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Journal of Mathematics and Statistical Science, 14-28 | Science Signpost Publishing

Bayesian Updating Based on Hausdorff Outer Measures and the Role of Emotions in Decision Process During the Therapeutic Phase of Alliance

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Abstract

A probabilistic approach of the diagnostic process is proposed in which the subject's degree of knowledge is represented with coherent upper conditional probabilities defined by Hausdorff outer measures.

Using this model the diagnosis is assumed to be positive when it produces a change, that is when the subject's level of knowledge is defined by an a posterior Hausdorff outer measure different from the initial Hausdorff outer measure.

The psychotherapist-patient system is interpreted as a complex system, whose evolution, representing the phase of alliance, is described by a finite family of similarities that, starting from certain initial conditions, evolve the system into the attractor. This set, characterized by its own complexity, measured in terms of the Hausdorff dimension, represents the unconscious of the therapist-patient system and it is characterized by symmetry and self-similarity.

Keywords: Coherent upper conditional probability, Hausdorff outer measures, fractal sets, therapeutic alliance, unconscious, emotions.

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Introduction

A psychological diagnosis is a complex process, related to the clinician's ability frame a set of behavioural symptoms and patient-reported experiences in a coherent system of classification that leads to an appropriate treatment. Throughout the process, the clinician has to represent the patient's discomfort, to use error due to the incompleteness of the information available, and make predictions about patient's well-being, and to manage unknown aspects.

One of the basic idea underlying clinical judgment is the concept of unconscious, defined as "the part of the mind which is inaccessible to the conscious mind but which affects behaviour and emotions" (Oxford Dictionary). Making of diagnosis cannot be seen only as a rational decision-making process involving problem-solving procedures that can be heuristically controlled and traced, but rather as a co-creation between clinician and patient of a shared framework, where subjective psychological experiences – including clinical symptoms – are considered and discussed by taking into account elements the patient is not yet aware of. According to this perspective, the therapeutic alliance, that is the interactive and collaborative relationship between patient and therapist, plays a central role in the diagnostic as well as in the treatment process: the session is considered crucial not as much as it allows to explore and ascertain actual evidences, but as it brings about a significant personal relationship between the therapist and the patient that prompts emotional reactions and allow the unconscious to emerge. Emotions, defined as "instinctive or intuitive feelings distinguished from reasoning or knowledge" (Oxford Dictionary), within the psychodynamic approach are the leading way to proceed to the unconscious where psychological problems can be adequately addressed and treated.

The resulting subjectivity in diagnostic field is not considered a disturbance variable to get around, but, according to a more complex vision of the therapeutic encounter, a resource to draw from. In addition, the early sessions with the patient play a crucial role in the making of a good therapeutic alliance that is likely to significantly affect the following stages of therapy.

The basic idea, developed on this paper, has been proposed in Doria, Angelucci (2015).

In our work therapeutic alliance is considered an opportunity to understand the logical asset of unconscious. We wish to describe in mathematical terms the diagnostic activity during the phase of the alliance, in view of the complex systems theory, characterized by a strong dependence on initial conditions and by the fact that their evolution brings the system into a set, defined the attractor of the system, characterized by its own complexity, measured in terms of Hausdorff dimension of the set.

We interpret the therapist-patient system as a complex system, whose evolution, representing the phase of alliance, is described by a finite family of similarities that, starting from certain initial conditions, evolve the system into the attractor; this set represents the state in which the therapist and patient find themselves after the phase of alliance.

If the attractor is a fractal set, that is a symmetric, self-similar set with non-integer Hausdorff dimension, it can be seen as the mathematical representation of the unconscious of the system. The unconscious is mathematically represented by a symmetric and self-similar set with zero probability, from which to update the conscious thought.

In order to make predictions in this complex system and to represent the subject's level of knowledge, we will use coherent upper conditional probabilities defined through Hausdorff outer measures.

This instrument allows us to evaluate the possibility of success of the therapist's diagnosis conditioned to the state in which the patient-therapist system is at the end of the phase of alliance and this probability depends on the complexity of such state. Furthermore, by defining the conditional probabilities using Hausdorff measures, we can update the knowledge conditionally even to states in which initially is given probability zero.

