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Gold Mining and Agricultural Sector: Effects on Labor and Exports

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Abstract

Burkina Faso is since 2009 become one of the greatest gold exporters in west Africa. Does it affects agriculture? The aim of this paper is to analyse what effects gold can have on labor used in main agricultural products and also their exports using simulation approach. Many assumptions are made to help us simulating the model. We expect to establish a mechanism of compensation to get back a level of production for which the variation is only due to absorption of gold, so there will be a high potential of export for the main agricultural products.

1. Introduction

Data on foreign trade in Burkina Faso shows that gold is become the leading export product since 2009[1], with a share of approximately 76.7 % of the total value exports in 2011. Cotton (11.3 %) comes in second place, followed by fruits with nuts dried or fresh and sesame seeds representing 5 % of exports by value.

The production of these agricultural goods requires intensive labor under used tools; in fact, because of their low income, most farmers can't modernize their production techniques. If labor is a key factor of production, capital and other specific factors must be taken into account also. A decrease in the farm labor

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[1] Report on "***Situation du commerce exterieur du Burkina Faso en 2011***" INSD.

significantly influences the production, and then affects the export[2].

Gold, which world price is very important, is a pull factor of that farm labor[3]. Simulating its effects may help to consider compensation scenarios to find a level of production where the only variation is due to the migration of labor from agriculture to gold.

2. The Model

The main export products included in the study are indexed 1, …, *L*. Gold is indexed by 0. Respectively denote by *P*, *X* and *C* production, export[4] and domestic consumption variables. We adopt a finite set *U* of disjoint localities taking into account the effects of immitation.

For each locality $u \in U$, we note $p^u = (p_0^u, p_1^u, \ldots, p_L^u)'$ the production vector in the locality *u*. We adopts a reduced cutting *i.e.*, $\exists \ell \in \{0, \ldots, L\}$ such as $p_\ell^u \neq 0$ (*significant production*). Similarly there $x^u = (x_0^u, x_1^u, \ldots, x_L^u)'$ is the exports vector of locality and domestic consumption of the production is noted $c^u = (c_0^u, c_1^u, \ldots, c_L^u)'$.

The production of locality u covers first, local consumption and the rest of the production is exported. With the time variable t[5], we have:

$$p^{u,t} = c^{u,t} + x^{u,t} \tag{1}$$

Production is a function of labor N, capital K and specific factor M; this last factor depends on the product only, not the locality.The production function f is then expressed:

$$p_\ell^u = f(N_\ell^u, K_\ell^u, M_\ell) \tag{2}$$

We sets K_ℓ^u and M_ℓ. Gold is mainly exported, so, $p_0^u = x_0^u$.

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[2] we place ourselves in a simple analysis where production is partly absorbed by domestic consumption, the rest being exported; stocks are negligible.

[3] In addition to the modern exploitation of gold, another type of exploitation is developped and draws the farm labor

[4] *re-exports are not included.*

[5] **t** represents the year and the stock at **t-1** is assumed to be negligible.

Labor used in the production grows at constant rate, *i.e*:

$$\sum_{u\in U}\sum_{\ell=0}^{L} N_{\ell}^{u,t+1} = (1+\rho)\sum_{u\in U}\sum_{\ell=0}^{L} N_{\ell}^{u,t} \tag{3}$$

At initial time t_0^u, $N_0^{u,t_0^u} \neq 0$: we begin with a moment where the gold production is not null in the locality u. only labor comes from other areas has been used for the production of gold (the assumed growth rate constant of the latter is denoted τ); no absorption was observed. At time t_0^u+1, the gold produced in the locality has absorbed a part of the agricultural workforce. There has been a significant change in the structure of the workforce in the locality.[6]

At time $t+1$, gold has absorbed a proportion λ_ℓ of labor used at $t \geq t_0^u$ in agricultural main good production ℓ and a proportion $1-\lambda_\ell$ of new workers. So,

$$N_0^{u,t+1} = \sum_{\ell=1}^{L}\left\{\lambda_\ell N_\ell^{u,t} + (1-\lambda_\ell)(N_\ell^{u,t+1} - N_\ell^{u,t})\right\} + (1+\tau)^{1+t-t_0^u} N_0^{u,t_0^u} \tag{4}$$

From one year to another, the labor used in the production of goods $\{1, \ldots, L\}$ decreases due to the strong attraction of gold.

