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On Generalized Some Inequalities for *s* Convex Functions

Mehmet Eyup Kiris

Department of Mathematics, Faculty of Science and Arts, Afyon Kocatepe University, Afyonkarahisar, Turkey. E-mail address: kiris@aku.edu.tr

Naki ÇALTINER

Institute of Science, Afyon Kocatepe University, Afyonkarahisar, Turkey. naki.caltiner@gmail.com

Abstract

In this paper, a general integral identity for differentiable mapping is derived. Then, we extend some estimates of the right hand and left hand side of a Hermite- Hadamard-Fejér type inequality for functions whose first derivatives absolute values are s convex. Some applications for special means of real numbers are also provided. The results presented here would provide extensions of those given in earlier works.

Keywords: Hermite-Hadamard-Fejer inequality, Trapezoid inequality, convex function, s - convex function, Hölder inequality.

1. Introduction

The following inequality is well known in the literature as the Hermite-Hadamard integral inequality (see, [2], [6]):

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2} \tag{1}$$

where $f : I \subset \mathbb{R} \to \mathbb{R}$ is a convex function on the interval I of real numbers and $a,b \in I$ with $a < b$.

Corresponding author: Naki ÇALTINER, Institute of Science, Afyon Kocatepe University, Afyonkarahisar, Turkey. naki.caltiner@gmail.com.

Definition 1. The function $f:[a,b]\subset\mathbb{R}\to\mathbb{R}$, is said to be convex if the following inequality holds

$$f(\lambda x+(1-\lambda)y)\leq\lambda f(x)+(1-\lambda)f(y)$$

for all $x,y\in[a,b]$ and $\lambda\in[0,1]$. We say that f is concave if $(-f)$ is convex.

Definition 2. A function $f:[0,\infty)\to\mathbb{R}$ is said to be $s-$convex in the second sense if

$$f\left(\lambda x+(1-\lambda)y\right)\leq\lambda^{s}f\left(x\right)+(1-\lambda)^{s}f\left(y\right)$$

for all $x,y\in[0,\infty)$, $\lambda\in[0,1]$ and for some fixed $s\in(0,1]$. This class of $s-$convex function is usually denoted by K_s^2.

It can be easily seen that for $s=1$, $s-$convexity reduces to ordinary convexity of functions defined on $[0,\infty)$.

The inequalities (1) have grown into a significant pillar for mathematical analysis and optimization, besides, by looking into a variety of settings, these inequalities are found to have a number of uses. What is more, for a specific choice of the function f, many inequalities with special means are obtainable. Hermite Hadamard's inequality (1), for example, is significant in its rich geometry and hence there are many studies on it to demonstrate its new proofs, refinements, extensions and generalizations. You can check ([1], [2], [6], [5] and [11]) and the references included there. In [1], Dragomir and Agarwal proved the following results connected with the right part of (1).

Lemma 1. Let $f:I^{\circ}\subseteq\mathbb{R}\to\mathbb{R}$ be a differentiable mapping on I°, $a,b\in I^{\circ}$ with $a<b$. If $f'\in L[a,b]$, then the following equality holds:

$$\frac{f\left(a\right)+f\left(b\right)}{2}-\frac{1}{b-a}\int_a^b f(x)dx=\frac{b-a}{2}\int_0^1(1-2t)f'(ta+(1-t)b)dt. \tag{2}$$

Theorem 1. Let $f:I^{\circ}\subseteq\mathbb{R}\to\mathbb{R}$ be a differentiable mapping on I°, $a,b\in I^{\circ}$ with $a<b$. If $\left|f'\right|$ is convex on $[a,b]$, then the following inequality holds:

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right| \le \frac{(b-a)}{8}\left(\left|f'(a)\right|+\left|f'(b)\right|\right). \tag{3}$$

Theorem 2. Let $f: I^\circ \subset \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I°, $a,b \in I^\circ$ with $a<b$, $f' \in L(a,b)$ and $p>1$. If the mapping $\left|f'\right|^{p/(p-1)}$ is convex on $[a,b]$, then the following inequality holds:

$$\left|f(a)+f(b)2-1b-a\int_a^b f(x)dx\right| \le \frac{b-a}{2(p+1)^{1/p}}\left(\frac{\left|f'(a)\right|^{p/(p-1)}+\left|f'(b)\right|^{p/(p-1)}}{2}\right)^{(p-1)/p}. \tag{4}$$

In [5], Kırmacı proved the following results connected with the left part of (1).

Lemma 2. Let $f: I^\circ \subset \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I°, $a,b \in I^\circ$ with $a<b$. If $f' \in L(a,b)$, then we have

$$\frac{1}{b-a}\int_a^b f(s)ds - f\left(\frac{a+b}{2}\right)$$
$$=(b-a)\left[\int_0^{\frac{1}{2}} tf'(ta+(1-t)b)dt+\int_{\frac{1}{2}}^1 (t-1)f'(ta+(1-t)b)dt\right].$$

Theorem 3. Let $f: I^\circ \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I°, $a,b \in I^\circ$ with $a<b$. If $\left|f'\right|$ is convex on $[a,b]$, then we have

$$\left|\frac{1}{b-a}\int_a^b f(s)ds - f\left(\frac{a+b}{2}\right)\right| \le \frac{b-a}{8}\left(\left|f'(a)\right|+\left|f'(b)\right|\right). \tag{5}$$

The most well-known inequalities related to the integral mean of a convex function are the Hermite Hadamard inequalities or its weighted versions, the so-called Hermite-Hadamard-Fejér inequalities (see, [7]-[16]). In [3], Fejer gave a weighted generalizatinon of the inequalities (1) as the following:

Theorem 4. $f:[a,b] \to \mathbb{R}$, be a convex function, then the inequality

$$f\left(\frac{a+b}{2}\right)\int_a^b w(x)dx \le \frac{1}{b-a}\int_a^b f(x)w(x)dx \le \frac{f(a)+f(b)}{2}\int_a^b w(x)dx \tag{6}$$

holds, where $w:[a,b] \to R$ is nonnegative, integrable, and symmetric about $x=\frac{a+b}{2}$.