With this contribution of ours we wish to analyze the diagnostic process through the use of the probabilistic subjective approach; to emphasize the importance of having an open attitude towards initially unexpected events, because they enable a deeper understanding of oneself and of the other; and finally, to grasp the importance of the active role of the psychotherapist in the choice of methods that will allow to shorten the gap in the therapeutic alliance phase with the single patient. The role of emotions is to understand the functioning of the unconscious, mathematically represented by a symmetric and self-similar set with zero probability, from which to update the conscious thought.

Making Decisions Under Conditions of Uncertainty: The Role of Emotions

In this section the role of the unconscious, which manifests itself during the therapeutic phase of alliance through emotions, is analysed. While the conscious, rational thinking is prominently conveyed by verbal cues and can be traced through logical sequencing, the unconscious unfolds and is manifested through non-verbal signals like emotions that do not necessarily follow a logical progression.

During the diagnostic process, the clinician has to make decisions about the clinical evaluation of the patient run often in a condition of uncertainty. Making the decision to sort out the patient into one or more diagnostic frameworks is a complex task in which the person-therapist is influenced by his/her own

feelings and cognitions, which lead step by step to assess, with a certain margin of error, the information that may result relevant for the person who asks for help.

The study of decisions has a long tradition especially in economics, and for a long time the mainstream theory of decision analysis under uncertainty has been the Expected Utility Theory (J. Von Neumann and O. Morgenstern, 1947).

Ramsey (1926) and De Finetti (1931, 1937) have affirmed the subjective value of probability, since it is related to the knowledge possessed by a particular individual and represents the degree of confidence that the subject has on the occurrence or non-occurrence of events.

The subjective point of view was axiomatized and fully developed by Savage (1954) in the Theory of Subjective Expected Utility, and by Anscombe-Aumann (1963). A Theory of Expected Utility without the axiom of additivity is proposed in Schmeidler (1989).

Studies conducted in the field of psychology of decision making have highlighted even more the subjective aspect with which individuals decide. In addition, more recent studies have led to reassess the role that emotions can play in the way in which people make decisions (Damasio, 1995; Epstein, 1994). We know that emotions can have a limiting as well as a facilitating effect, leading the individual in the best direction rather than sidetracking him/her (*Affect Heuristic*, Slovic, Finucane, Peters, and MacGregor, 2002). Emotions work at an unconscious level of reasoning and affect the conscious reasoning by inducing people to pay attention to specific information.

Several authors have considered the role of the unconscious process in building an effective therapeutic alliance with the patient at an early stage of therapy. Bion (1970), insisted in the position that the clinician's attitude is to activate a vigilant avoidance of the memory of past knowledge of the patient and urging expand the ability *to sense the evolution*, so as to be able to go into the unknown of each therapy session. Lacan (1969-70) underlined the importance of considering the *analyst's desire* to be free of demand, even that of the patient's good, otherwise the risk is to flatten on the other. The analyst, says the author, must be careful to respond to the patient's wishes and not to his questions.

Matte Blanco (1975) produced one of the most comprehensive descriptions of the structure and functioning of the unconscious with the purpose to account for the non-logical aspects of human thought, as they can be detected in serious mental disorders. He drew a distinction between the logical conscious thought, structured on the categories of time and space and ruled by the Aristotle's principle of non-contradiction, which he defined "asymmetrical thought", and the unconscious thought, which he defined "symmetrical thought", based upon the principles of abstraction and the principle of generalization.

According to this author, both types of thoughts combine in the different experiences of human thinking thus yielding to a bi-logic asset. In order to observe how the unconscious works, the analyst has to pay attention to the emotions and to the peculiar exchange the conscious thought is able to have with the poorly organized set of physical sensations and emotions. Emotions are the way to reach the unconscious, they function the same way as the unconscious and are the means to decode it. Emotions do not involve thinking. Thinking is a process and develops over time, it is something that happens while feeling is experienced as something that does not happen but simply is. The experience of feeling the emotions thus cannot be immediately reduced to thought and thinking. Feeling and thinking represent a constitutive conflict of human beings since they are of bi-logical nature. Such constitutive conflict can become pathological if the contrast or conflict between these ways of being symmetric and asymmetric, that is conscious or unconscious, becomes too strong.

