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A System Reliability Model with Priority to Repair over Preventive Maintenance Under Different Weather Conditions

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Abstract

The main emphasis of this paper is to obtained profit analysis of a two unit cold standby system with priority to repair activity over the preventive maintenance of the units working under different weather conditions. The both units are identical in nature which may fail directly from normal mode. There is a single server who visits the system immediately whenever required and works only in normal weather conditions. The operative unit under goes for preventive maintenance after a maximum operation time. Repair of the unit is done by the server at its complete failure. The unit works as new after maintenance and repair. The time to failure of the unit follows negative exponential distribution while the distributions of preventive maintenance and repair times are taken as arbitrary with different probability density functions. All random variables are statistically independent. The expressions for several reliability measures are derived in steady state using regenerative point technique and semi-Markov process. The graphical behavior of MTSF, availability and profit function have been observed with respect to preventive maintenance rate for arbitrary values of other parameters and costs.

Key words: Cold Standby system / Reliability Model / Preventive Maintenance/ Repair /Priority/ Weather Conditions and Profit Analysis.

Introduction

The repairable systems of two or more identical units have been investigated stochastically in detail by the scholars including Gopalan and Naidu (1982), Goyal and Murari (1984). Goel, Sharma and Gupta (1985) analyzed cost of a two-unit cold standby system under different weather conditions. Gupta and Goel (1991) discussed profit analysis of a two-unit cold standby system with abnormal weather condition. Also, the continued operation in abnormal weather and ageing deteriorate the system which may even cause some serious faults. It is proved that preventive maintenance can slow the deterioration process of a repairable system and restore the system in a younger age or state. Thus, the method of preventive maintenance can be used to improve reliability and profit of such systems. Malik and Barak [2007] analyzed a single server system operating under different weather conditions. Recently, Malik and Barak [2013], discussed a reliability model of a cold standby system with preventive maintenance and repair. However, the concepts of preventive maintenance and repair have not been used simultaneously so far in the reliability modeling of two unit cold standby systems under different weather conditions.

Keeping the above study of standby system working in different weather conditions with various facilities in mind, a reliability model for a two-unit cold standby system is developed in which operative unit under goes for preventive maintenance after a maximum operation time with priority to repair over preventive maintenance. The unit has two modes-operative and complete failures. The repair/preventive maintenance activity of the unit is done only in normal weather conditions by a perfect server who visits the system immediately whenever required. The unit works as new after preventive maintenance and repair. All random variables are statistically independent. The failure time of the unit follows negative exponential distribution, while the distributions of preventive maintenance and repair times are taken as arbitrary with different probability density functions. The switch devices are perfect. The expressions for several reliability measures such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), steady state availability, busy period of the server due to preventive maintenance and repair, expected number of visits of the server for conducting preventive maintenance and repair and profit function are derived using semi-Markov process and regenerative point technique. The graphical behavior of some important reliability indices have been observed with respect to preventive maintenance rate for fixed values of other parameters and costs.

Notations

Any system is working under prescribed conditions called working in normal weather otherwise called working in abnormal conditions.

E={S_0, S_1, S_2, S_3, S_{14}}:The set of regenerative states

O/Cs : The unit is operative/cold stand by

α_0 : Maximum constant rate of Operation time

λ : Constant failure rate of the unit.