In [7], some inequalities of Hermite-Hadamard-Fejer type for differentiable convex mappings were proved using the following lemma.

Lemma 3. Let $f: I^{\circ} \subset \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I°, $a,b \in I^{\circ}$ with $a<b$, and $w:[a,b] \to [0,\infty)$ be a differentiable mapping. If $f' \in L[a,b]$, then the following equality holds:

$$\frac{f(a)+f(b)}{2}\int_a^b w(x)dx - \int_a^b f(x)w(x)dx = \frac{(b-a)^2}{2}\int_0^1 p(t)f'(ta+(1-t)b)dt \tag{7}$$

for each $t \in [0,1]$, where

$$p(t) = \int_t^1 w(as+(1-s)b)ds - \int_0^t w(as+(1-s)b)ds.$$

In [17], some inequalities on Generalized some Inequalities for Convex Functions were proved using the following lemma.

Lemma 4. Let $f: I^{\circ} \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I°, $a,b \in I^{\circ}$ with $a<b$ and let $g:[a,b] \to \mathbb{R}$. If $f', g \in L[a,b]$, then for all $x \in [a,b]$, the following identity holds:

$$\int_a^b P_\lambda(x,t) f'(t)dt \tag{8}$$

$$= (1-\lambda) f(x)\int_a^b g(s)ds + \lambda\left[f(a)\int_a^x g(s)ds + f(b)\int_x^b g(s)ds\right] - \int_a^b g(s)f(s)ds$$

where

$$P_\lambda(x,t) := \begin{cases} (1-\lambda)\int_a^t g(s)ds + \lambda\int_x^t g(s)ds & ,a \le t < x \\ (1-\lambda)\int_b^t g(s)ds + \lambda\int_x^t g(s)ds & ,x \le t \le b. \end{cases}$$

for $\lambda \in [0,1]$.

In this article, using functions whose derivatives absolute values are s-convex, we obtained new inequalities of Hermite-Hadamard-Fejer type. The results presented here would provide extensions of those given in earlier works.

2. Main Results

Using this Lemma 4 we can obtain the following general integral inequalities:

Theorem 5. Let $f: I^{\circ} \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable mapping on I°, $a,b \in I^{\circ}$ with $a<b$ and let $g:[a,b] \rightarrow \mathbb{R}$ be continuous on $[a,b]$. If $|f'|$ is $s-$ convex on $[a,b]$, then, for all $x \in [a,b]$, the following inequalities hold:

$$\Bigg|(1-\lambda) f(x)\int_a^b g(s)ds - \int_a^b g(s) f(s) ds \tag{9}$$

$$+\lambda\left[f(a)\int_a^x g(s)ds + f(b)\int_x^b g(s)ds \right]\Bigg|$$

$$\leq \frac{\|g\|_{[a,x],\infty}}{(b-a)^s}\Bigg\{\Bigg[(1-\lambda)\left|f'(a)\right|\frac{\left\{(b-a)^{s+2}-(b-x)^{s+1}\left[b-a(s+2)+x+sx\right]\right\}}{(s+1)(s+2)}+$$

$$(1-\lambda)\left|f'(b)\right|\frac{(x-a)^{s+2}}{s+2}+$$

$$\lambda\left|f'(a)\right|\frac{\left\{(b-x)^{s+2}-(b-a)^{s+1}\left[a+b+as-(s+2)x\right]\right\}}{(s+1)(s+2)}+$$

$$\lambda\left|f'(b)\right|\frac{(x-a)^{s+2}}{(s+1)(s+2)}\Bigg]\Bigg\}+$$

$$\frac{\|g\|_{[x,b],\infty}}{(b-a)^s}\Bigg\{\Bigg[(1-\lambda)\left|f'(a)\right|\frac{(b-x)^{s+2}}{s+2}+$$

$$(1-\lambda)\left|f'(b)\right|\frac{\left\{(b-a)^{s+2}+(x-a)^{s+1}\left[a-b(s+2)+x+sx\right]\right\}}{(s+1)(s+2)}+$$

$$\lambda\left|f'(a)\right|\frac{(b-x)^{s+2}}{(s+1)(s+2)}+$$

$$\lambda\left|f'(b)\right|\frac{\left\{(x-a)^{s+2}+(b-a)^{s+1}\left[a+b+bs-(s+2)x\right]\right\}}{(s+1)(s+2)}\Bigg]\Bigg\}$$

$$
=\frac{\|g\|_{[a,b],\infty}}{(b-a)^{s}}\Bigg\{\Bigg[(1-\lambda)\left|f'(a)\right|\frac{\left\{(b-a)^{s+2}-(b-x)^{s+1}\left[b-a(s+2)+x+sx\right]\right\}}{(s+1)(s+2)}+
$$

$$
(1-\lambda)\left|f'(b)\right|\frac{(x-a)^{s+2}}{s+2}+
$$

$$
\lambda\left|f'(a)\right|\frac{\left\{(b-x)^{s+2}-(b-a)^{s+1}\left[a+b+as-(s+2)x\right]\right\}}{(s+1)(s+2)}+
$$

$$
\lambda\left|f'(b)\right|\frac{(x-a)^{s+2}}{(s+1)(s+2)}\Bigg]+\Bigg[(1-\lambda)\left|f'(a)\right|\frac{(b-x)^{s+2}}{s+2}+
$$

$$
(1-\lambda)\left|f'(b)\right|\frac{\left\{(b-a)^{s+2}+(x-a)^{s+1}\left[a-b(s+2)+x+sx\right]\right\}}{(s+1)(s+2)}+
$$

$$
\lambda\left|f'(a)\right|\frac{(b-x)^{s+2}}{(s+1)(s+2)}+
$$

$$
\lambda\left|f'(b)\right|\frac{\left\{(x-a)^{s+2}+(b-a)^{s+1}\left[a+b+bs-(s+2)x\right]\right\}}{(s+1)(s+2)}\Bigg]\Bigg\}
$$

where $\lambda\in[0,1]$ and $\|g\|_{[a,b],\infty}=s\in[a,b]\sup|g(s)|$.