Updating the Level of Knowledge and Making a Successful Diagnosis

In this section we describe, in mathematical terms, the diagnostic process in psychotherapy in the light of what was previously discussed.

The complex phenomenon taken into consideration is the encounter between therapist and patient and the problem we want to analyze is how to assess the likelihood of success in the diagnosis made by the therapist.

We assume that a psychotherapy produces a positive outcome when it yields a change in the patient, that is when it is able to modify the level of information and knowledge that the patient has about his/her situation. This knowledge is not to be intended in a superficial and abstract way, but most of all as the patient's ability to evaluate certain situations he/she's faced with, to focus on his/her discomfort, to make predictions of well-being, to handle emotions.

Psychotherapy yields positive outcomes when it is able to change the *degree of confidence* the patient has about the events. Similarly, also the therapist, before meeting with a patient, will have his/her own degree of confidence on the possible outcome of the encounter. The degree of confidence that one has about a given situation is obviously of subjective nature.

We choose to represent mathematically the level of information a person has on a given situation with a *measure of uncertainty or a coherent conditional probability* as defined in de Finetti's subjective approach (de Finetti, 1970).

Because of incomplete and inaccurate information, it seems appropriate to use as a measure of

uncertainty the coherent imprecise probabilities (Walley 1991), which consist of probabilities assigned using intervals and not by a single number. They are extensions of de Finetti's coherent probabilities to broader domains.

The use of probabilities assigned by intervals allows to quantify the uncertainty present in the phenomenon itself, due to the complexity of several factors: the complexity of the therapist and the patient (ability to go deeply into the experience, to predict well-being, etc.), the therapist-patient relationship, the therapist's lack of information about the patient, etc.

We call "*a priori* measure of uncertainty" the level of knowledge each subject (patient and therapist) has before the psychotherapy. Obviously, each represents his/her own degree of confidence by means of his/her own measure of uncertainty.

After the phase of the alliance, the therapist-patient system will be in a state denoted by the set B; the measure of uncertainty that quantifies the level of knowledge each subject has on *a posteriori* situations is a coherent upper probability conditioned to the state B.

We use the model of coherent conditional upper probabilities defined by Hausdorff outer measures (Doria, 2012; 2011), therefore we affirm that the phase of the alliance yields a positive diagnosis if the patient's *a priori* and *a posteriori* measures of uncertainty are different, that is if they are defined by different Hausdorff outer measures. For the definition of Hausdorff outer measure and its property see Falconer (1986) and Roger (1970).

The diagnosis is positive if during the phase of the alliance it leads the therapist-patient system in a state B in which, for both subjects, the a priori measure of uncertainty is equal to zero. This way, each subject, in order to define his/her own a posteriori measure of uncertainty, uses an upper conditional probability defined by a new Hausdorff outer measure.

The therapist's role in modifying the patient's measure of uncertainty is to make evolve the system in an attractor with a Hausdorff dimension different from that of Ω, through the choice of similarities that describe the phase of the alliance.

Iterated Functions Systems Describing the Relational Dynamics Between Therapist and Patient in the Phase of the Alliance

Consider a metric space $(\boldsymbol{\Omega}, d)$ where $\boldsymbol{\Omega} \subset \mathbb{R}^{\mathbf{2}}$ is the space of elementary events, each element $\omega \in \boldsymbol{\Omega}$ represents a possible out come of the therapist-patient relationship and d is a metric.

Let E be a subset of Ω that represents the state in which the system finds the therapist-patient relationship as a result of their first meeting. The set E represents the initial condition of the system. Note that different outcomes of the meeting may bring the system in the same state described by E.

