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# Spectral Analysis of the Sum of Infinitesimal Perturbations in the Mathematical Models of Dynamic Systems

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## Abstract

Continuous changes are happening in the dynamic systems observed in nature: time, configuration of objects and the mass of these objects. On the basis, the certain parameters of the system also change over time.

In this paper, we consider the sum of infinitesimal perturbations of the boundary values of observations used in mathematical models. For each time point, a specific set of numbers corresponds that can be determined from a comparative analysis of observation results and calculations by related models. Observations can be divided into the following parts:1) those that may be determined by mathematical modeling; 2) those that take hidden part in the observed processes. They usually are elusive. Over the time, they can cause a variety of resonance phenomenaor processes, such as chaos and catastrophes; 3) errors of the equipment used: measurement of time; measurement of the distance between the bodies of the system and their masses; 4) errors of performers of the work.

As an example, this paper presents the results of a comparative analysis of the coordinates $\alpha$ and $\delta$, for resonant asteroids of Hecuba family. Their boundary values can be represented by the expressions:

$$\begin{aligned} &-390^{s},00 \le \varepsilon(\Delta\alpha_{TO}) \le 615^{s}02, \\ &-1812''00 \le \varepsilon(\Delta\delta_{TO}) \le 1029''00;\ [10:22\ (1940-1962)] \end{aligned} \qquad \text{(I)}$$

where, the number of observations subjected to comparative analysis and years covered by these observations are indicated in brackets. In the expressions (I), $\varepsilon$ indicates compliance of this parameter to the parameter $\varepsilon$ in the A.M. Lyapunov's theorems on stability. It follows that as long as the sum of infinitesimal perturbations vary within the boundaries of (I), they are stable in the sense A.M. Lyapunov. In the case of violation of the borders (I), additional perturbation forces (or objects) are creeping into Solar system and they can cause a variety of resonance phenomena such as chaos or catastrophes.

*Keywords*: dynamic systems, mathematical modeling, the sum of infinitesimal perturbations, boundary values of problems, resonant asteroids of Hecuba family, observations, comparative analyzes.

# 1. Introduction

The processes and objects observed in nature vary with the time. Their masses, links, mutual distances and characteristics are changing. Observations can be divided into the following parts:1) those that can be found by mathematical modeling. They always depend on time and reflect the main laws of motions (or process). As an example, we can indicate the well-known laws of nature such as Kepler's laws and the gravity law; 2) those that take part hiddenly in the observed processes. They usually are elusive. Over time, they can cause a variety of resonance phenomena or processes, such as chaos and catastrophes. Such phenomena are self-organizing and self-regulating abnormal phenomena such as tsunamis and other variety of atmospheric phenomena. They originate at particular conditions, develop, and eventually disposed by energy dissipation. The sum of infinitesimal perturbations plays a significant role in dynamic, economic, and other processes. It is always present in chaos and catastrophe.

The observed processes are being formed and are under the influence of the evolution of both systems of internal elements and a range of external objects and processes.

The interactions between the elements and link with neighboring systems are reflected by the amount of infinitesimal perturbations individual assessments for which are associated with many complex processes. These are physical principles (forces) that constitute the sum of the elusive infinitesimal perturbations. They reflect changes of both the internal and the external structures of dynamical systems [1, 2, 3, 4, 5, 8].

Cyclic character of changes in terms of sums of infinitesimal disturbances often leads to resonances, which eventually become apparent as chaos and catastrophe. They are often observed in nature. Large forest can burn from small spark. This is how the chaos and catastrophe are born.

Thus, the theory of chaos and catastrophe is reduced to uniting into a single system of the sums of infinitesimal perturbations.

Deviations from cyclic character for variety of processes lead to deviations from the normal course and this eventually leads to abnormal phenomenain nature.

In the mathematical modeling, the selection of set of initial conditions (values) has a special significance. Therefore, the solution can be reduced to a search of such valuesor conditions that could guarantee the existence (or non-existence) of solutions (or chaos and catastrophes). In general, the solution of this problemis reduced to solving the following two problems: 1) is it possible to find such initial boundary conditions (values), using the known observational data, which could guarantee the existence (or non-existence) of solutions to the problem (or the chaos and disasters)? 2) is it possible to find a solution to the problem (or time, place and forces of chaos and possible accidents) from the analysis of the solutions of system of differential equations for the problem?