Proof. We take absolute of equation (8). Using bounded of the mapping g and the $s-$convexity of $\left|f'\right|$, we find that

$$
|I|=\left|(1-\lambda)f(x)\int_a^b g(u)du-\int_a^b g(u)f(u)du+\lambda\left[f(a)\int_a^x g(u)du+f(b)\int_x^b g(u)du\right]\right|
$$

$$
\le\int_a^b\left|P_\lambda(x,t)\right|\left|f'(t)\right|dt
$$

$$
=\int_a^x\left((1-\lambda)\left|\int_a^t g(u)du\right|+\lambda\left|\int_x^t g(u)du\right|\right)\left|f'(\frac{b-t}{b-a}a+\frac{t-a}{b-a}b)\right|dt+
$$

$$
\int_x^b\left((1-\lambda)\left|\int_b^t g(u)du\right|+\lambda\left|\int_x^t g(u)du\right|\right)\left|f'(\frac{b-t}{b-a}a+\frac{t-a}{b-a}b)\right|dt
$$

$$
\le\int_a^x\left((1-\lambda)\int_a^t|g(u)|du+\lambda\int_x^t|g(u)|du\right)\left|f'(\frac{b-t}{b-a}a+\frac{t-a}{b-a}b)\right|dt+
$$

$$
\int_x^b\left((1-\lambda)\int_b^t|g(u)|du+\lambda\int_x^t|g(u)|du\right)\left|f'(\frac{b-t}{b-a}a+\frac{t-a}{b-a}b)\right|dtdt
$$

$$\leq \|g\|_{[a,x],\infty}\int_a^x [(1-\lambda)(t-a)+\lambda(x-t)][(\frac{b-t}{b-a})^s \left|f'(a)\right|+(\frac{t-a}{b-a})^s \left|f'(b)\right|]dt+$$

$$\|g\|_{[x,b],\infty}\int_x^b [(1-\lambda)(b-t)+\lambda(t-x)][(\frac{b-t}{b-a})^s \left|f'(a)\right|+(\frac{t-a}{b-a})^s \left|f'(b)\right|]dt$$

$$=\frac{\|g\|_{[a,x],\infty}}{(b-a)^s}\Big[(1-\lambda)\left|f'(a)\right|\int_a^x (t-a)(b-t)^s dt+(1-\lambda)\left|f'(b)\right|\int_a^x (t-a)^{s+1}dt+$$

$$\lambda\left|f'(a)\right|\int_a^x (x-t)(b-t)^s dt+\lambda\left|f'(b)\right|\int_a^x (x-t)(t-a)^s dt\Big]+$$

$$\frac{\|g\|_{[x,b],\infty}}{(b-a)^s}\Big[(1-\lambda)\left|f'(a)\right|\int_x^b (b-t)^{s+1}dt+(1-\lambda)\left|f'(b)\right|\int_x^b (b-t)(t-a)^s dt$$

$$+\lambda\left|f'(a)\right|\int_x^b (t-x)(b-t)^s dt+\lambda\left|f'(b)\right|\int_x^b (t-x)(t-a)^s dt\Big]$$

Since;

$$\int_a^x (t-a)(b-t)^s dt=\frac{1}{(s+1)(s+2)}\left\{(b-a)^{s+2}-(b-x)^{s+1}\left[b-a(s+2)+x+sx\right]\right\}$$

$$\int_a^x (t-a)^{s+1}dt=\frac{(x-a)^{s+2}}{s+2}$$

$$\int_a^x (x-t)(b-t)^s dt=\frac{1}{(s+1)(s+2)}\left\{(b-x)^{s+2}-(b-a)^{s+1}\left[a+b+as-(s+2)x\right]\right\}$$

$$\int_a^x (x-t)(t-a)^s dt=\frac{(x-a)^{s+2}}{(s+1)(s+2)}$$

$$\int_x^b (b-t)^{s+1}dt=\frac{(b-x)^{s+2}}{s+2}$$

$$\int_x^b (b-t)(t-a)^s dt=\frac{1}{(s+1)(s+2)}\left\{(b-a)^{s+2}+(x-a)^{s+1}\left[a-b(s+2)+x+sx\right]\right\}$$

$$\int_x^b (t-x)(b-t)^s dt=\frac{(b-x)^{s+2}}{(s+1)(s+2)}$$

$$\int_x^b (t-x)(t-a)^s dt=\frac{1}{(s+1)(s+2)}\left\{(x-a)^{s+2}+(b-a)^{s+1}\left[a+b+bs-(s+2)x\right]\right\}$$