A function $\boldsymbol{\psi}: \boldsymbol{\Omega} \longrightarrow \mathbb{R}$ is a contraction if there exists a positive constant $c < 1$ such that

$$\boldsymbol{d(\psi(\omega_1)\psi(\omega_2)) < cd(\omega_1, \omega_2)}$$

That is, $\boldsymbol{\psi}$ is a function that shortens the distances. A *similarity* is a particular contraction, that is a function that, in addition to shortening the distances, maintains unchanged some geometric properties, such as the parallelism and the corners.

The dynamic with which therapist and patient interact in the phase of the alliance is represented by a family of similarities $\{\boldsymbol{\psi_i}\}_{\boldsymbol{i=1}}^{\boldsymbol{m}}$, also known in the literature as an iterated functions system.

The idea of representing the therapist-patient relationship during the alliance through a family of similarities is motivated by the fact that at this stage therapist and patient must approach, thus shortening the distance between them. Furthermore, both the therapist and the patient interact while maintaining some of their relational characteristics that are present in different situations, for example in groups of persons consisting of more than two elements, which means there are characteristics, attitudes and dynamics that individuals implement at different scales.

Applying iteratively a family of similarities to a set that represents the initial condition, a system evolves towards an invariant and self-similar set, called attractor. Moreover, the attractor of a system of similarities has Hausdorff dimension equal to the similarity measure.

The Hausdorff dimension of the attractor of a family of similarity is given by the sole value s that satisfies the following equation

$$\sum_{\boldsymbol{i=1}}^{\boldsymbol{m}} (\boldsymbol{r_i})^{\boldsymbol{s}} = \mathbf{1} \tag{1}$$

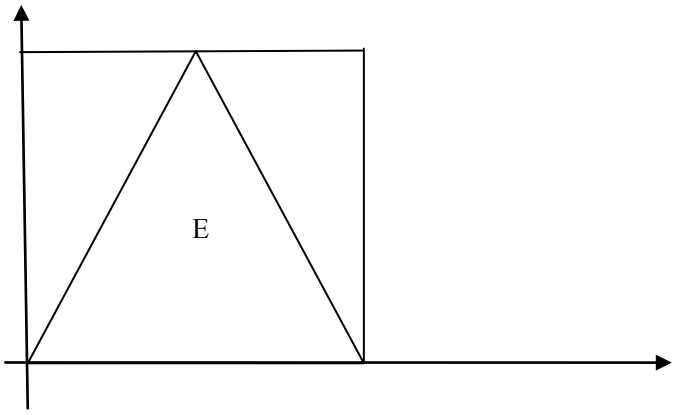
where $\boldsymbol{r_i}$ represent the relations of similarities of functions $\boldsymbol{\psi_i}$ for $\boldsymbol{i = 1, \ldots m}$.

The Model of Sierpinski Triangle

Let's try to clarify what above said by introducing a classic example of an attractor of a family of similarities known as *Sierpinski triangle*.

Let $\boldsymbol{\Omega = [0,1] \times [0,1]}$ (the square in the figure) e let E be the triangle of vertices O(0,0), A(1,0),

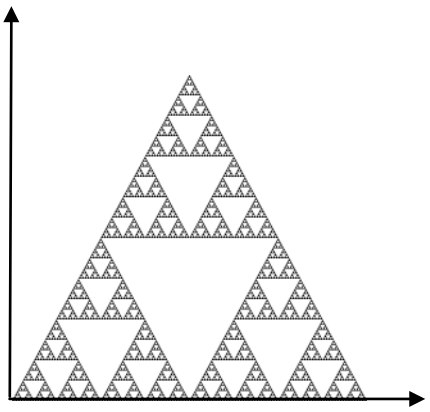
$B\left(\frac{1}{2}, 1\right)$. The set *E* represents the initial condition of the patient-therapist system after their first meeting. All points $\omega \in \Omega$ represent the possible states in which the system is to be situated after the first meeting.