Thus, $t \geq \max_{u\in U} t_0^u$,

$$\sum_{u\in U}\sum_{\ell=1}^{L} N_{\ell}^{u,t+1} < \sum_{u\in U}\sum_{\ell=1}^{L} N_{\ell}^{u,t} \tag{5}$$

As $K_\ell^{u,t}$ and M_ℓ^t are fixed, this reduction in farm labor affects the production; we get:

$$\sum_{u\in U}\sum_{\ell=1}^{L} p_{\ell}^{u,t+1} < \sum_{u\in U}\sum_{\ell=1}^{L} p_{\ell}^{u,t} \tag{6}$$

The production covers the first domestic consumption (gold is totally exported), only the rest meets the needs of foreign markets:

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[6] a judicious choice of group of localities can help us taking into account the effects of immitation.

$$\sum_{u\in U}\sum_{\ell=1}^{L} p_\ell^{u,t} = \sum_{u\in U}\sum_{\ell=1}^{L} c_\ell^{u,t} + \sum_{u\in U}\sum_{\ell=1}^{L} x_\ell^{u,t} \tag{7}$$

We will estimate for each product $\ell \in \{1, \ldots, L\}$, function $(g_\ell)_{\ell\in\{1,\ldots,L\}}$ such that:

$$\sum_{u\in U} c_\ell^{u,t} = g_\ell\Big(\sum_{u\in U} p_\ell^{u,t}, R^t\Big) \tag{8}$$

The functions g are independent of time. We note

The demographic growth rate is assumed to be constant.

Then we have the following relationship:

$$\begin{aligned}\left\{\sum_{\ell=1}^{L}\sum_{u\in U} x_\ell^{u,t+1} - \sum_{\ell=1}^{L}\sum_{u\in U} x_\ell^{u,t}\right\} &= \left\{\sum_{\ell=1}^{L}\sum_{u\in U} p_\ell^{u,t+1} - \sum_{\ell=1}^{L}\sum_{u\in U} p_\ell^{u,t}\right\} \\ &\quad - \left\{\sum_{\ell=1}^{L} g_\ell\Big(\sum_{u\in U} p_\ell^{u,t+1}, R^{t+1}\Big) - \sum_{\ell=1}^{L} g_\ell\Big(\sum_{u\in U} p_\ell^{u,t}, R^t\Big)\right\}\end{aligned} \tag{9}$$

To get back a level of production (thus a maximum potential export) by offsetting factors K_ℓ^u et M^ℓ, we must choose $(k,m)\in[0,1]\times[0,1]$ such that:

$$\begin{aligned}&\sum_{\ell=1}^{L}\sum_{u\in U} f(N_\ell^{u,t+1}, K_\ell^u + k.K_\ell^u, M_\ell + m.M_\ell) = \\ &-\left\{\sum_{\ell=1}^{L}\sum_{u\in U} f(N_\ell^{u,t+1}, K_\ell^u, M_\ell) - \sum_{\ell=1}^{L}\sum_{u\in U} f(N_\ell^{u,t}, K_\ell^u, M_\ell)\right\}\end{aligned} \tag{10}$$