then we obtain that

$$
\begin{aligned}
|I| \le {} & \frac{\|g\|_{[a,x],\infty}}{(b-a)^s}\Bigg[(1-\lambda)\left|f'(a)\right|\frac{1}{(s+1)(s+2)}\left\{(b-a)^{s+2}-(b-x)^{s+1}\left[b-a(s+2)+x+sx\right]\right\}+ \\
& (1-\lambda)\left|f'(b)\right|\frac{(x-a)^{s+2}}{s+2}+ \\
& \lambda\left|f'(a)\right|\frac{1}{(s+1)(s+2)}\left\{(b-x)^{s+2}-(b-a)^{s+1}\left[a+b+as-(s+2)x\right]\right\}+ \\
& \lambda\left|f'(b)\right|\frac{(x-a)^{s+2}}{(s+1)(s+2)}\Bigg]+ \\
& \frac{\|g\|_{[x,b],\infty}}{(b-a)^s}\Bigg[(1-\lambda)\left|f'(a)\right|\frac{(b-x)^{s+2}}{s+2}+ \\
& (1-\lambda)\left|f'(b)\right|\frac{1}{(s+1)(s+2)}\left\{(b-a)^{s+2}+(x-a)^{s+1}\left[a-b(s+2)+x+sx\right]\right\}+ \\
& \lambda\left|f'(a)\right|\frac{(b-x)^{s+2}}{(s+1)(s+2)}+ \\
& \lambda\left|f'(b)\right|\frac{1}{(s+1)(s+2)}\left\{(x-a)^{s+2}+(b-a)^{s+1}\left[a+b+bs-(s+2)x\right]\right\}\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
= {} & \frac{\|g\|_{[a,x],\infty}}{(b-a)^s}\Bigg[\frac{\left|f'(a)\right|}{(s+1)(s+2)}\Big\{(1-\lambda)\left\{(b-a)^{s+2}-(b-x)^{s+1}\left[b-a(s+2)+x+sx\right]\right\}+ \\
& \lambda\left\{(b-x)^{s+2}-(b-a)^{s+1}\left[a+b+as-(s+2)x\right]\right\}\Big\}+ \\
& \frac{\left|f'(b)\right|(x-a)^{s+2}}{s+2}\left\{(1-\lambda)+\frac{\lambda}{s+1}\right\}\Bigg]+ \\
& \frac{\|g\|_{[x,b],\infty}}{(b-a)^s}\Bigg[\frac{\left|f'(a)\right|(b-x)^{s+2}}{s+2}\left\{(1-\lambda)+\frac{\lambda}{s+1}\right\}+ \\
& \frac{\left|f'(b)\right|}{(s+1)(s+2)}\Big\{(1-\lambda)\left\{(b-a)^{s+2}+(x-a)^{s+1}\left[a-b(s+2)+x+sx\right]\right\}+ \\
& \lambda\left\{(x-a)^{s+2}+(b-a)^{s+1}\left[a+b+bs-(s+2)x\right]\right\}\Big\}\Bigg]
\end{aligned}
$$

$$=\frac{\|g\|_{[a,x],\infty}}{(b-a)^s(s+1)(s+2)}\Big[\left|f'(a)\right|\Big\{\Big\{(b-a)^{s+2}-(b-x)^{s+1}\big[b-a(s+2)+x+sx\big]\Big\}-$$
$$\lambda\Big\{\Big[(b-x)^{s+1}(2b+-a(s+2)++x+sx)-(b-a)^{s+1}\big[2a+as-(s+2)x\big]\Big]\Big\}\Big\}+$$
$$\left|f'(b)\right|(x-a)^{s+2}\{s+1-\lambda s\}\Big]+$$
$$\frac{\|g\|_{[x,b],\infty}}{(s+2)(s+1)(b-a)^s}\left|f'(a)\right|(b-x)^{s+2}\{s+1-\lambda s\}\Big]+$$
$$\left|f'(b)\right|\Big\{\Big\{(b-a)^{s+2}+(x-a)^{s+1}\big[a-b(s+2)+x+sx\big]\Big\}-$$
$$\lambda\Big\{(x-a)^{s+1}(-2a+b(s+2)-sx)+(b-a)^{s+1}\big[2a+bs-(s+2)x\big]\Big\}\Big\}\Big]$$

$$=\frac{\|g\|_{[a,b],\infty}}{(b-a)^s(s+1)(s+2)}\Big[\left|f'(a)\right|\Big\{\Big\{(b-a)^{s+2}-(b-x)^{s+1}\big[b-a(s+2)+x+sx\big]\Big\}-$$
$$\lambda\Big\{\Big[(b-x)^{s+1}(2b-a(s+2)+x+sx)-(b-a)^{s+1}\big[2a+as-(s+2)x\big]\Big]\Big\}\Big\}+$$
$$\left|f'(b)\right|(x-a)^{s+2}\{s+1-\lambda s\}\Big]+\Big[\left|f'(a)\right|(b-x)^{s+2}\{s+1-\lambda s\}+$$
$$\left|f'(b)\right|\Big\{\Big\{(b-a)^{s+2}+(x-a)^{s+1}\big[a-b(s+2)+x+sx\big]\Big\}-$$
$$\lambda\Big\{(x-a)^{s+1}(-2a+b(s+2)-sx)+(b-a)^{s+1}\big[2a+bs-(s+2)x\big]\Big\}\Big]$$

Hence, this completes the proof. □

Remark 1. Under the same assumptions of Theorem 5 with $s=1$; then the following identity holds:

$$\left|(1-\lambda)f(x)\int_a^b g(u)du-\int_a^b g(u)f(u)du+\lambda\Big[f(a)\int_a^x g(u)du+f(b)\int_x^b g(u)du\Big]\right|$$

$$\leq\frac{\|g\|_{[a,b],\infty}}{6(b-a)}\Big\{\left|f'(a)\right|(x-a^2)\big((1-\lambda)(3b-a-2x)+\lambda(3b-2a-x)\big)$$
$$+\left|f'(a)\right|(2-\lambda)(b-x)^3+\left|f'(b)\right|(2-\lambda)(x-a)^3+$$
$$\left|f'(b)\right|(b-x)^2\big((1-\lambda)(b-3a+2x)+\lambda(2b-3a+x)\big)\Big\}$$

which is proved by Erden and Sarikaya in [8]

Corollary 1. Let $\lambda = 1$ in Theorem 5.Then we have

$$\Bigg|(1-\lambda)\, f(x)\int_a^b g(s)ds - \int_a^b g(s) f(s)\, ds$$

$$+\lambda\left[f(a)\int_a^x g(s)ds + f(b)\int_x^b g(s)ds \right]\Bigg|$$

$$\leq \frac{\|g\|_{[a,b],\infty}}{(b-a)^s}\Bigg\{\left|f'(a)\right|\frac{\left\{(b-x)^{s+2}-(b-a)^{s+1}\left[a+b+as-(s+2)x\right]\right\}}{(s+1)(s+2)}+$$

$$\left|f'(b)\right|\frac{(x-a)^{s+2}}{(s+1)(s+2)}\Bigg]+$$

$$\Bigg[\left|f'(a)\right|\frac{(b-x)^{s+2}}{(s+1)(s+2)}+$$

$$\left|f'(b)\right|\frac{\left\{(x-a)^{s+2}+(b-a)^{s+1}\left[a+b+bs-(s+2)x\right]\right\}}{(s+1)(s+2)}\Bigg]\Bigg\}$$

$$=\frac{\|g\|_{[a,b],\infty}}{(b-a)^s(s+1)(s+2)}\Big\{\left|f'(a)\right|\left\{2(b-x)^{s+2}-(b-a)^{s+1}\left[a+b+as-(s+2)x\right]\right\}+$$

$$\left|f'(b)\right|\left\{2(x-a)^{s+2}+(b-a)^{s+1}\left[a+b+bs-(s+2)x\right]\right\}\Big\}$$

