hoem *et.al* [8], compared the variations in fits of the cubic splines, the Gamma, the Hadwiger, the Beta and the Brass functions in smoothing human fertility, using contemporary Denish fertility data and documented that among the models, the cubic spline fitted best. The Gamma and the Hadwiger functions were second and still fit the data well, while the Beta and the Brass functions were less accurate.

Schmertmann [23] in 2003 fitted a quadratic spline to the age pattern of fertility, but this model required 13 parameters to estimate making it a bit complex. In the year 2000, Gage, [4] extended the application of the Gamma distribution function, the Hadwiger function and the Brass polynomial to several non-human mammalian populations (primates , Asian elephants, and Przewalskis horse (an extinct species)). He tested all the three models and documented that Gamma model provided the best fit for fecundity of the non human populations. Otumba in 2012 [15], developed a model for optimal fish harvesting using Leslie Matrix. The fish species fecundity data was observed to follow a Gamma distribution. Jenna *et.al* [10] in 2015 used a multi level longitudinal data to investigate the fertility response to unanticipated mortality shock that had resulted from the 2004 Tsunami. They observed a positive association between exposure to the 2004 Tsunami and subsequent fertility.

For all the models described above, modeling fertility rate with emphasis on interference effect is scarce in literature. This study models the fertility rates for the data sets of Kenya both in the presence and in the absence of interference with an aimof determining the effect of interference on the fertility rate.

2. Data and Methodology

The following data sets were used in this study:

- The Kenya 2003 Demographic and Health survey data
- The Kenya 2009 Demographic and Health survey data
- The Kenya 2014 Demographic and Health survey data

2.1. Fitting model to data

Fitting model involves a process of selection of the best fitting distribution function from a predefined family of distributions. This practice requires judgment and expertise and generally follows an iterative process of model choice, parameters estimation, and quality of fit evaluation. In R software environment, which was developed by the R core team [21], the package 'fitdistrplus' provides functions for fitting distributions to different types of data sets (continuous, censored or non censored data and discrete data). The package also allows for different estimation methods (maximum likelihood, moment matching, and maximum goodness of fit estimation).

In our study we used the maximum likelihood estimation method to estimate the parameters of the Gamma, Weibull and Lognormal distribution functions.

The Gamma distribution is defined by;

$$f(x) = \frac{1}{\Gamma\alpha\beta^{\alpha}} x^{\alpha-1} e^{\frac{-x}{\beta}} \quad \alpha > 0, \quad \beta > 0$$

where; α is the shape parameter, $\frac{1}{\beta}$ is the rate parameter, β being the scale parameter.

The Weibull distribution is defined by;

$$g(x) = \alpha\beta(\beta x)^{\alpha-1} e^{-(\beta x)^{\alpha}}, \quad for \quad x > 0, \quad \alpha > 0, \quad \beta > 0$$

where, α is the shape parameter , β is the scale parameter.

The Lognormal distribution is defined by;

$$q(x) = \frac{1}{\sqrt{2\pi\sigma^2}x} exp[-\frac{(ln(x)-\mu)^2}{2\sigma^2}], \quad for \quad x > 0, \quad \sigma > 0, \quad -\infty < \mu < \infty$$

Where μ is the shape parameter (the mean of the random variables logarithm), σ is the scale parameter (the standard deviation of the random variables logarithm). Akaike's information Criteria (AIC) was also used to estimate the quality of fit of each model. AIC is a measure of the relative quality of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models. AIC is founded on information theory and it offers a relative estimate of the information lost when a given model is used to represent the process that generates the data hence from among the candidate models, the model that minimizes the information loss (the model with the lowest AIC value), is selected to be the best fitting model to the set of data.

3. Results and Discussion

3.1. Kenya fertility rate before and after the 2008 Post election violence Interference

Samir [22] reported that in January 2008, Kenya underwent a post election violence following the 30th December, 2007 results of a hotly-contested presidential election. Opposition leader Raila Odinga and his supporters rejected the declared victory of incumbent president Mwai Kibaki, alleging it was the result of rigging. Protests went into widespread violence as decades of ethnic rivalry blew out of control. The violence claimed the lives of more than 1,200 people and about 600,000 people displaced into temporary camps. We carried out an analysis of the fertility rate of Kenya in the years 2003, 2009 and 2014 DHS data sets [18, 19, 20], using R statistical software tool and displayed the results in Table 1 below.