The phase of the alliance is described by the following family of similarities $\{\psi_i\}_{i=1}^{3}$:

$$\psi_1: \begin{cases} X = \frac{1}{2}x \\ Y = \frac{1}{2}y \end{cases} \qquad \psi_2: \begin{cases} X = \frac{1}{2}x \\ Y = \frac{1}{2}y + \frac{1}{2} \end{cases} \qquad \psi_2: \begin{cases} X = \frac{1}{2}x + \frac{1}{2} \\ Y = \frac{1}{2}y + \frac{1}{2} \end{cases}.$$

Iteratively applying the three transformations to the initial triangle *E* you get the attractor of the system of similarities, known as the Sierpinsky triangle shown in the following figure:



The Hausdorff dimension of the Sierpinski triangle coincides with the size of the similarity dimension *s* and is calculated by solving the equation

$$3\left(\frac{1}{2}\right)^{s} = 1$$

that is $s = \frac{\log 2}{\log 3} < 2$.

The Sierpinski triangle is a fractal, because its Hausdorff dimension is non-integer and it is self-similar because its geometric structure is repeated identically at different scales. This property is also expressed by saying that *a single part reproduces the whole*. In addition, the Sierpinski triangle presents symmetries.

This set, according to the mathematical representation given of the diagnostic process, is the attractor of the patient-therapist system and its points represent the states in which the system is to be after the phase of the alliance.

An observation should be added. Mathematically, the attractor is described by applying to infinity the contractions to the initial set *E*. However, already after a relatively small number of iterations, the structure of the attractor we are going to build is clearly outlined, with particular reference to the properties of symmetry and self-similarity. Obviously, the phase of the alliance does not last an infinite time, therefore the attractor can be interpreted as the set toward which the system tends if the diagnostic process was prolonged indefinitely.

The Attractor and the Representation of the Unconscious

According to the mathematical representation of the diagnostic process highlighted in this work, we find similarities with the idea of Matte-Blanco (1975) that emotion can undergo endless measurements.

For Matte Blanco, the conscious and unconscious are two different modes of being, asymmetric and in becoming the first, symmetric and static the second.

In accordance with studies on schizophrenia made by Matte Blanco, the fundamental principles of the unconscious are the *generalization principle* and the *symmetric principle*. The generalization principle claims that the unconscious treats any single thing as it were a member of a class which contains other members. In turn, this class is considered as a subclass of another more general class, and so on.

According to the symmetric principle, in the realm of the unconscious, every relation is symmetric (as in the mathematical sense of this term).

When the attractor of the system is a fractal set, i.e. a symmetric and self-similar set with non-integer Hausdorff dimension, we can interpret this set as a mathematical representation of the unconscious of the

patient-therapist system.

The unconscious is mathematically represented by a symmetric and self-similar set with zero probability, from which to update the conscious thought.

We believe that if the same emotion (e.g., fear) can affect the unconscious of each person in a different way, then the unconscious of different persons may be represented by different fractals obtained as attractors of different families of similarities.

For each patient, a different family of similarities describing the evolution of the therapist-patient relationship is considered (because the therapist-patient system is different), and then different attractors are achieved. The same emotion marks in a different way the unconscious of different patients and to measure it means to make use of different Hausdorff measures depending on the Hausdorff dimension of the attractor reached.

Coherent Upper Conditional Probabilities Assigned by Hausdorff outer Measures as a Mathematical Tool to Represent the Change

In this section we analyze how the process of upgrading the level of knowledge can be described by a model of conditional probabilities defined by Hausdorff outer measures (Doria 2007, 2012).

This model describes the variation of the measurement of uncertainty adopted by a subject when new information represented by the *B* set is assumed. The upgrade procedure is equal for both the patient and the therapist.

The set *B* represents the state in which the patient-therapist system is to be at the end of the phase of the alliance, that is the attractor of the system of similarities.