Remark 2. Under the same assumptions of Remark1 with $\lambda = 1$; then the following identity holds:

$$\left| f(a)\int_a^x g(s)ds + f(b)\int_x^b g(s)ds - \int_a^b g(s)f(s)ds \right|$$

$$\leq \frac{\|g\|_{[a,x],\infty}}{(b-a)}\left[\frac{|f'(a)|(x-a)^2(3b-2a-x)+|f'(b)|(x-a)^3}{6}\right]$$

$$+\frac{\|g\|_{[x,b],\infty}}{(b-a)}\left[\frac{|f'(a)|(b-x)^3+|f'(b)|(b-x)^2(2b-3a+x)}{6}\right]$$

$$\leq \frac{\|g\|_{[a,b],\infty}}{6(b-a)}\left\{|f'(a)|\left[(x-a)^2(3b-2a-x)+(b-x)^3\right]\right.$$

$$\left.+|f'(b)|\left[(b-x)^2(2b-3a+x)+(x-a)^3\right]\right\}$$

which is proved by Tseng et. al in [11].

Corollary 2. Let $\lambda = 0$ in Theorem 5.Then we have

$$\left|(1-\lambda) f(x)\int_a^b g(s)ds - \int_a^b g(s)f(s)ds\right.$$

$$\left.+\lambda\left[f(a)\int_a^x g(s)ds + f(b)\int_x^b g(s)ds\right]\right|$$

$$\leq \frac{\|g\|_{[a,b],\infty}}{(b-a)^s}\left\{|f'(a)|\left[\frac{\left\{(b-a)^{s+2}-(b-x)^{s+1}[b-a(s+2)+x+xs]\right\}}{(s+1)(s+2)}+\frac{(b-x)^{s+2}}{(s+2)}\right]+\right.$$

$$\left.|f'(b)|\left[\frac{(x-a)^{s+2}}{(s+2)}+\frac{\left\{(b-a)^{s+2}+(x-a)^{s+1}[a-b(s+2)+x+sx]\right\}}{(s+1)(s+2)}\right]\right\}$$

$$=\frac{\|g\|_{[a,b],\infty}}{(b-a)^s(s+1)(s+2)}\left\{|f'(a)|\left\{(b-a)^{s+2}-(b-x)^{s+1}[b-a(s+2)+x+xs]+(s+1)(b-x)^{s+2}\right\}+\right.$$

$$\left.|f'(b)|\left\{(b-a)^{s+2}+(x-a)^{s+1}[a-b(s+2)+x+sx]+(s+1)(x-a)^{s+2}\right\}\right.$$

Remark 3. Under the same assumptions of remark1 with $\lambda = 0$; then the following identity holds:

$$\left|(1-\lambda)\, f(x)\int_a^b g(s)ds - \int_a^b g(s) f(s)\, ds + \lambda\left[f(a)\int_a^x g(s)ds + f(b)\int_x^b g(s)ds\right]\right|$$

$$\leq \frac{\|g\|_{[a,b],\infty}}{(b-a)}\left\{ \left|f'(a)\right|\left[\frac{\left\{(b-a)^3-(b-x)^2\left[b-3a+2x\right]\right\}}{6}+\frac{(b-x)^3}{3}\right]\right.$$

$$\left.\left|f'(b)\right|\left[\frac{(x-a)^3}{3}+\frac{(b-a)^3-(x-a)^2\left[a-3b+2x\right]}{6}\right]\right\}$$

$$=\frac{\|g\|_{[a,b],\infty}}{6(b-a)}\left\{ \left|f'(a)\right|\left[(b-a)^3-(b-x)^2\left[b-3a+2x\right]+2(b-x)^3\right]+\right.$$

$$\left.\left|f'(b)\right|\left[(b-a)^3-(x-a)^2\left[a-3b+2x\right]+2(x-a)^3\right]\right\}$$

which is proved by Sarikaya and Erden in [8]

Remark 4. Let $s=1$, $0\leq\alpha\leq 1$ and $x=\alpha a+(1-\alpha)b$ in Theorem 5. Then we have

$$\left|(1-\lambda)\, f(\alpha a+(1-\alpha)b)\int_a^b g(s)ds + \lambda f(a)\int_a^{\alpha a+(1-\alpha)b} g(s)ds \right. \tag{10}$$

$$\left. + f(b)\int_{\alpha a+(1-\alpha)b}^{b} g(s)ds - \int_a^b g(s) f(s)\, ds\right|$$

$$\leq \|g\|_{[a,\alpha a+(1-\alpha)b],\infty}(b-a)^2\left(\left|f'(b)\right|\frac{(2-\lambda)(1-\alpha)^3}{6}\right.$$

$$\left.+\left|f'(a)\right|\frac{(1-\alpha)^2\left[(1-\lambda)(2\alpha+1)+\lambda(2+\alpha)\right]}{6}\right)$$

$$+\|g\|_{[\alpha a+(1-\alpha)b,b],\infty}(b-a)^2\left(\left|f'(a)\right|\frac{(2-\lambda)\alpha^3}{6}\right.$$

$$\left.+\left|f'(b)\right|\frac{\alpha^2\left[(1-\lambda)(3-2\alpha)+\lambda(3-\alpha)\right]}{6}\right)$$

$$\leq \|g\|_{[a,b],\infty}(b-a)^2$$

$$\times\left\{\left|f'(a)\right|\frac{(1-\alpha)^2\left[(1-\lambda)(2\alpha+1)+\lambda(2+\alpha)\right]+(2-\lambda)\alpha^3}{6}\right.$$

$$\left.+\left|f'(b)\right|\frac{\alpha^2\left[(1-\lambda)(3-2\alpha)+\lambda(3-\alpha)\right]+(2-\lambda)(1-\alpha)^3}{6}\right\}$$

for $\lambda\in[0,1]$, which is proved by Sarikaya and Erden in [8]