3. The Program and the Basic Equations

Combining equations (3) and (4), we obtain the relation for $t \geq \max_{u\in U} t_0^u$:

$$\begin{aligned}&\sum_{\ell=1}^{L}\sum_{u\in U}(2-\lambda_\ell)N_\ell^{u,t+1} = \sum_{\ell=1}^{L}\sum_{u\in U}(\rho + 2(1-\lambda_\ell))N_\ell^{u,t} + (1+\rho)\sum_{u\in U} N_0^{u,t} - \\ &\sum_{u\in U}(1+\tau)^{1+t-t_0^u} N_0^{u,t_0^u}\end{aligned} \tag{11}$$

with $\rho < \lambda_\ell$

Equations (7), (2) and (8) give:

$$\sum_{u\in U}\sum_{\ell=1}^{L} x_\ell^{u,t} = \sum_{\ell=1}^{L}\sum_{u\in U} f(N_\ell^{u,t}, K_\ell^u, M_\ell) - \sum_{\ell=1}^{L} g_\ell\left(\sum_{u\in U} f(N_\ell^{u,t}, K_\ell^u, M_\ell), R^t\right) \tag{12}$$

By integrating the exports , program to solve is to choose $(k,m) \in [0,1]\times[0,1]$ such that:

$$\begin{aligned}\sum_{\ell=1}^{L}\sum_{u\in U} f(N_\ell^{u,t+1}, K_\ell^u + k.K_\ell^u, M_\ell + m.M_\ell) = -\left\{\sum_{\ell=1}^{L}\sum_{u\in U} x_\ell^{u,t+1} - \sum_{\ell=1}^{L}\sum_{u\in U} x_\ell^{u,t}\right\} \\ -\left\{\sum_{\ell=1}^{L} g_\ell\left(\sum_{u\in U} f(N_\ell^{u,t+1}, K_\ell^u, M_\ell), (1+r)\times R^t\right) - \sum_{\ell=1}^{L} g_\ell\left(\sum_{u\in U} f(N_\ell^{u,t}, K_\ell^u, M_\ell), R^t\right)\right\}\end{aligned} \tag{13}$$

There are many pairs (k,m) that satisfy the program. Econometric tools can estimate functions of production and consumption under certain assumptions. Information on agricultural exports will determine the "***law***" of process

$$\left\{\sum_{\ell=1}^{L}\sum_{u\in U} x_\ell^{u,t+1} - \sum_{\ell=1}^{L}\sum_{u\in U} x_\ell^{u,t}\right\}$$

4. Simulation

We will simulate the program on several functions of production and consumption and using the law followed by agricultural exports.

**** Effect on Farm Labor***

The effect is explained by a farm labor workers who desire gold workers living conditions. We derive the expressions as if they were continuous variables.

$$\sum_{\ell=1}^{L}\sum_{u\in U} \frac{\partial(N_\ell^{u,t})}{\partial(\sum_{u\in U} N_0^{u,t})} = -(1+\rho)\times cardU \times \sum_{\ell=1}^{L} \frac{1}{\rho + 2(1-\lambda_\ell)} \tag{14}$$

***** Effect on Agricultural Exports***

The effect is justified by the strong attration of farm labor to the gold, agricultural production decreases and thus the potential export.

$$\frac{\partial(\sum_{u\in U} x_\ell^{u,t})}{\partial(\sum_{u\in U} N_0^{u,t})} = -(1+\rho)\sum_{u\in U}\frac{1}{\rho+2(1-\lambda_\ell)} \times \left[\frac{\partial f(.)}{\partial N_\ell^{u,t}} - \frac{\partial g_\ell(.)}{\partial^1()}\bigg|_{f(.)} \times \frac{\partial f(.)}{\partial N_\ell^{u,t}}\right] \quad (15)$$

The notation $\partial^1()$ means that the function g_ℓ is derived following its first variable and card *U* is the number of locations, $\ell \in \{1,\ldots,L\}$.

*** ***Compensation Mechanism***

The problem is to propose a mechanism of compensation in terms of equivalent of other factors to find maximun level of agricultural products for export.

- **Choose function forms**
- **Use the law of the series of agricultural exports**
- **Set the labor used in the production of gold** $\sum_{u\in U} N_0^{u,t}$
- **Set reasonable values of labor for each agricultural product** $N_\ell^{u,t}$ **taking into account equation (14)**
- **Deduce from equation (11), the possible values of** $N_\ell^{u,t+1}$
- **Simulate several values of** K_ℓ^u **and** M_ℓ**, deduce ratio k, m**

4.1 Assumptions

In this paper, used functions are Cobb-Douglas type. The parameters of these functions will be selected taking into account the weight that is given more to this factor as another factor. We will often make use of the normal distribution. Other parameters that are needed to run the model are reasonably taken based data sources that we have.