The degree of confidence of a subject on an event *A* is represented (before the meeting) with an interval whose endpoints are called upper and lower unconditional probabilities (Walley 1991). This interval represents the belief interval that an individual gives to the event *A* based on one's own experience

$$\left[\underline{\boldsymbol{P}}(\boldsymbol{A}|\boldsymbol{\Omega});\ \overline{\boldsymbol{P}}(\boldsymbol{A}|\boldsymbol{\Omega})\right].$$

To some states *A* the subject will assign zero value to lower and upper probabilities because he/she considers them to be "very unlikely". The upper a priori probability of an event *A*, whose conditioning is Ω and in the following will be also indicated with

$$\overline{\boldsymbol{P}}(\boldsymbol{A}) = \overline{\boldsymbol{P}}(\boldsymbol{A}|\boldsymbol{\Omega}).$$

The upper and lower probabilities are linked by conjugacy relation

$$\underline{\boldsymbol{P}}(\boldsymbol{A}|\boldsymbol{\Omega}) = \mathbf{1} - \overline{\boldsymbol{P}}(\boldsymbol{A^c}|\boldsymbol{\Omega})$$

where $\boldsymbol{A^C}$ is the complementary event of *A,* that is the event is true if *A* is false and it is false when *A* is true.

If the set Ω has positive and finite Hausdorff measure in its Hausdorff dimension t, then the a priori probability of an event *A* is defined by choosing the set Ω as a conditioning set. Denoted with t the Hausdorff dimension of Ω, the a priori probability is defined by

$$\overline{\boldsymbol{P}}(\boldsymbol{A}) = \overline{\boldsymbol{P}}(\boldsymbol{A}|\boldsymbol{\Omega}) = \frac{\boldsymbol{h^t}(\boldsymbol{A} \cap \boldsymbol{\Omega})}{\boldsymbol{h^t}(\boldsymbol{\Omega})}$$

where the symbol ∩ represents the logical product between events, that is the event that is true if both events are true.

The measure of uncertainty representing the level of knowledge of the subject before the meeting is defined by means of the Hausdorff measure $\boldsymbol{h^t}$, where *t* is the Hausdorff dimension of Ω.

Following the first meeting, the therapist-patient system is found in the state described by the *E* set. From this set *E*, which represents the *initial condition*, the therapist-patient system during the phase of the alliance, mathematically represented by a family of similarities, evolves towards its attractor *B*.

Let's see how in this model the measure of uncertainty adopted by a subject when a new piece of information, represented by the *B* set, is assumed is likely to vary.

Note that, under appropriate mathematical assumptions (validity of the Open Set Condition), there exists a real number $\boldsymbol{s} \geq \mathbf{0}$ such that the attractor of a system of contractions has Hausdorff measure *s*-dimensional positive and finite. If we interpret the conditioning event as the attractor of a complex dynamic, expressed by a family of similarities, and if we denote with *s* the Hausdorff dimension of the set *B* and with $\boldsymbol{h^s}$ the s-dimensional Hausdorff measure, thus we can define the conditional or *a posteriori* probability of an event *A* given *B* in the following way

$$\overline{\boldsymbol{P}}(\boldsymbol{A}|\boldsymbol{B}) = \frac{\boldsymbol{h^s}(\boldsymbol{A} \cap \boldsymbol{B})}{\boldsymbol{h^s}(\boldsymbol{B})}$$

where the event $\boldsymbol{A} \cap \boldsymbol{B}$ is the logical product of $\boldsymbol{A}$ and $\boldsymbol{B}$ events, that is the event that is true if both $\boldsymbol{A}$ and $\boldsymbol{B}$ are true.

This formula can be seen as a generalization of Bayes Theorem, used in the Probability Theory to update probabilities when a new piece of information, represented by the event *B*, is acquired.

The innovative aspect consists in the fact that the measure that is used to define the conditional probability depends on the complexity of the conditioning event.

It is noteworthy to note that if the Hausdorff dimension of the set $\boldsymbol{A \cap B}$ is lower than $\boldsymbol{s}$, that is the Hausdorff dimension of *B*, we have:

$$\boldsymbol{h^s(A \cap B) = 0}$$

therefore $\boldsymbol{\overline{P}(A|B) = 0}$.