β / β_1 : Abnormal weather rate / Normal weather rate

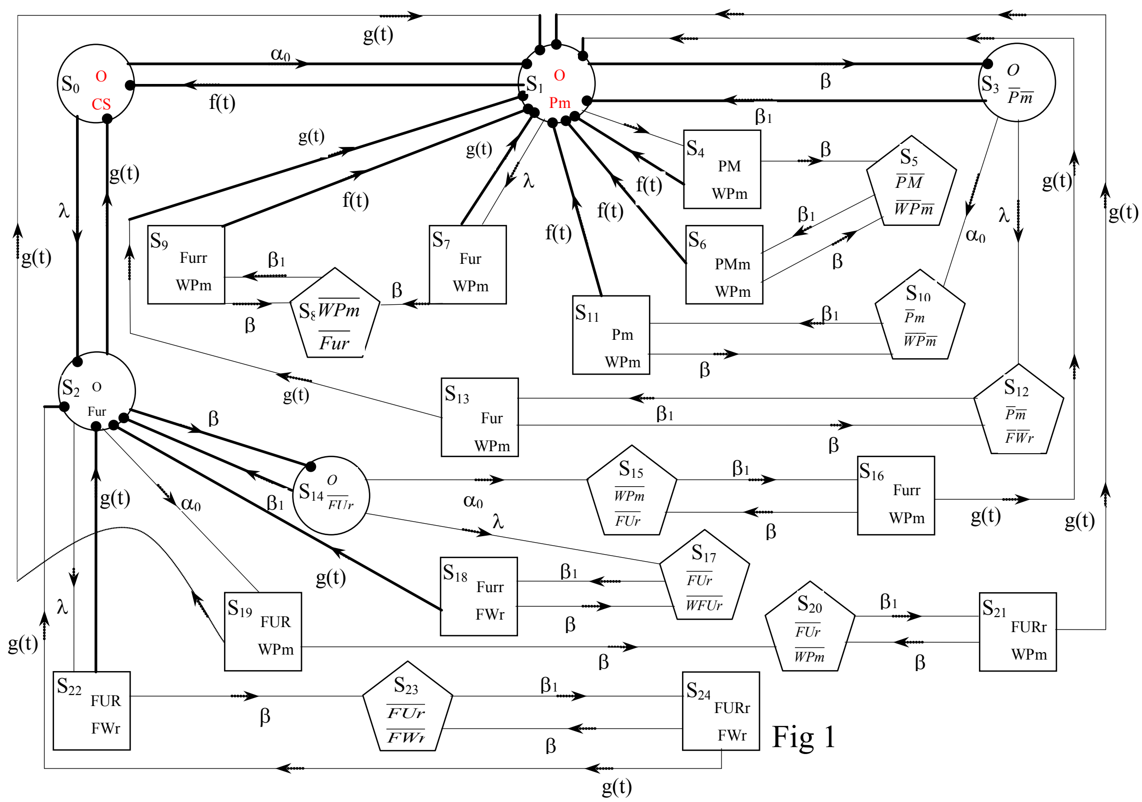

Fig 1

f(t)/F(t): pdf /cdf of preventive maintenance time

g(t)/G(t): pdf /cdf of repair time of a failed unit

$P_m / WP_m / (\overline{P}_m / \overline{WP}_m)$: The unit is under preventive maintenance/waiting for preventive maintenance / (stopped due to abnormal weather conditions)

$PM / FUR / (\overline{PM} / \overline{FUR})$: The unit is continuously under preventive maintenance/under repair from previous state/(stopped due to abnormal weather conditions)

$FU_r / Fwr / (\overline{FU_r} / \overline{Fw_r})$: The failed unit under repair/waiting for repair / (stopped due to abnormal weather conditions)

$PMm / FURr$: The total time that unit is continuously under preventive maintenance/ under repair from the previous state

M_{ij}: The unconditional mean time taken by the system to transit from any regenerative state S_i when it (time) is counted from epoch of entrance in to that state S_j. Mathematically it can be written as

$$m_{ij} = \int_0^{\infty} t d[Q_{ij}(t)] = -q_{ij}^{*'}(0)$$

μ_i : The mean Sojourn time in state S_i this is given by $\mu_i = E(t) = \int_0^\infty P(T > t)dt = \sum_j m_{ij}$, where T denotes the time to system failure

⊗/⊕: Symbol for Laplace Stieltjes convolution/Laplace convolution

~/*: Symbol for Laplace Steltjes transform/ Laplace transform

$^-$: used to stopped all mechanical activity due to abnormal weather

′(desh): Used to represent alternative result

Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for non-zero elements in particular case: let $f(t) = \theta e^{-\theta t}$ and $g(t) = \phi e^{-\phi t}$

$$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)\,dt \tag{1}$$

$$p_{01} = \frac{\alpha_0}{\alpha_0 + \lambda},\ p_{02} = \frac{\lambda}{\alpha_0 + \lambda},\ p_{10} = \frac{\theta}{\lambda + \beta + \theta + \alpha_0},\ p_{13} = \frac{\beta}{\lambda + \beta + \theta + \alpha_0},\ p_{14} = \frac{\alpha_0}{\lambda + \beta + \theta + \alpha_0},$$

$$p_{17} = \frac{\lambda}{\lambda + \beta + \theta + \alpha_0},\ p_{20} = \frac{\phi}{\lambda + \beta + \phi + \alpha_0},\ p_{2,14} = \frac{\beta}{\lambda + \beta + \phi + \alpha_0},\ p_{2,19} = \frac{\alpha_0}{\lambda + \beta + \phi + \alpha_0},\ p_{2,22} = \frac{\lambda}{\lambda + \beta + \phi + \alpha_0},$$

$$p_{41} = p_{61} = p_{11,1} = \frac{\theta}{\theta + \beta},\ p_{31} = p_{14,2} = \frac{\beta_1}{\lambda + \beta_1 + \alpha_0},$$

$$p_{45} = p_{65} = p_{11,10} = \frac{\beta}{\theta + \beta},\ p_{3,10} = p_{14,15} = \frac{\alpha_0}{\lambda + \beta_1 + \alpha_0},\ p_{3,12} = p_{14,17} = \frac{\lambda}{\lambda + \beta_1 + \alpha_0}$$

$$p_{7,1} = p_{9,1} = p_{13,1} = p_{16,1} = p_{18,2} = p_{19,1} = p_{21,1} = p_{22,2} = p_{24,2} = \frac{\phi}{\phi + \beta},$$

$$p_{7,8} = p_{9,8} = p_{13,12} = p_{16,15} = p_{18,17} = p_{19,20} = p_{21,20} = p_{22,23} = p_{24,23} = \frac{\beta}{\phi + \beta}$$

