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Anti-Hausdorff I-Spaces

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Abstract

This is the second paper on I-spaces. Here anti-Hausdorffness has been introduced for I-spaces and many topological theorems related to anti-Hausdorffness have been generalized to I-spaces, as an extension of study of infratopological spaces.

Keywords: I-space, Trivial anti-Hausdorff I-space, Non-trivial anti-Hausdorff I-space, I-Continuous image, Quotient I-space, Irreducible spaces, Infratopological spaces.

1. Introduction

In a previous paper [1] we have introduced I-spaces and studied some of their properties. In this paper we use the terminology of [1]. Some study of these spaces was done previously in ([2], [3], [4], [5]) in less general form. These spaces were called infratopological spaces. Anti-Hausdorff topological spaces and anti-Hausdorff U-spaces were introduced and studied in [6] and [7] respectively. In this paper the concept of anti-Hausdorff I-spaces have been introduced and a few important properties of such spaces have been studied. A number of interesting examples have been constructed to prove non- trivialness of such results.

2. Anti-Hausdorff I-Spaces

We have generalized some results on anti-Hausdorff topological spaces in [6] to I-spaces. We recall that an I- space X is a non-empty set X together with a collection $\mathscr{I}$ of subsets of X such that (i) $\mathscr{I}$ is closed under finite intersections, and (ii) X and Φ belong to $\mathscr{I}$.

Definition 2.1. An I- space X with $|X| \geq 2$ is said to be anti-Hausdorff, if for no pair of distinct points x, y in X, there exist I- open sets G and H such that $x \in G$, $y \in H$, $G \cap H = \Phi$, i.e., if no pair of distinct points can be separated by disjoint I-open sets. Here, $|X|$ denotes the number of elements of X. An anti-Hausdorff I-space which is not a topological space and hence, not a U-space will be called a non-trivial anti-Hausdorff I-space. Otherwise it is called trivial. It is easily seen that anti-Hausdorff I-spaces with 1 or 2 elements are trivial spaces.

Definition 2.2. If (X, $\mathscr{I}$) is an I-space and $\Phi \neq A \subseteq X$, then $\mathscr{I}_A = \{A \cap G \mid G \in \mathscr{I}\}$ is an I-structure in A. For, $\bigcap_{\alpha}(A \cap G_{\alpha}) = A \cap \left(\bigcap_{\alpha} G_{\alpha}\right) and \bigcap_{\alpha} G_{\alpha} \in \mathscr{I}$. Thus (A, $\mathscr{I}_A$) is an I-space, and is called an I-subspace of (X, $\mathscr{I}$).

Example 2.1. Let X = {a, b, c, d}, $\mathscr{I}$ = {X, Φ, {a},{a, b},{a, c}, {a, c, d}}. Then (X, $\mathscr{I}$) is non-trivial anti- Hausdorff I- space.

Example 2.2. Let X = {a, b, c, d,e} and $\mathscr{I}_1$ = {X, Φ, {b},{a, b},{b, d}},
$\mathscr{I}_2$ = {X, Φ,{b},{b, c},{a, b, d}}. Then (X, $\mathscr{I}_1$) and (X, $\mathscr{I}_2$) are non-trivial anti-Hausdorff I- spaces.

Example 2.3. Let X = $\mathbb{N}$, $\mathscr{I}$ = {X, Φ, $2\mathbb{N}$, $5\mathbb{N}$}. Then (X, $\mathscr{I}$) is a non-trivial anti- Hausdorff space.

We introduce the concept of irreducibility as in [6], [7] and [8].

Definition 2.3. An I-space X is said to be irreducible if every pair of non-empty I-open sets in X intersect.

Thus an I-space X is irreducible if, for every pair of non-empty I-open sets V, W in X, $V \cap W \neq \Phi$.

We now prove a theorem which corresponds to Theorem of 4.2 of [6] & Theorem 2.5 of [7].

Theorem 2.1. Let X be an I-space. The following statements are equivalent:

(i) X is anti-Hausdorff,

(ii) X is irreducible,

(iii) The union of two distinct non- empty I-open sets in X is connected,

(iv) Every non- empty I-open set in X is dense in X,

Proof: We first prove (i) $\Leftrightarrow$ (ii).

To prove (i) $\Rightarrow$ (ii) let X be an anti- Hausdorff. If possible suppose that X is not irreducible. Then there exist non-empty I-open sets V and W in X such that $V \cap W = \Phi$. Since V and W are non-empty, there exist $x \in V$ and $y \in W$. Since $V \cap W = \Phi$, $x \neq y$. X being anti-Hausdorff, this is a contradiction. Therefore X is irreducible.

We now prove (ii) $\Rightarrow$ (i). Let X be irreducible. If possible, let X be not anti-Hausdorff.