The a priori probability of an event *B* is defined similarly, although by using $\boldsymbol{\Omega}$ as the conditioning set Ω. Denoted with *t* the Hausdorff dimension of Ω, the a priori probability is defined by

$$\boldsymbol{\overline{P}(B) = \overline{P}(B|\Omega) = \frac{h^t(B \cap \Omega)}{h^t(\Omega)}}.$$

If the upper a priori probability assigned to *B* is positive, that is $\boldsymbol{\overline{P}(B|\Omega) > 0}$, it implies that the Hausdorff dimension of the set *B* is equal to *t*, that is it is equal to the Hausdorff dimension of Ω.

In this case the measurement of a posteriori uncertainty of information *B* is always defined by the outer Husdorff *t*-dimensional measure. Then no change occurs in the assessment of the events by the subject.

Otherwise, if the system is in a state *B* represented by a set which has Hausdorff dimension equal to *s* < *t*, that is the a priori probability of the event *B* is zero, arises the problem to update the information conditionally on the event *B*. The conditional probability will be defined by the *s*- dimensional Hausdorff measure, no longer by the *t*-order Hausdorff dimensional measure. Then another measure needs to be used to represent the subject's level of knowledge conditioned to the state *B* in which the system is found.

The measure of uncertainty, representing the partial knowledge that the subject has, is to be modified and then to the individual events is given a different degree of confidence.

Therefore the events with a zero-value a priori probability determine the change of the measure of uncertainty that represents the level of knowledge of the subject.

We focus the attention on the following property of coherent upper conditional probability:

$$\boldsymbol{\overline{P}(B|B) = 1}$$

which assures that the mathematical representation of the subject's level of knowledge does not contradict some signals of the unconscious processes, such as sleep, sleepwalking, dreaming, delirium, and comas that should not be contradicted during the phase of alliance.

Symmetric property of coherent upper conditional probabilities defined by Hausdorff outer measures have been investigated in Doria (2014).

Conclusions

In this work we present a probabilistic approach of the diagnostic process in which the subject's degree of knowledge is represented with coherent conditional probabilities defined by Hausdorff outer measures. Using this model, the diagnosis is assumed to be positive when it produces a change, that is when the subject's level of knowledge is defined by an a posteriori Hausdorff measure different from the initial Hausdorff measure.

We believe that one of the roles of the therapist in the phase of the alliance is to shorten the distance between him/her and the patient so that he/she can upgrade the level of cognitive and emotional understanding of the problem the patient asks for his/her help for. The first step that the therapist must take is to realize that he/she him/herself is a complex system and small perturbations to the initial state, i.e. the encounter with the patient, can bring to totally unpredictable states, from which he/she has to assess the probability of success of the diagnosis.

The phase of the psychotherapist-patient alliance can be interpreted as the phase in which the complexity is likely to increase. It should be noted that, during the alliance, the therapist has a part in increasing the complexity of the system through his/her ability to predict the well-being for the patient, to empathize with patient's discomfort, to understand his/her own emotions, and to prioritize what forces to modify his/her judgment rather than what confirms it.

In the mathematical model, the role of the therapist is represented by choosing a particular system of contractions, the similarities, that keep unchanged some geometric properties.
These invariance of geometric properties aims to describe the fact that some features of the therapist are repeated at different scales, influencing the diagnostic attitude.

By iterating these contractions, the patient-therapist system can reach a state represented by a self-similar symmetric set having zero probability with respect to the Hausdorff measure that defines the initial level of knowledge. This set, called the attractor of the system can be interpreted as the unconscious of the system. The Bayesian updating based on Hausdorff outer measure permits to condition the coherent upper probability to this new event with zero probability and to update the knowledge.

The goal of the phase of the alliance is therefore to have the patient to confront with a set having initial probability of zero value.

The proposed model highlights the importance of getting in tune with the bi-logical structures in the first sessions between therapist and patient because they help to focus the symmetries and the self-similarities that are useful to discover deeper emotional aspects.

The human mind produces conscious and unconscious thought and the role of emotions is to understand the functioning of the unconscious, mathematically represented by a symmetric and self-similar set with zero probability, by which update the conscious thought.

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Published: Volume 2017, Issue 1 / January 25, 2017