$$p_{11\bullet 4} = \frac{\theta\alpha_0}{(\beta + \theta)(\lambda + \theta + \beta + \alpha_0)},\ p_{11\bullet 4(5,6)''} = \frac{\beta\alpha_0}{(\beta + \theta)(\lambda + \theta + \beta + \alpha_0)},\ p_{11\bullet 7} = \frac{\phi\lambda}{(\beta + \phi)(\lambda + \theta + \beta + \alpha_0)}$$

$$p_{11\bullet 7(8,9)''} = \frac{\beta\lambda}{(\beta + \phi)(\lambda + \theta + \beta + \alpha_0)},\ p_{21\bullet 19} = \frac{\alpha_0\phi}{(\beta + \phi)(\lambda + \phi + \beta + \alpha_0)},$$

$$p_{21\bullet 19(20,21)''} = \frac{\alpha_0\beta}{(\beta + \phi)(\lambda + \phi + \beta + \alpha_0)},\ p_{22\bullet 22} = \frac{\phi\lambda}{(\beta + \phi)(\lambda + \phi + \beta + \alpha_0)},\ p_{22\bullet 22(23,24)''} = \frac{\beta\lambda}{(\beta + \phi)(\lambda + \phi + \beta + \alpha_0)},$$

$$p_{3,1(10,11)''} = p_{14,1(15,16)''} = \frac{\alpha_0}{\lambda + \beta_1 + \alpha_0},\ p_{3,1(12,13)''} = p_{14,2(17,18)''} = \frac{\lambda}{\lambda + \beta_1 + \alpha_0},\ p_{4,1(5,6)''} = p_{7,1(8,9)''} = \frac{\beta}{\theta + \beta},$$

$$p_{19,1(20,21)''} = p_{22,2(23,24)''} = \frac{\beta}{\phi + \beta}$$

$$p_{5,1(5,6)^n} = p_{8,1(8,9)^n} = p_{10,1(10,11)^n} = p_{12,1(12,13)^n} = p_{15,1(15,16)^n} = p_{17,2(17,18)^n} = p_{20,1(20,21)^n} = 1$$

$$p_{56} = p_{89} = p_{10,11} = p_{12,13} = p_{15,16} = p_{17,18} = p_{20,21} = p_{23,24} = 1 \tag{2}$$

The mean sojourn times $(\mu_i \; and \; \mu_i')$ is the state S_i are

$$\mu_0 = \frac{1}{\alpha_0 + \lambda}, \mu_1 = \frac{1}{\alpha_0 + \lambda + \beta + \theta}, \mu_2 = \frac{1}{\alpha_0 + \lambda + \beta + \phi}, \; \mu_3 = \mu_{14} = \frac{1}{\alpha_0 + \lambda + \beta_1},$$

$$\mu_5 = \mu_8 = \mu_{10} = \mu_{12} = \mu_{15} = \mu_{17} = \mu_{20} = \mu_{23} = \frac{1}{\beta_1}, \; \mu_4 = \mu_6 = \mu_{11} = \frac{1}{\beta + \theta}$$

$$\mu_7 = \mu_9 = \mu_{13} = \mu_{16} = \mu_{18} = \mu_{19} = \mu_{21} = \mu_{22} = \mu_{24} = \frac{1}{\phi + \beta} \tag{3}$$

and

$$\mu_1' = \frac{[\theta\phi\beta_1 + (\lambda\theta + \alpha_0\phi)(\beta + \beta_1)}{(\lambda + \beta + \theta + \alpha_0)}, \mu_2' = \frac{\phi\beta_1 + (\lambda + \alpha_0)(\beta + \beta_1)}{\phi\beta_1(\lambda + \beta + \theta + \alpha_0)}, \mu_4' = \frac{\beta + \beta_1}{\theta\beta_1}, \quad \mu_7' = \frac{(\beta + \beta_1)}{\theta\beta_1},$$

$$\mu_3' = \frac{\theta\phi\beta\beta_1 - \theta\phi\beta_1(\lambda + \alpha_0) + (\lambda(\varphi + \beta) + \alpha_0(\theta + \beta))(\beta + \beta_1))}{\theta\phi\beta\beta_1(\lambda + \beta_1 + \alpha_0)}, \mu_{14}' = \frac{\phi\beta\beta_1 - \phi\beta_1(\lambda + \alpha_0) + (\lambda + \alpha_0)(\beta + \beta_1)(\phi + \beta)}{\phi\beta\beta_1(\lambda + \beta_1 + \alpha_0)}$$

$$\mu_7' = \mu_{19}' = \mu_{22}' = \frac{(\beta + \beta_1)}{\phi\beta_1}, \mu_5' = \mu_{10}' = \frac{(\theta + \beta + \beta_1)}{\theta\beta_1}, \mu_8' = \mu_{12}' = \mu_{15}' = \mu_{17}' = \mu_{20}' = \frac{(\phi + \beta + \beta_1)}{\phi\beta_1} \tag{4}$$

Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the c.d.f. of first passage time from the regenerative state *i* to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$;

$$\phi_i(t) = \sum_j Q_{i,j}(t) \otimes \phi_j(t) + \sum_k Q_{i,k}(t), \qquad where \quad i = 0,1,2,3 \quad and \quad 14 \tag{5}$$

where *j* is an un-failed regenerative state to which the given regenerative state i can transit to regenerative state to regenerative state and *k* is failed state to which the state i can transit directly. Taking L.S.T. of above relation (5) and solving for $\widetilde{\phi}_0(t)$. We have

$$R^*(s) = \frac{1 - \widetilde{\phi}_0(s)}{s} \tag{6}$$

The reliability of the system model can be obtained by taking L.S. inverse transformation of (6). The mean time to system failure (MTSF) is given by

$$MTSF = \lim_{s \to 0} \frac{1 - \widetilde{\phi}_0(s)}{s} = \frac{\begin{bmatrix} \{(\alpha_0(\phi + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1)(\lambda + \alpha_0 + \beta_1 + \beta)\} \\ + \{(\lambda(\theta + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1)(\lambda + \alpha_0 + \beta_1 + \beta)\} \\ + \left[\{(\phi + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1)\}\{(\theta + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1)\}\right] \end{bmatrix}}{\begin{bmatrix} (\lambda + \alpha_0)\{(\phi + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1)\}\{(\theta + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1)\} \\ - \theta\alpha_0(\lambda + \alpha_0 + \beta_1)(\phi + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1) \\ - \phi\lambda(\lambda + \alpha_0 + \beta_1)(\theta + \lambda + \alpha_0 + \beta)(\lambda + \alpha_0 + \beta_1) - \beta\beta_1) \end{bmatrix}} \tag{7}$$

Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state i at $t = 0$. The recursive relations for $A_i(t)$ are given as

$$A_0(t) = M_0(t) + q_{0,1}(t) \oplus A_1(t) + q_{0,2}(t) \oplus A_2(t)$$

$$A_1(t) = M_1(t) + q_{1,0}(t) \oplus A_0(t) + [q_{1,1;4}(t) + q_{1,1;4(5,6)^n}(t) + q_{1,1;7}(t) + q_{1,1;7(8,9)^n}(t)] \oplus A_1(t) + q_{1,3}(t) \oplus A_3(t)$$

$$A_2(t) = M_2(t) + q_{20}(t) \oplus A_0(t) + [q_{2,1;19}(t) + q_{2,1;19(20,21)^n}(t)] \oplus A_1(t) + [q_{2,2;22}(t) + q_{2,2,22(23,24)^n}(t)] \oplus A_2(t) + q_{2,14}(t) \oplus A_{14}(t)$$

$$A_3(t) = M_3(t)) + [q_{3,1}(t) + q_{3,1;(10,11)^n}(t) + q_{3,1(12,13)^n}(t)] \oplus A_1(t)$$

$$A_{14}(t) = M_{14}(t) + q_{14,1;(15,16)^n}(t) \oplus A_1(t) + [q_{14,2}(t) + q_{14,2;(17,18)^n}(t)] \oplus A_2(t) \tag{8}$$

where $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(\alpha_0+\lambda)t},\ M_1(t) = e^{-(\alpha_0+\lambda+\beta)t}\overline{F(t)},\ M_2(t) = e^{-(\alpha_0+\beta+\lambda)t}\overline{G(t)} \text{ and } M_3(t) = M_{14}(t) = e^{-(\alpha_0+\beta_1+\lambda)t} \tag{9}$$

Taking L.T. of above relations (8) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{sN}{D}, \text{ where} \tag{10}$$

$$N = \frac{\begin{bmatrix} \theta(\lambda+\alpha_0+\beta_1)^2(\phi+\lambda+\alpha_0+\beta\ ) - \theta\beta\beta_1(\lambda+\alpha_0+\beta_1) + (\alpha_0(\lambda+\alpha_0+\beta+\beta_1) \\ +\alpha_0(\alpha_0+\lambda+\beta+\beta_1)(\alpha_0+\lambda+\beta_1)(\alpha_0+\lambda+\beta+\phi)) - \alpha_0\beta\beta_1(\alpha_0+\lambda+\beta+\beta_1) \end{bmatrix}}{(\lambda+\alpha_0)(\lambda+\alpha_0+\beta_1)^2(\lambda+\alpha_0+\theta+\beta)(\lambda+\alpha_0+\phi+\beta)}$$

$$D = \frac{\begin{gathered}\left[\beta_1\theta^2\phi(\lambda+\alpha_0+\beta_1)\{(\lambda+\alpha_0+\beta_1)(\phi+\lambda+\alpha_0+\beta)\} - \{\lambda(\lambda+\alpha_0+\beta+\beta_1)+\beta\beta_1\}\right] \\ +\alpha_0(\beta+\beta_1)\left[\{(\lambda+\alpha_0+\beta_1)(\phi+\alpha_0+\lambda+\beta) - \beta\beta_1\}\{(\lambda+\alpha_0+\beta_1)(\alpha_0\phi+\lambda\theta+\phi\theta)+\beta(\alpha_0\phi+\lambda\theta)\}\right] \\ +\lambda\theta^2(\lambda+\alpha_0+\beta_1)\left[(\lambda+\alpha_0+\beta+\beta_1)\{\beta_1\phi+(\alpha_0+\lambda)(\beta+\beta_1)+\beta\phi(\lambda+\alpha_0)\}\right]\end{gathered}}{\theta\phi\beta_1(\lambda+\alpha_0)(\lambda+\alpha_0+\beta_1)^2(\lambda+\alpha_0+\theta+\beta)(\lambda+\alpha_0+\phi+\beta)}$$