Remark 5. Under the same assumptions of remark4 with $\lambda=1$; then the following identity holds:

$$\left|f(a)\int_a^{\alpha a+(1-\alpha)b}g(s)ds+f(b)\int_{\alpha a+(1-\alpha)b}^{b}g(s)ds-\int_a^b g(s)f(s)ds\right|$$

$$\leq\|g\|_{[a,\alpha a+(1-\alpha)b],\infty}(b-a)^2\left[\frac{\left|f'(b)\right|(1-\alpha)^3+\left|f'(a)\right|(1-\alpha)^2(2+\alpha)}{6}\right]$$

$$+\|g\|_{[\alpha a+(1-\alpha)b,b],\infty}(b-a)^2\left[\frac{\left|f'(a)\right|\alpha^3+\left|f'(b)\right|\alpha^2(3-\alpha)}{6}\right]$$

$$\leq\|g\|_{[a,b],\infty}(b-a)^2\left(\frac{\left|f'(a)\right|\left[(1-\alpha)^2(2+\alpha)+\alpha^3\right]+\left|f'(b)\right|\left[\alpha^2(3-\alpha)+(1-\alpha)^3\right]}{6}\right)$$

which is proved by Tseng et. al in [11].

Remark 6. Let $s=1$ $g:[a.b]\to\mathbb{R}$ be symmetric to $\frac{a+b}{2}$ and $\alpha=\frac{1}{2}$ in Remark 4. Then we have the inequalities

$$\left|(1-\lambda)f\left(\frac{a+b}{2}\right)\int_a^b g(s)ds+\lambda\frac{f(a)+f(b)}{2}\int_a^b g(s)ds-\int_a^b g(s)f(s)ds\right| \tag{11}$$

$$\leq\|g\|_{\left[a,\frac{a+b}{2}\right],\infty}(b-a)^2\left(\frac{4+\lambda}{48}\left|f'(a)\right|+\frac{2-\lambda}{48}\left|f'(b)\right|\right)$$

$$+\|g\|_{\left[\frac{a+b}{2},b\right],\infty}(b-a)^2\left(\frac{2-\lambda}{48}\left|f'(a)\right|+\frac{4+\lambda}{48}\left|f'(b)\right|\right)$$

$$\leq\frac{\|g\|_{[a,b],\infty}(b-a)^2\left(\left|f'(a)\right|+\left|f'(b)\right|\right)}{8}$$

which is "**weighted trapezoid**" inequality provided that $\left|f'\right|$ is convex on $[a,b]$.

Theorem 6. Let $f:I^\circ\subseteq\mathbb{R}\to\mathbb{R}$ be a differentiable mapping on I° and let $f'\in L[a,b]$, $a,b\in I^\circ$ with $a<b$, and let $g:[a,b]\to\mathbb{R}$ be continuous on $[a,b]$. If $\left|f'\right|^q$ is s convex on $[a,b]$, $q>1$, then for all $x\in[a,b]$, the following inequalities hold: for $\lambda\in[0,1]\setminus\left\{\frac{1}{2}\right\}$

$$\left|(1-\lambda)\,f(x)\int_a^b g(s)ds+\lambda\left[f(a)\int_a^x g(s)ds+f(b)\int_x^b g(s)ds\right]-\int_a^b g(s)f(s)ds\right| \qquad (12)$$

$$\leq\left(\frac{(1-\lambda)^{p+1}-\lambda^{p+1}}{(p+1)(1-2\lambda)}\right)^{\frac{1}{p}}\|g\|_{[a,b],\infty}(b-a)^{\frac{1}{q}}\left[(x-a)^{p+1}+(b-x)^{p+1}\right]^{\frac{1}{p}}$$

$$\times\left[\frac{\left|f'(a)\right|^q+\left|f'(b)\right|^q}{s+1}\right]^{\frac{1}{q}}$$

and for $\lambda=\frac{1}{2}$

$$\left|\frac{f(x)}{2}\int_a^b g(s)ds+\frac{1}{2}\left[f(a)\int_a^x g(s)ds+f(b)\int_x^b g(s)ds\right]-\int_a^b g(s)f(s)ds\right|$$

$$\leq\frac{\|g\|_{[a,b],\infty}(b-a)^{\frac{1}{q}}}{2}\left[\frac{\left|f'(a)\right|^q+\left|f'(b)\right|^q}{s+1}\right]^{\frac{1}{q}}\left[(x-a)^{p+1}+(b-x)^{p+1}\right]^{\frac{1}{p}}$$

where $\frac{1}{p}+\frac{1}{q}=1$ and $\|g\|_\infty=s\in[a,b]\sup\left|g(s)\right|$.

Proof. We take absolute value of (12). Using Hölder’s inequality and the s convexity of $\left|f'(t)\right|^q$, we find that

$$\left|(1-\lambda) f(x)\int_a^b g(u)du+\lambda\left[f(a)\int_a^x g(u)du+f(b)\int_x^b g(u)du\right]-\int_a^b g(u)f(u)ds\right|$$

$$\leq \int_a^b |P_\lambda(x,t)||f'(t)|dt \leq \left(\int_a^b |P_\lambda(x,t)|^p dt\right)^{\frac{1}{p}}\left(\int_a^b |f'(t)|^q dt\right)^{\frac{1}{q}}$$

$$\leq \left(\int_a^x \left|(1-\lambda)\int_a^t g(u)du+\lambda\int_x^t g(u)du\right|^p dt+\int_x^b\left|(1-\lambda)\int_b^t g(u)du+\lambda\int_x^t g(u)du\right|^p du\right)^{\frac{1}{p}}$$

$$\times\left(\int_a^b\left[\left(\frac{b-t}{b-a}\right)^s|f'(a)|^q+\left(\frac{t-a}{b-a}\right)^s|f'(b)|^q\right]dt\right)^{\frac{1}{q}}$$

$$=\left(\|g\|^p_{[a,x],\infty}\int_a^x\left[(1-\lambda)(t-a)+\lambda(x-t)\right]^p dt\right.$$

$$\left.+\|g\|^p_{[x,b],\infty}\int_x^b\left[(1-\lambda)(b-t)+\lambda(t-x)\right]^p dt\right)^{\frac{1}{p}}$$

$$\times(b-a)^{\frac{1}{q}}\left[\frac{|f'(a)|^q+|f'(b)|^q}{s+1}\right]^{\frac{1}{q}}.$$