Table 1. Fertility rate of Kenya in the years 2003, 2009 and 2014.

Year	Fertility rate
2003	3.507
2009	5.277
2014	3.410

From Table 1, the fertility rate of Kenya in the year 2003 was 3.507. This rate rose up to 5.277 in the year 2009, and later on reduced to 3.410 in the year 2014. We observed from Table 1 that the fertility rate had increased in the Kenya 2009 data set which had the Post election violence interference effect.

4. Modeling Both Interference Free and Interference Effect Contained Data

4.1. Introduction

In this section, fertility data sets for Kenya 2003 (interference free data set) and Kenya 2009 (data set that contained effect of interference), were modeled and Akaikes information Criteria (AIC) used to investigate among the probability distributions (Gamma, Weibull and Lognormal), the probability distribution that provides the best fit to the data sets.

5. Kenya 2003 and Kenya 2009 Fertility Data Model Choice

Descriptive statistics and Graphical techniques were used to identify the candidate distribution for the Kenya 2003 and Kenya 2009 fertility data sets. Histograms for the two data sets were plotted displayed below

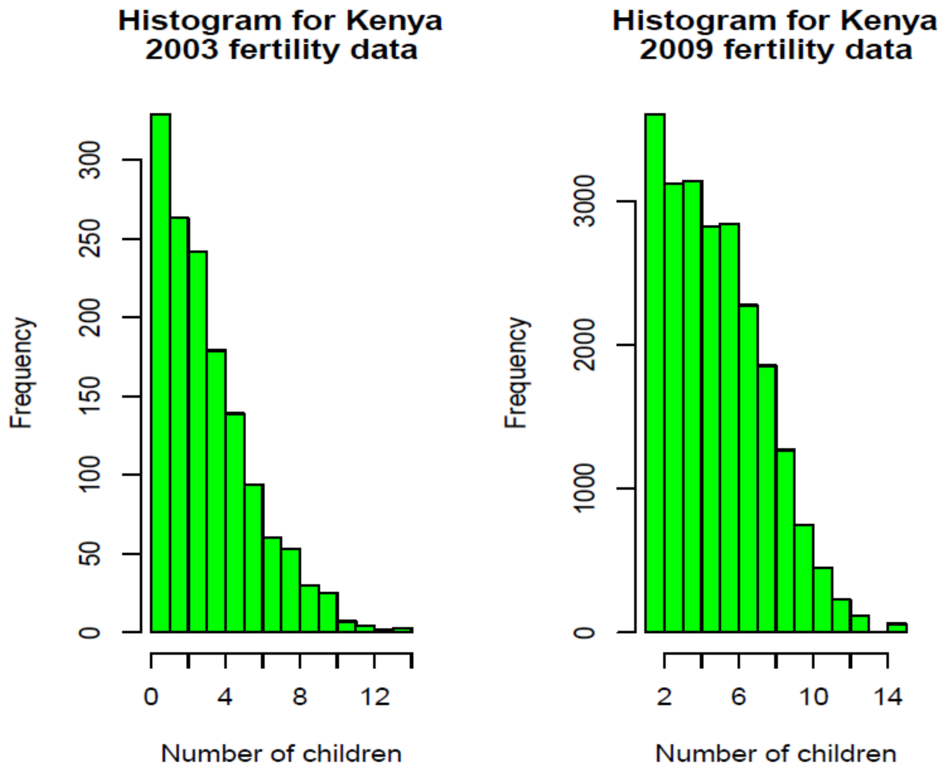

Figure 1. Histograms for Kenya 2003 and Kenya 2009 fertility data sets.

Figure 1 above shows that both the histograms were positively skewed. The candidate distributions for both the Kenya 2003 and Kenya 2009 fertility data sets were positively skewed.