Busy Period Analysis for Server

(a) Let $B_i^p(t)$ be the probability that the server is busy in preventive maintenance of the unit at an instant 't' given that system entered state i at $t = 0$. The recursive relations for $B_i^p(t)$ are as follows:

$$B_0^p(t) = q_{0,1}(t) \oplus B_1^p(t) + q_{0,2}(t) \oplus B_2^p(t)$$

$$B_1^p(t) = W_1(t) + q_{1,0}(t) \oplus B_0^p(t) + [q_{1,1;4}(t) + q_{1,1;4(5,6)^n}(t) + q_{1,1;7}(t) + q_{1,1;7(8,9)^n}(t)] \oplus B_1^p(t) + q_{1,3}(t) \oplus B_3^p(t)$$

$$B_2^p(t) = q_{20}(t) \oplus B_0^p(t) + [q_{2,1;19}(t) + q_{2,1;19(20,21)^n} + q_{2,2;22}(t) + q_{2,2,22(23,24)^n}(t)] \oplus B_2^p(t) + q_{2,14}(t) \oplus B_{14}^p(t)$$

$$B_3^p(t) = W_3(t) + [q_{3,1}(t) + q_{3,1;(10,11)^n}(t) + q_{3,1(12,13)^n}(t)] \oplus B_2^p(t)$$

$$B_{14}^p(t) = q_{14,1;(15,16)^n}(t) \oplus B_1^p(t) + [q_{14,2}(t) + q_{14,2;(17,18)^n}(t)] \oplus B_2^p(t) \tag{11}$$

where, $W_i(t)$ be the probability that the server is busy in state S_i due to preventive maintenance up to time 't' without making any transition to any other regenerative state or before returning to the same via one or more non-regenerative states and

$$W_1(t) = \frac{1}{\lambda+\beta+\theta+\alpha_0}\left[1+\frac{\alpha_0}{\theta}\left(1-\left\langle\frac{\beta}{\beta+\theta}\right\rangle^N\right)\right], W_3(t) = \frac{\alpha_0}{\theta(\lambda+\beta_1+\alpha_0)}\left(1-\left\langle\frac{\beta}{\beta+\theta}\right\rangle^N\right)$$

(b) Let $B_i^R(t)$ be the probability that the server is busy in repair of the unit at an instant 't' given that system entered state i at t=0. The recursive relations for $B_i^R(t)$ are as follows:

$$B_0^R(t) = q_{0,1}(t)\oplus B_1^R(t) + q_{0,2}(t)\oplus B_2^R(t)$$

$$B_1^R(t) = R_1(t) + q_{1,0}(t)\oplus B_0^R(t) + [q_{1,1;4}(t) + q_{1,1;4(5,6)^n}(t) + q_{1,1;7}(t) + q_{1,1;7(8,9)^n}(t)]\oplus B_1^R(t) + q_{1,3}(t)\oplus B_3^R(t)$$

$$B_2^R(t) = R_2(t) + q_{20}(t)\oplus B_0^R(t) + [q_{2,1;19}(t) + q_{2,1;19(20,21)^n}]\oplus B_1^R(t) + [q_{2,2;22}(t) + q_{2,2,22(23,24)^n}(t)]\oplus B_2^R(t)$$
$$+ q_{2,14}(t)\oplus B_{14}^R(t)$$

$$B_3^R(t) = R_3(t) + [q_{3,1}(t) + q_{3,1;(10,11)^n}(t) + q_{3,1(12,13)^n}(t)]\oplus B_1^K(t)$$

$$B_{14}^R(t) = R_{14}(t) + q_{14,1;(15,16)^n}(t)\oplus B_1^R(t) + [q_{14,2}(t) + q_{14,2;(17,18)^n}(t)]\oplus B_2^R(t) \quad (12)$$