Now, we make change of variable

$$\begin{aligned}(1-\lambda)(t-a)+\lambda(x-t)&=u \qquad dt=\frac{du}{1-2\lambda}\\ (1-\lambda)(b-t)+\lambda(t-x)&=v \qquad dt=\frac{dv}{2\lambda-1}.\end{aligned} \tag{13}$$

From (13), it follows that

$$\left|(1-\lambda) f(x)\int_a^b g(s)ds+\lambda\left[f(a)\int_a^x g(s)ds+f(b)\int_x^b g(s)ds\right]-\int_a^b g(s)f(s)ds\right|$$

$$\leq\left(\frac{(1-\lambda)^{p+1}-\lambda^{p+1}}{(p+1)(1-2\lambda)}\right)^{\frac{1}{p}}\left[\|g\|_{[a,x],\infty}^{p}(x-a)^{p+1}+\|g\|_{[x,b],\infty}^{p}(b-x)^{p+1}\right]^{\frac{1}{p}}$$

$$\times(b-a)^{\frac{1}{q}}\left[\frac{|f'(a)|^{q}+|f'(b)|^{q}}{s+1}\right]^{\frac{1}{q}}.$$

$$\leq\left(\frac{(1-\lambda)^{p+1}-\lambda^{p+1}}{(p+1)(1-2\lambda)}\right)^{\frac{1}{p}}\|g\|_{[a,b],\infty}\left[(x-a)^{p+1}+(b-x)^{p+1}\right]^{\frac{1}{p}}$$

$$\times(b-a)^{\frac{1}{q}}\left[\frac{|f'(a)|^{q}+|f'(b)|^{q}}{s+1}\right]^{\frac{1}{q}}.$$

we obtain the inequality (12).

For $\lambda=\frac{1}{2}$, because of Lemma 1, and using Hölder’s inequality and the convexity of $|f'(t)|^{q}$, we find that

$$\left|\frac{f(x)}{2}\int_a^b g(s)ds+\frac{1}{2}\left[f(a)\int_a^x g(s)ds+f(b)\int_x^b g(s)ds\right]-\int_a^b g(s)f(s)ds\right|$$

$$\leq\int_a^b\left|P_{\frac{1}{2}}(x,t)\right|\left|f'(t)\right|dt\leq\left(\int_a^b\left|P_{\frac{1}{2}}(x,t)\right|^{p}dt\right)^{\frac{1}{p}}\left(\int_a^b\left|f'(t)\right|^{q}dt\right)^{\frac{1}{q}}$$

$$\leq\frac{1}{2}\left(\|g\|_{[a,x],\infty}^{p}\int_a^x\left[(t-a)+(x-t)\right]dt+\|g\|_{[x,b],\infty}^{p}\int_x^b\left[(b-t)+(t-x)\right]dt\right)^{\frac{1}{p}}$$

$$\times(b-a)^{\frac{1}{q}}\left[\frac{|f'(a)|^{q}+|f'(b)|^{q}}{s+1}\right]^{\frac{1}{q}}.$$

Hence, the proof is completed. □

Remark 7. Under the same assumptions of Theorem 6 with $s=1$

$$\left|(1-\lambda)\, f(x)\int_a^b g(s)ds+\lambda\left[f(a)\int_a^x g(s)ds+f(b)\int_x^b g(s)ds\right]-\int_a^b g(s)f(s)ds\right| \tag{14}$$

$$\leq\left(\frac{(1-\lambda)^{p+1}-\lambda^{p+1}}{(p+1)(1-2\lambda)}\right)^{\frac{1}{p}}(b-a)^{\frac{1}{q}}\left[(x-a)^{p+1}+(b-x)^{p+1}\right]^{\frac{1}{p}}$$

$$\times\left[\frac{|f'(a)|^q+|f'(b)|^q}{2}\right]^{\frac{1}{q}}\|g\|_{[a,b],\infty}$$

and for $\lambda=\frac{1}{2}$

$$\left|\frac{f(x)}{2}\int_a^b g(s)ds+\frac{1}{2}\left[f(a)\int_a^x g(s)ds+f(b)\int_x^b g(s)ds\right]-\int_a^b g(s)f(s)ds\right|$$
$$\leq\frac{\|g\|_{[a,b],\infty}(b-a)^{\frac{1}{q}}}{2}\left[\frac{|f'(a)|^q+|f'(b)|^q}{2}\right]^{\frac{1}{q}}\left[(x-a)^{p+1}+(b-x)^{p+1}\right]^{\frac{1}{p}}$$

which is proved by Sarikaya and Erden in [8]

Corollary 3. Under the same assumptions of Theorem 6 with $\lambda=1$

$$\left|\left[f(a)\int_a^x g(s)ds+f(b)\int_x^b g(s)ds\right]-\int_a^b g(s)f(s)ds\right| \tag{15}$$

$$\leq\left(\frac{1}{(p+1)}\right)^{\frac{1}{p}}\|g\|_{[a,b],\infty}(b-a)^{\frac{1}{q}}\left[(x-a)^{p+1}+(b-x)^{p+1}\right]^{\frac{1}{p}}$$

$$\times\left[\frac{|f'(a)|^q+|f'(b)|^q}{s+1}\right]^{\frac{1}{q}}$$

Remark 8. Under the same assumptions of Theorem 6 with $\lambda=1$ and $s=1$; then the following inequality holds:

$$\left| f(a)\int_a^x g(s)ds + f(b)\int_x^b g(s)ds - \int_a^b g(s)f(s)ds \right|$$

$$\le \frac{\|g\|_{[a,b],\infty}(b-a)^{\frac{1}{q}}}{(p+1)^{\frac{1}{p}}}\left[(x-a)^{p+1}+(b-x)^{p+1}\right]^{\frac{1}{p}}$$

$$\times\left[\frac{|f'(a)|^q+|f'(b)|^q}{2}\right]^{\frac{1}{q}}$$

which is proved by Tseng et. al in [11].