## 2. On the Boundary Values Limitations in Dynamic Systems

The universe exists as a single automated system. Each time moment of the system corresponds to a unified system of statistical data. It consists of many sets of inter connected automated subsystems, with many bodies and features. The values of the observations, regardless of which object or process they correspond to at different times, in different locations (places) of space, unequivocally reflect all that happened and is happening in the universe. However accounting of this actions is connected with many difficulties. They are fully reflected in the estimates of the values of the observations. We study all observed processes with the help of observations. Observations can be divided into four parts. 1) The amount of actions that can be studied using methods of mathematical modeling. It allows disclosure of laws governing the formation, evolution and prediction of the motion or process under consideration. Their behavior depends on the evolution of parameters of the system under consideration and time. 2) The amount of actions, which is present hiddenly in the studied motion or process. This amount and its additive are generally elusive and display themselves in the form of resonance phenomena such as chaos or catastrophe. Their existence depends on variations in the parameters of the system and time. 3) The amount of activity of the auxiliary equipment (apparatus) in the form of sums of errors of the instrument: measurement of time, measurement of distance and measurement of mass. They are present in all movements and research processes. They depend on the changes in the parameters of the equipment in use and do not depend ontime. 4) The sum of performers' work errors. They do not depend on time.

Forecasting the each following position of the object of dynamic system can be determined from the equation:

$$\alpha(t+1) = f(x(t), y(t), z(t)),$$
$$\delta(t+1) = \varphi(x(t), y(t), z(t))$$

i.e. form the assessment of system of previous observations. They can be represented as:

$$\left.\begin{aligned} &\alpha(t-1) = f(x(t-2), y(t-2), z(t-2)),\\ &\alpha(t) = f(x(t-1), y(t-1), z(t-1)),\\ &\alpha(t+1) = f(x(t), y(t), z(t),). \end{aligned}\right\} \tag{2}$$

for times $t = t-1,\ t$ and $t+1$. Similar formulas are valid for the $\delta(t-1),\ \delta(t)$ and $\delta(t+1)$.

Such observed dynamic processes can also be presented as a system of differential equations [4, 5]:

$$\begin{aligned} &\frac{d^2\alpha}{dt^2} = F(\frac{d\alpha}{dt}, g(\alpha, t)) + R\ (\alpha, t),\\ &\frac{d^2\delta}{dt^2} = K(\frac{d\delta}{dt}, h(\delta, t)) + \Phi(\delta, t) \end{aligned} \tag{3}$$

where

$$\begin{aligned} &\alpha \in [\alpha_i, \alpha_{i+1}],\ \delta \in [\delta_1, \delta_{i+1}];\ t \in [t_i, t_{i+1}],\ \ (i = 1, 2, ..., n);\\ &F(\frac{d\alpha}{dt}, g(\alpha, t)), K(\frac{d\delta}{dt}, h(\delta, t)) \end{aligned} \tag{4}$$

represent the sum of the effects that allows imulation, i.e. allow to obtain the solution of the problem in closed form; $R(\alpha, t),\ \Phi(\delta, t) -$ sums of subtle hidden infinitesimal disturbances, which can cause of many subtle (hidden) chaotic processes.

As an example, we will consider the ranges of variations of assessments of boundaries of observations minus calculation of results (i.e. O − C; where O and C are the results of observations and calculations, respectively) for the resonant asteroids of Hecuba family (108).

Intermediate orbit of resonance asteroids of Hecuba family (108) was built in [7], based on the solution of the generalized problem of three fixed centers [6].

Comparative analysis of the observational data and calculation of results are given in Table 1