$R_i(t)$ be the probability that the server is busy in state S_i due to repair up to time 't' without making any transition to any other regenerative state or before returning to the same via one or more non-regenerative states and

$$R_1(t) = \frac{\lambda}{\phi(\lambda+\beta_1+\alpha_0+\theta)}\left(1-\left\langle\frac{\beta}{\beta+\theta}\right\rangle^N\right), R_2(t) = \frac{1}{\lambda+\beta+\theta+\alpha}\left[1+\frac{\lambda+\alpha_0}{\theta}\left(1-\left\langle\frac{\beta}{\beta+\theta}\right\rangle^N\right)\right]$$

$$R_3(t) = \frac{\lambda}{\phi(\lambda+\beta_1+\alpha_0)}\left(1-\left\langle\frac{\beta}{\beta+\theta}\right\rangle^N\right) \text{and } R_{14}(t) = \frac{\lambda+\alpha_0}{\phi(\lambda+\beta_1+\alpha_0)}\left(1-\left\langle\frac{\beta}{\beta+\theta}\right\rangle^N\right) \quad (13)$$

Taking L.T of above relations (11) and (12) and solving for $B_0^{*p}(t)$ and $B_0^{*R}(t)$, the time for which server is busy due to preventive maintenance and repair respectively is given by

$$B_0^p(t) = \lim_{s\to 0} sB_0^{*p}(t) = \frac{M_1^p}{D}, \text{ and } B_0^R(t) = \lim_{s\to 0} sB_0^{*R}(t) = \frac{M_2^R}{D}, \quad (14)$$

where $M_1^p(t) = \dfrac{\alpha_0\left[\{(\phi+\lambda+\beta+\alpha_0)(\alpha_0+\lambda+\beta_1)-\beta\beta_1\}\{\theta(\alpha_0+\lambda+\beta_1)(\beta+\theta)^N+\alpha_0((\beta+\theta)^N-\beta^N)(\lambda+\alpha_0+\beta+\beta_1)\}\right]}{\theta(\theta+\lambda+\beta+\alpha_0)(\phi+\lambda+\beta+\alpha_0)(\lambda+\alpha_0)(\lambda+\beta_1+\alpha_0)^2(\beta+\theta)^N}$

$$M_2^R(t) = \frac{\lambda\left[\begin{array}{l}\alpha_0((\beta+\phi)^N-\beta^N)(\lambda+\beta+\beta_1+\alpha_0)\{(\alpha_0+\lambda+\beta_1)(\lambda+\beta+\phi+\alpha_0)-\beta\beta_1\}+[\theta(\alpha_0+\lambda+\beta_1)^2\\ \times\{\phi(\beta+\phi)^n+(\lambda+\alpha_0)((\beta+\phi)^N-\beta^N)\}+\beta(\alpha_0+\lambda)(\alpha_0+\lambda+\beta_1)((\beta+\phi)^N-\beta^N)\end{array}\right]}{\phi(\theta+\lambda+\beta+\alpha_0)(\phi+\lambda+\beta+\alpha_0)(\lambda+\alpha_0)(\lambda+\beta_1+\alpha_0)^2(\beta+\phi)^N} \quad (15)$$

Expected Number of visits due to Repair and Preventive Maintenance of the Units

Let $N_i^P(t)$ and $N_i^R(t)$ be the expected number of preventive maintenance and repair of unit by the server in (0, *t*] given that the system entered the regenerative state *i* at *t* = 0. The recursive relations for $N_i^P(t)$ and $N_i^R(t)$ are given as

$$N_0^K(t) = Q_{01}(t) \otimes (N_1^K(t) + \delta_{PK}) + Q_{02}(t) \otimes (N_2^K(t) + \delta_{RK})$$

$$N_1^K(t) = Q_{10}(t) \otimes N_0^K(t) + [Q_{1,1;4}(t) + Q_{1,1;4(5,6)^n}(t) + Q_{1,1;7}(t) + Q_{1,1;7(8,9)^n}(t)] \otimes N_1^K(t) + Q_{13}(t) \otimes N_3^K(t)$$

$$N_2^K(t) = Q_{20}(t) \otimes N_0^K(t) + [Q_{2,1;19}(t) + Q_{2,1;19(20,21)^n}(t)] \otimes N_1^K(t) + [Q_{2,2;22}(t) + Q_{2,2;22(23,24)^n}(t)] \otimes N_2^K(t) + Q_{2,14}(t) \otimes N_{14}^P(t)$$

$$N_3^K(t) = [Q_{3,1}(t) + Q_{3,1;(10,11)^n}(t) + Q_{3,1;(12,13)^n}(t)] \otimes N_1^K(t)$$

$$N_{14}^K(t) = Q_{14,1;(15,16)^n}(t) \otimes N_1^K(t) + [Q_{14,2} + Q_{14,2;(17,18)^n}(t)] \otimes N_2^K(t) \quad (16)$$

$$where \quad K = \begin{cases} P, for\ preventive\ \ maintenance\ of\ the\ \ units \\ R, for\ repair\ of\ the\ units \end{cases}$$