Corollary 4. Under the same assumptions of Theorem 6 with $\lambda = 0$; then the following inequality holds:

$$\left| f(x)\int_a^b g(s)ds - \int_a^b g(s)f(s)ds \right| \tag{16}$$

$$\le \left(\frac{1}{(p+1)}\right)^{\frac{1}{p}}\|g\|_{[a,b],\infty}(b-a)^{\frac{1}{q}}\left[(x-a)^{p+1}+(b-x)^{p+1}\right]^{\frac{1}{p}}$$

$$\times\left[\frac{|f'(a)|^q+|f'(b)|^q}{s+1}\right]^{\frac{1}{q}}$$

Remark 9. Under the same assumptions of Theorem 6 with $\lambda = 0$ and $s = 1$; then the following inequality holds:

$$\left| f(x)\int_a^b g(s)ds - \int_a^b g(s)f(s)ds \right|$$

$$\le \frac{(b-a)^{\frac{1}{q}}}{(p+1)^{\frac{1}{p}}}\left[(x-a)^{p+1}+(b-x)^{p+1}\right]^{\frac{1}{p}}$$

$$\times\left[\frac{|f'(a)|^q+|f'(b)|^q}{2}\right]^{\frac{1}{q}}\|g\|_{[a,b],\infty}$$

which is "**weighted Ostrowski**" inequality provided that $|f'|^q$ is s convex on $[a,b]$.

Remark 10. Let $s=1,\ 0\le\alpha\le 1$ and $x=\alpha a+(1-\alpha)b$ in Theorem 6. Then the following inequality holds:

$$\Bigg|(1-\lambda)\,f\left(\alpha a+(1-\alpha)b\right)\int_a^b g(s)ds+\lambda f\left(a\right)\int_a^{\alpha a+(1-\alpha)b} g(s)ds \tag{17}$$

$$+f\left(b\right)\int_{\alpha a+(1-\alpha)b}^{b} g(s)ds-\int_a^b g(s)f\left(s\right)ds\Bigg|$$

$$\le\left(\frac{(1-\lambda)^{p+1}-\lambda^{p+1}}{(p+1)(1-2\lambda)}\right)^{\frac{1}{p}}\left(b-a\right)^2\left[(1-\alpha)^{p+1}+\alpha^{p+1}\right]^{\frac{1}{p}}$$

$$\times\left[\frac{\left|f'\left(a\right)\right|^q+\left|f'\left(b\right)\right|^q}{2}\right]^{\frac{1}{q}}\left\|g\right\|_{[a,b],\infty}$$

for $\lambda\in[0,1]$ which is proved by Sarikaya and Erden in [8]

Remark 11. If we take $\lambda=1$ in (17), we get

$$\left|f\left(a\right)\int_a^{\alpha a+(1-\alpha)b} g(s)ds+f\left(b\right)\int_{\alpha a+(1-\alpha)b}^{b} g(s)ds-\int_a^b g(s)f\left(s\right)ds\right|$$

$$\le\frac{\left(b-a\right)^2}{(p+1)^{\frac{1}{p}}}\left[(1-\alpha)^{p+1}+\alpha^{p+1}\right]^{\frac{1}{p}}\left[\frac{\left|f'\left(a\right)\right|^q+\left|f'\left(b\right)\right|^q}{2}\right]^{\frac{1}{q}}\left\|g\right\|_{[a,b],\infty}$$

which is proved by Tseng et. al in [11].

Remark 12. If we take $g:[a.b]\to\mathbb{R}$ be symmetric to $\frac{a+b}{2}$ and $\alpha=\frac{1}{2}$ in Remark 10. Then we have the inequality

$$\left|(1-\lambda)\,f\left(\frac{a+b}{2}\right)\int_a^b g\left(s\right)ds+\lambda\frac{f\left(a\right)+f\left(b\right)}{2}\int_a^b g(s)ds-\int_a^b g(s)f\left(s\right)ds\right| \tag{18}$$

$$\leq\left(\frac{(1-\lambda)^{p+1}-\lambda^{p+1}}{(p+1)(1-2\lambda)}\right)^{\frac{1}{p}}\frac{(b-a)^2}{2}\left[\frac{\left|f'(a)\right|^q+\left|f'(b)\right|^q}{2}\right]^{\frac{1}{q}}\|g\|_{[a,b],\infty}.$$

which is proved by Sarikaya and Erden in [8]

Remark 13. ***weighted trapezoid*** If we take $\lambda=1$ in (18), we obtain

$$\left|\frac{f(a)+f(b)}{2}\int_a^b g(s)ds-\int_a^b g(s)f(s)ds\right|$$

$$\leq\frac{(b-a)^2}{2(p+1)^{\frac{1}{p}}}\left[\frac{\left|f'(a)\right|^q+\left|f'(b)\right|^q}{2}\right]^{\frac{1}{q}}\|g\|_{[a,b],\infty}$$

which is proved by Tseng et. al in [11].

Remark 14. If we take $\lambda=0$ in (18), we get

$$\left|f\left(\frac{a+b}{2}\right)\int_a^b g(s)ds-\int_a^b g(s)f(s)ds\right|$$

$$\leq\frac{(b-a)^2}{2(p+1)^{\frac{1}{p}}}\left[\frac{\left|f'(a)\right|^q+\left|f'(b)\right|^q}{2}\right]^{\frac{1}{q}}\|g\|_{[a,b],\infty}$$

which is "**weighted midpoint**" inequality provided that $\left|f'\right|^q$ is s-convex on $[a,b]$ and $f'\in L(a,b)$ where $p>1$.

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