5.1. Skewness-Kurtosis Plots for Kenya 2003 and Kenya 2009 fertility data sets

A skewness-kurtosis plot [3] was done for Kenya 2003 and for Kenya 2009. Values of skewness and kurtosis were computed on bootstrap samples and reported on skewness-kurtosis plot as shown in Figures 2A and 2B below. Positively skewed candidate distributions were then identified.

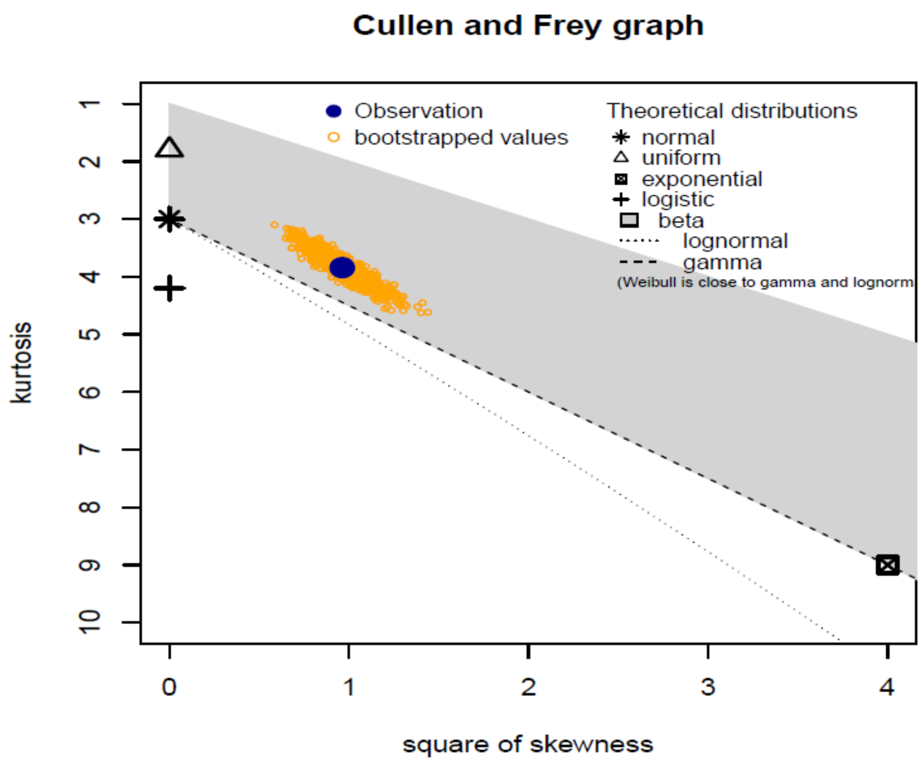

Figure 2A. Skewness-kurtosis plot for Kenya 2003 fertility data

Figure 2A Summary Statistics

Estimated skewness: 1.12774

Estimated kurtosis: 4.051652

Figure 2A shows that the skewness is non Zero and positive (1.13) and kurtosis is leptokurtic (4.05) for the Kenya 2003 data. The non zero skewness from the graph reveals lack of symmetry of the empirical distribution.

From the Cullen and Frey graph (Figure 2A) the skewness and the kurtosis combination of the Kenya 2003 data is (1.13, 4.05). On comparing the (square of skewness, kurtosis) combination of the Kenya 2003 data set(1.28, 4.05) with the (square of skewness, kurtosis) combinations that can be assumed by other distributions, we observe a consistency of the Kenya 2003 data with the Gamma, Weibull and Lognormal distributions whose (square of skewness, kurtosis) were about (1.28, 4.8), (1.28, 5.0) and (1.28, 5.2) respectively. The candidate models for Kenya 2003 data, were identified as Gamma, Lognormal and Weibull.