4.2 Numerical Solve

In a simple analysis, functions used for the simulation are Cobb-Douglas. The choice of other functions can lead to a complex equation whose analytical solution is not easy; in this case, a numerical method is then recommended.

4.3 The Results

Using equation (azieq14) we can setting labor values for gold and agricultural products and simulate parameters λ_ℓ. Knowing $\lambda_1,\ldots,\lambda_L$ we cand find the loss of production in each agricultural product. Another way to have the parameters $\lambda_1,\ldots,\lambda_L$ is to realize a survey. In our context (Burkina Faso), if very difficult to do that in "exploitation artisanale".

```
# Generating randoms values for agricultural products
L<-2 # 2 products
U<-13
# agricultural product 1
Agr_Labor_1<-matrix(nrow=U,ncol=3)
set.seed(12345) # reproducible results
for (i in 1:U) {
Agr_Labor_1[i,1]<-i
Agr_Labor_1[i,2]<-round(rnorm(1,50000,100),0)
# normal distribution with rounding
Agr_Labor_1[i,3]<-round(rnorm(1,40000,100),0)
# normal distribution with rounding
}
Agr_Labor_1<-as.data.frame(Agr_Labor_1)
names(Agr_Labor_1)<-c("Region","Num_t","Num_t+1")
head(Agr_Labor_1)
## Region Num_t Num_t+1
## 1 1 50059 40071
## 2 2 49989 39955
## 3 3 50061 39818
## 4 4 50063 39972
## 5 5 49972 39908
## 6 6 49988 40182
# agricultural product 2
Agr_Labor_2<-matrix(nrow=U,ncol=3)
for (i in 1:U) {
```

```
Agr_Labor_2[i,1]<-i
Agr_Labor_2[i,2]<-round(rnorm(1,10000,60),0)
# normal distribution with rounding
Agr_Labor_2[i,3]<-round(rnorm(1,5000,60),0)
# normal distribution with rounding
}
Agr_Labor_2<-as.data.frame(Agr_Labor_2)
names(Agr_Labor_2)<-c("Region","Num_t","Num_t+1")
head(Agr_Labor_2)
## Region Num_t Num_t+1
## 1 1 9971 5037
## 2 2 10037 4990
## 3 3 10049 5132
## 4 4 10123 5098
## 5 5 10015 5029
## 6 6 9981 4900
# Gold
Gold_Labor<-matrix(nrow=1,ncol=2)
Gold_Labor[1,1]<-round(rnorm(1,100000,250),0)
Gold_Labor[1,2]<-round(rnorm(1,150000,250),0)
Gold_Labor<-as.data.frame(Gold_Labor)
names(Gold_Labor)<-c("Num_t","Num_t+1")
head(Gold_Labor)
## Num_t Num_t+1
## 1 1e+05 150088
```

	Product	Variation
1	1.00	−2.59
2	2.00	−1.30

Combining equation (14) and the 2 variations found previously, we have:

$$\begin{cases} (13 + a_1) * \rho - 2 * a_1 * \lambda_1 \ + 2 * a_1 + 13 = 0 \\ (13 + a_2) * \rho - 2 * a_2 * \lambda_2 + 2 * a_2 + 13 = 0 \\ \rho < \lambda_1 \text{ and } \rho < \lambda_2 \end{cases}$$

```
# 13 is the number of region
a1<-MyVariation$Variation[1]
a2<-MyVariation$Variation[2]
a1
## [1] -2.591
a2
## [1] -1.298
```

References

[1]. Roger D. PENG, (2013), Computing for data analysis, course, coursera.org.

[2]. INSD (2012), Situation du commerce ext´erieur du Burkina Faso en 2011, Report, edition 2012, P.7-8.