**Table 1**

| n | Date (UT) | $\alpha_0$ | $\Delta\alpha_K$ | $\Delta\alpha_{TU}$ | $\Delta\alpha_1$ | $\delta_0$ | $\Delta\delta_K$ | $\Delta\delta_{TU}$ | $\Delta\delta_1$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1940.X.4.88553 | $22^h.16^m.52^s.98$ | $057^s.82$ | $020^s.02$ | $940^s$ | $11°.47'.26''.7$ | $0831''$ | $0093''$ | $340''$ |
| 2 | 1941.XI.9.78519 | $02^h.19^m.16^s.04$ | $642^s.36$ | $615^s.16$ | $830^s$ | $18°.51'.50''.74$ | $3673''$ | $1029''$ | $270''$ |
| 3 | 1947.X.26.13665 | $03^h.53^m.30^s.27$ | $437^s.73$ | $429^s.73$ | $875^s$ | $25°.40''.27''.23$ | $1353''$ | $-387$ | $310''$ |
| 4 | 1949.II.22.93811 | $09^h.31^m.24^s.80$ | $184^s.54$ | $148^s.88$ | $-905^s$ | $17°.56'.06''.13$ | $-1368''$ | $-1026''$ | $-245''$ |
| 5 | 1951.VII.28.59292 | $20^h.30^m.26^s.42$ | $-370^s.42$ | $-390^s.00$ | $-928^s$ | $-23°.43'.59''.9$ | $1338''$ | $0480''$ | $194''$ |
| 6 | 1952.XI.24.94576 | $00^h.22^m.56^s.32$ | $-080^s.32$ | $-067^s.12$ | $735^s$ | $03°.39'.59''.9$ | $0720''$ | $0240''$ | $225''$ |
| 7 | 1953.XI.30.65556 | $05^h.00^m.45^s.42$ | $026^s.48$ | $018^s.48$ | $680^s$ | $20°.03'.34''.67$ | $-092''$ | $-1812''$ | $207''$ |
| 8 | 1955.II.18.6 | $11^h.11^m.58^s.26$ | $-063^s.10$ | $-058^s.14$ | $-661^s$ | $06°.28'.16''.36$ | $842''$ | $0604''$ | $-215''$ |
| 9 | 1957.VII.26.01942 | $21^h.35^m.54^s.95$ | $-102^s.95$ | $-130^s.95$ | $-728^s$ | $-18°.09'.24''.17$ | $-654''$ | $-213''$ | $-187''$ |
| 10 | 1962.VII.6.88889 | $17^h.48^m.49^s.68$ | $-005^s.30$ | $-002^s.00$ | $-628^s$ | $-29°.43'.09''.16$ | $-243''$ | $177''$ | $198''$ |

where the first column displays numbers of observations used, that are listed in the third column; the dates of observations are given in the second column; the fourth column shows their relevant data of Keplerian orbit; the fifth column shows the results of the calculation for the right ascension $\delta$, according to formulas of intermediate orbit of the corresponding model of the generalized problem of three fixed centers [6]. In the sixth column, their respective results of by F. Isayeva [9], that are obtained based on the model of averaged planar three-body problem, based on the scheme Delaunay Hill. In columns 7, 8, 9, 10 of the table, the corresponding results are shown for the equatorial coordinates of inclination, Hecuba (108).

From the analysis of results shown in Table1, we obtain:

$$\begin{aligned} &-390^s, 00 \leq \varepsilon(\Delta\alpha_{TO}) \leq 615^s 02, \\ &-1812'' 00 \leq \varepsilon\ (\Delta\delta_{TO}) \leq 1029'' 00;\ [10:22(1940-1962)] \end{aligned} \tag{5}$$

In (5) $\varepsilon$ indicates compliance of this parameter to the parameter $\varepsilon$ in the theorems about the stability by A.M. Lyapunov [10]. It follows that as long as the sum of infinitesimal perturbations $\varepsilon$ of the solar system varies within the range (5), it is stable in the sense A.M. Lyapunov. When the range (5) is violated, additional perturbation forces (or objects) are creeping into Solar system and they can cause a variety of resonance phenomena such as chaos or catastrophes.

## 3. Conclusion

In this paper, qualitative analysis of observations in dynamic systems is performed. Observations are split into the following components. 1) The part that expresses the basic laws of problems and can be mathematically modeled. These are the basic laws of physics, chemistry, biology, economics, etc. 2) the part that take part hiddenly in the observed processes. They are usually elusive. Over time, they can cause a variety of resonance phenomena or processes such as chaos and catastrophes; 3) error of the equipment used: measuring time, measuring the distance between the bodies of the system and their masses; 4) errors of performers of the work.

2) +3) +4) is the sum of infinitesimal perturbations. It can be presentin the following form for the resonant asteroid of Hecuba family (108):

$$
\begin{aligned}
&-390^{s},00 \le \varepsilon(\Delta\alpha) \le 615^{s}02, \\
&-1812''00 \le \varepsilon(\Delta\delta) \le 1029''00; \left[10\ (1940-1962)\right].
\end{aligned}
$$

It follows that as long as the sum of infinitesimal perturbations varies within indicated ranges the Hecuba family (108) of resonance asteroids is stable in the sense of Lyapunov [10]. When these ranges are violated in Hecuba family (108) of resonance asteroids, additional perturbation forces are creeping into the system and they can cause a variety of resonance phenomena such as chaos or catastrophes.

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