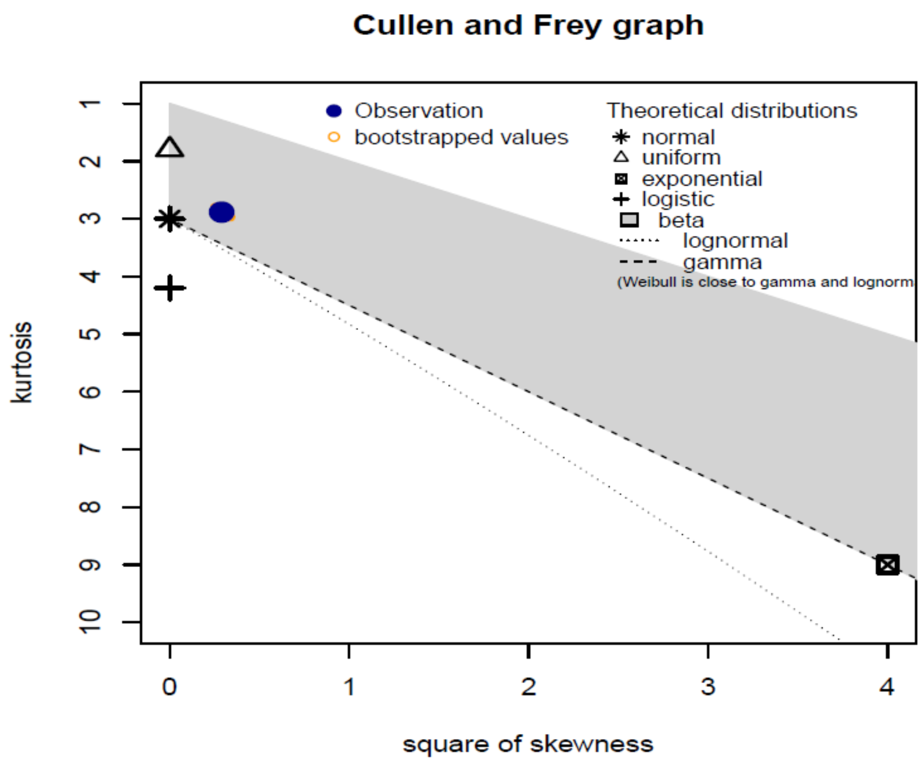

Figure 2B. Skewness-kurtosis plot for Kenya 2009 fertility data

Figure 2B summary statistics

Estimated skewness: 0.5378432

Estimated kurtosis: 2.884296

From the Cullen and Frey graph in Figure 2B, the skewness and the kurtosis combination of the Kenya 2009 data is (0.54, 2.88). On comparing the (square of skewness, kurtosis) combination of the Kenya 2009 data set(0.29, 2.88) with the (square of skewness, kurtosis) combinations that can be assumed by other distributions, we observe a consistency of the Kenya 2009 data with the Gamma, Weibull and Lognormal distributions whose (square of skewness, kurtosis) were about (0.29, 3.3), (0.29, 3.4) and (0.29, 3.5) respectively. The candidate models for Kenya 2009 data, were Gamma, Lognormal and Weibull.

5.2. Gamma, Weibull and Lognormal distributions fits to Kenya 2003 and Kenya 2009 fertility data

The Gamma, Weibull and Lognormal models were fitted to the Kenya 2003 and Kenya 2009 data sets and their parameters estimates obtained by Maximum likelihood estimation method. The parameter estimates and the Akaike's Information Criteria (AIC) were determined as shown in Table 2 below.

Table 2. Fitted model Parameter estimates for Gamma, Weibull and Lognormal distributions to Kenya 2003 and to Kenya 2009 fertility data sets

Data set	Distribution	parameter	estimate	AIC
Kenya 2003 fertility data set	Gamma	shape	1.9894204	6070.594
		rate	0.5782404	
	Weibull	shape	1.486736	6084.772
		scale	3.818015	
	Lognormal	meanlog	0.9636709	6201.48
		sdlog	0.8060106	
Kenya 2009 fertility data set	Gamma	shape	3.7706601	104686.9
		rate	0.7155335	
	Weibull	shape	2.059387	105522.9
		scale	5.964338	
	Lognormal	meanlog	1.5235397	105776.3
		sdlog	0.5512704	

From Table 2, Gamma distribution fitted with the lowest AIC values for both Kenya 2003 and Kenya 2009 data sets. Gamma distribution therefore lost the least information when used to generate the Kenya 2003 and Kenya 2009 fertility data sets respectively,This was according to its low AIC values compared to Weibull and Lognormal models. Gamma distribution was thus the best model for both of the data sets.