Taking L.S.T of relations (16) and, solving for $\tilde{R}_0^R(s)$ and $\tilde{R}_0^P(s)$. The expected no of repair and preventive maintenance per unit time are respectively of given by

$$R_0^P(\infty) = \lim_{s \to 0} s\tilde{R}_0^P(s) = \frac{N_3^P}{D} \text{ and } R_0^R(\infty) = \lim_{s \to 0} s\tilde{R}_0^R(s) = \frac{N_4^R}{D} \quad (17)$$

where

$$N_3^P = \frac{\theta\alpha_0[(\lambda+\alpha_0+\beta_1)(\phi+\lambda+\alpha_0+\beta_1) - (\lambda^2+\lambda\alpha_0+\beta\lambda+\beta_1\lambda+\beta\beta_1)]}{(\lambda+\alpha_0)(\lambda+\alpha_0+\beta_1)(\lambda+\alpha_0+\theta+\beta)(\lambda+\alpha_0+\phi+\beta)}$$

$$N_4^R = \frac{\lambda\theta[(\lambda+\alpha_0+\beta_1)(\phi+\lambda+\alpha_0+\beta) - (\lambda^2+\lambda\alpha_0+\beta\lambda+\beta_1\lambda+\beta\beta_1)]}{(\lambda+\alpha_0)(\lambda+\alpha_0+\beta_1)(\lambda+\alpha_0+\theta+\beta)(\lambda+\alpha_0+\phi+\beta)},$$

and D has already defined.

Profit Analysis

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^R - K_2B_0^P - K_3R_0^R - K_4R_0^P \quad (18)$$

assuming that

K_0= (5,000): Revenue per unit up-time of the system
K_1=(400): Cost per unit time for which server is busy due preventive maintenance
K_2= (500): Cost per unit time for which server is busy due to repair
K_3= (350): Cost per unit time repair
K_4= (300): Cost per unit time preventive

Conclusion

The model is a case study of water supply system with particular values to the parameters like (α, β, β_1, λ, and θ).The graphs for mean time to system failure, availability and profit function have been drawn with respect to preventive maintenance rate as shown in the figures 2-4 respectively.

MTSF: As shown in the figure 2; line1, line 3, line 4, line 5 and line 6 all are coincide and increasing with small change in the parameters $(\alpha_0 = 5, \beta = .45, \beta_1 = .55, \lambda = .01, \phi = 2.5)$ on increasing of preventive maintenance rate θ. But the line 2 is decline the value when maximum operation time α_0 increasing. The graphical behavior indicates that the abnormal weather cannot effect at high level of MTSF only the effect of maximum operation time affected MTSF.

Availability: In figure 3, the trend of the graph of availability shows that when the server stopped all activity in abnormal weather with priority to repair over the preventive maintenance declines the total availability of the system not less than .54, in this situation the system is not useful up to the capacity of the system.

Profit: the figure 4 highlight the behave of the profit which depend upon the availability of the system the trend of graph are highly increasing when preventive maintenance rate θ as well as normal weather rate β_1 is increasing up to 56.086 to 1943.429 and decline but increasing when maximum operation time α_0 is increasing 5 to 7 in the range -71.131 to 1146.269.

Hence, the study reveals that a cold standby system with two identical units working under different weather conditions and server works only in normal weather conditions would be less reliable and profitable to use if its preventive maintenance is conducted before a pre-specific period of operation rather than to increase normal weather rate/repair rate of the system.

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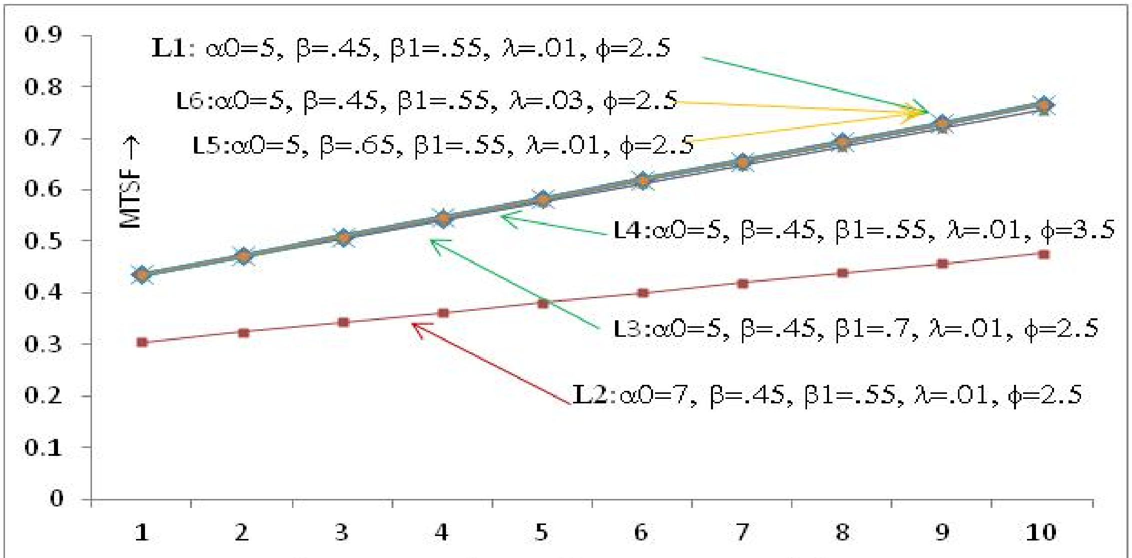