5.3. Quality and Goodness of fit test for Models to Kenya 2003 and Kenya 2009 fertility data sets

The density functions of the Gamma, Weibull and Lognormal and the histogram of Kenya 2003 and Kenya 2009 data sets were plotted and given in Figure 3 below, for quality of fit assessment. Alongside, *Q-Q* plots for Gamma, Weibull and Lognormal for Kenya 2003 and Kenya 2009 were also plotted in Figure 3, for graphical testing for the goodness of fit.

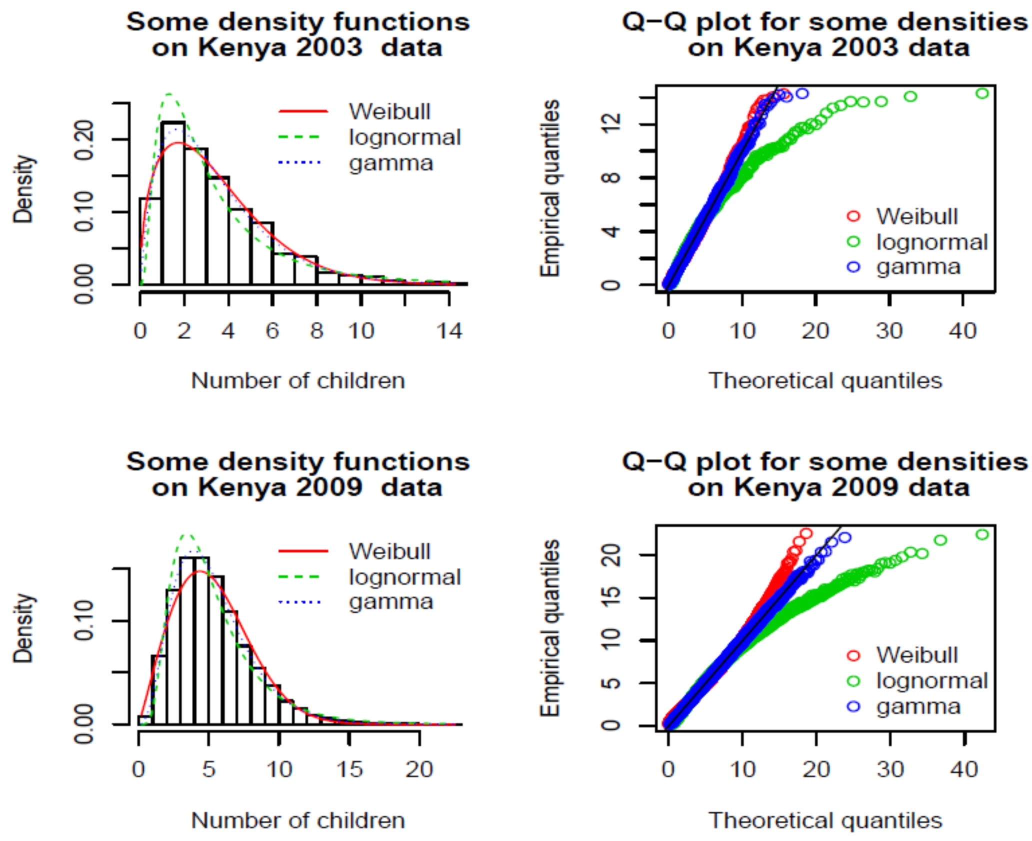

Figure 3. Density plots of some distributions on Histogram for Kenya 2003 and Kenya 2009 data sets

From Figure 3, Gamma distribution as mentioned earlier fits the data best.

Gamma distribution fits the data best to both the Kenya 2003 and Kenya 2009 fertility data sets, as most of the Gamma point lied along the empirical straight line. Gamma distribution was observed to be the best fitting model both for the Kenya 2003 and for the Kenya 2009 fertility data sets.

6. Conclusions

6.1. Gamma distribution fitted on Kenya 2003 versus Gamma distribution fitted on Kenya 2009 fertility rate data

The Table 3 below shows the summary of parameter estimates for Gamma distribution fits to both Kenya 2003 and Kenya 2009 data sets.

Table 3. Summary of parameter estimates for Gamma fit on data sets of Kenya 2003 and Kenya 2009

Year	shape parameter	rate parameter
003	1.989	0.578
009	3.771	0.716

From Table 3, we observe an increase in both the shape and the rate parameters of the Gamma distribution.

The shape parameter α increased by 89.6 percent in the year 2009 compared to the year 2003.

The rate parameter $\frac{1}{\beta}$ increased by 23.7 percent in the year 2009 than in the year 2003.

The scale parameter β decreased by 19.1 percent in the year 2009 than in the year 2003.

Figure 4 below shows a graph of Gamma distribution fitted to Kenya 2003 and also to Kenya 2009 fertility data sets on the same scale.

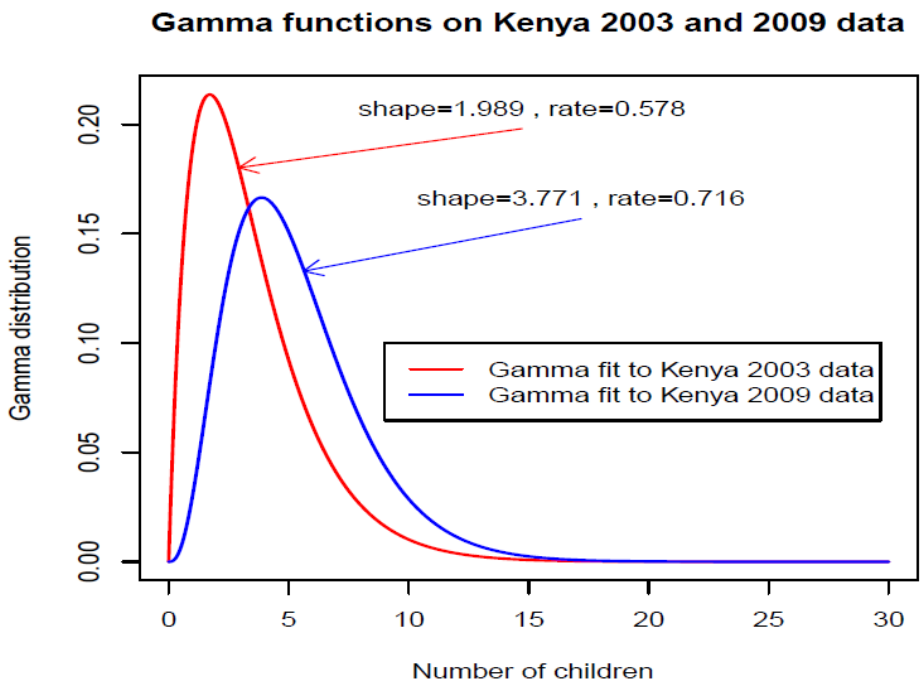

Figure 4. Gamma fit on Kenya 2003 and on Kenya 2009 data on the same scale

From Figure 4 above, the peakedness of the Gamma distribution fitted to Kenya 2009 data had decreased and its range had also become broader as compared to the Gamma fitted to the Kenya 2003 one.

Fertility data sets for Kenya were modeled before and after interference. The model parameters were estimated by the maximum likelihood estimation method. Using Akaike's Information Criteria, (AIC), it was established that amongst the distributions studied; Gamma, Weibull and Lognormal, Gamma gave the best fit for both the fertility rate data sets, interference simply shifts the Gamma distribution parameters. Also, in our analysis results in Table 1, fertility rates had increased in the presence of interference effect. We therefore concluded that presence of interference effect in a country increases its fertility rate.

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