Fig. 2 : Preventive Maintanence Rate (Θ) →

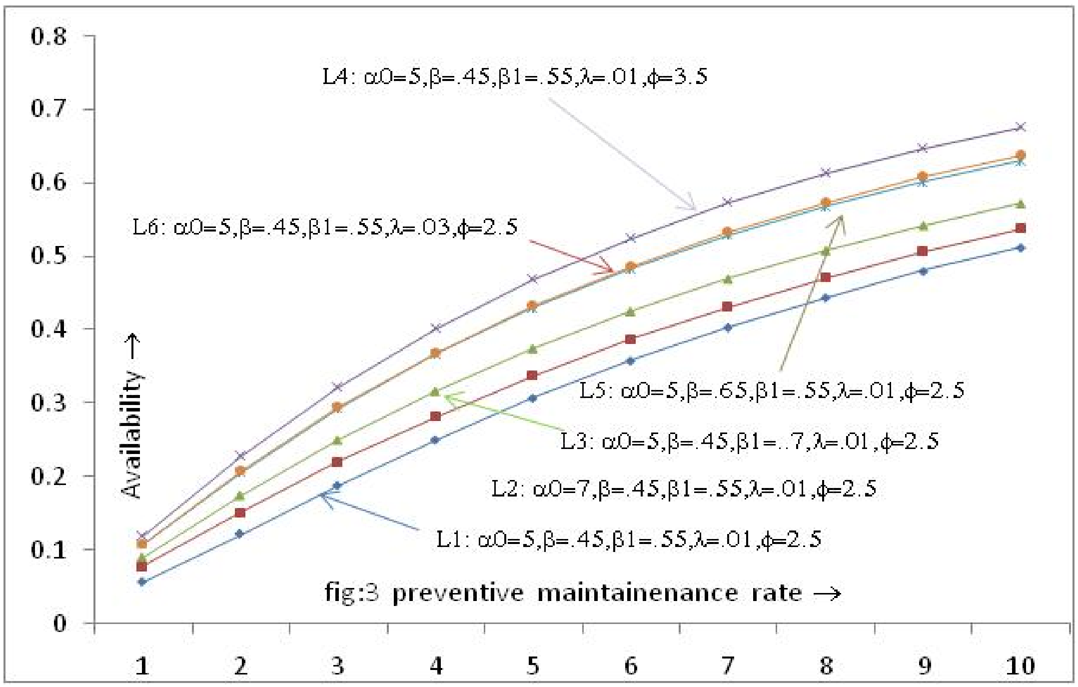

fig:3 preventive maintainenance rate →

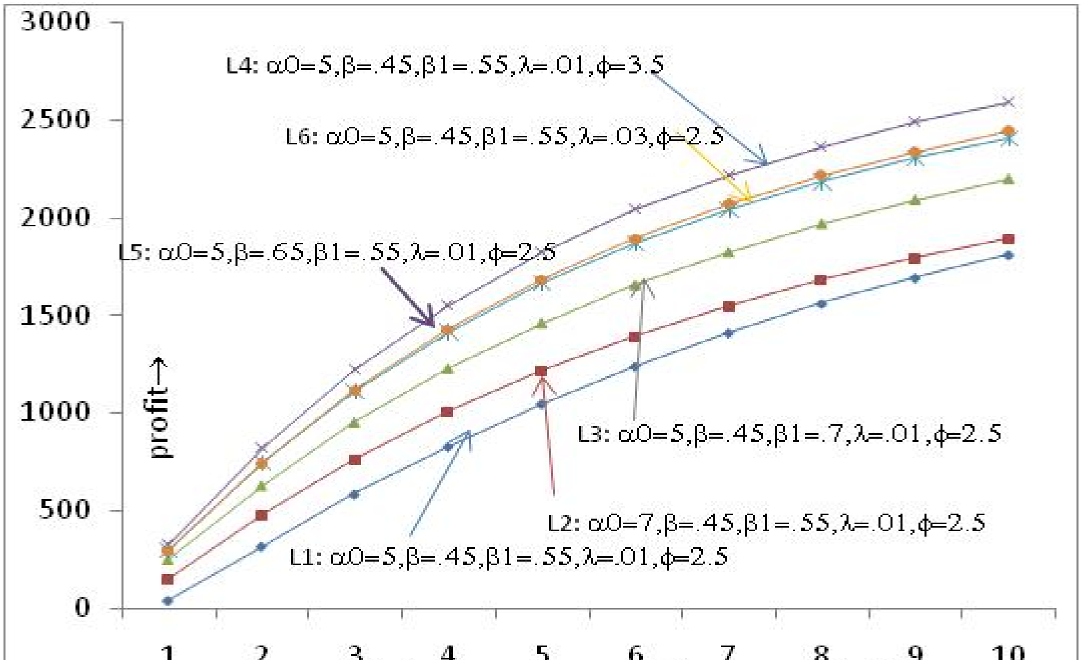

fig4: preventive maintenance rate θ→