Then there exist x, y $\in X$ with $x \neq y$ and I-open sets V and W in X with $V \cap W = \Phi$ and $x \in V$, $y \in W$. Since V and W are non-empty, this is a contradiction to the fact that X is irreducible.

Hence X is anti-Hausdorff.

To prove (ii) $\Rightarrow$ (iii), Suppose X is irreducible and also suppose (iii) does not hold. Then there exists non-empty distinct I-open sets V_1 and V_2 in X, such that $V_1 \cup V_2 = V$(say) is disconnected. Then $V = V_3 \cup V_4$, for some non-empty I-open sets such that $V_3 \cap V_4 = \Phi$. This contradicts (ii) and hence to our hypothesis. Hence the union of distinct non-empty two I-open set in X is connected.

To prove (iii) $\Rightarrow$ (ii), Suppose (iii) holds. If possible, let X be not irreducible. Then there exists non-empty I-open sets V_1, V_2 in X such that $V_1 \cap V_2 = \Phi$. Then $V_1 \cup V_2$ is disconnected. This contradicts (iii). Hence X is irreducible.

We now prove (ii) $\Leftrightarrow$ (iv). Let X be a irreducible. Let V be a non- empty I-open set in X and let $x \in X$. Let W be an I-open set in X such that $x \in W$. Then $W \neq \Phi$. Since X is irreducible,

$V \cap W \neq \Phi$. So, $x \in \overline{V}$. Thus $X = \overline{V}$, i.e., V is dense in X. Thus (ii) $\Rightarrow$ (iv).

Conversely, suppose that every non-empty I-open set in X is dense in X. Let V and W be two non-empty I-open sets in X and let $x \in V$. Since $\overline{W} = X$ and V is a neighborhood of x, $V \cap W \neq \Phi$. So X is irreducible.

Therefore (iv) $\Rightarrow$ (ii).

The proof of the theorem is thus complete.

Theorem 2.2. Every I-continuous image of an anti- Hausdorff I-space is an anti-Hausdorff I-space.

Proof: Let X, Y be two I-spaces where X is anti-Hausdorff I-space. Let f be a I-continuous map of X onto Y. Let y_1 and y_2 be two distinct points of Y, and let $H_1$and H_2 be two I-open sets in Y such that $y_1 \in H_1$, $y_2 \in H_2$. Since f is onto there exist x_1, x_2 in X such that $f(x_1) = y_1$, $f(x_2) = y_2$.

Let $G_1 = f^{-1}(H_1)$, $G_2 = f^{-1}(H_2)$. Since f is I-continuous, both G_1 and G_2 are I-open sets. Since X is anti-Hausdorff I-space, $G_1 \cap G_2 \neq \Phi$. Let $x \in G_1 \cap G_2$, then $f(x) \in H_1 \cap H_2$. Thus $H_1 \cap H_2 \neq \Phi$. So, Y is anti-Hausdorff I-space.

Definition 2.4. Let (X, $\mathscr{I}$) be an I-space and R an equivalence relation on X. The equivalence class for each $x \in X$ is denoted by $\bar{x}$. We define an I-structure $\overline{\mathscr{I}}$ on the collection of equivalence classes $\frac{X}{\mathrm{R}}$ of X with respect to R as follows. Any subset $\overline{V}$ of $\frac{X}{\mathrm{R}}$ will be a member of $\overline{\mathscr{I}}$ iff $\{x \in X | \bar{x} \in \overline{V}\} \in \mathscr{I}$, i.e., for every I-open set V, the collection of equivalence classes of the elements is I-open in $\frac{X}{\mathrm{R}}$, and these are the only I-open members of $\frac{X}{\mathrm{R}}$. That $\overline{\mathscr{I}}$ is an I-structure is obvious. This I-structure $\overline{\mathscr{I}}$ is called the identification I-structure or the quotient I-structure on X, and ($\frac{X}{\mathrm{R}}$, $\overline{\mathscr{I}}$) is called the identification I-space or the quotient I-space of X with respect to R.

Example 2.4. Let X = $\mathbb{R}$, $\mathscr{I} = \{\mathbb{R}, \Phi\} \cup \{(n.i, n.(i+1)) \mid n \in \mathbb{Z}, i = 0,1,2,\ldots\ldots 9\} \cup \{(n.j, n.(j+2)) \mid n \in \mathbb{Z}, j = 0,1,2,\ldots\ldots,8\}$ (Here decimal representation has been used.). Then (X, $\mathscr{I}$) is a non-trivial anti-Hausdorff I-space.

Let ρ be the relation on $\mathbb{R}$ given by x ρ y, iff $x - y \in \mathbb{Z}$. Then ρ is an equivalence relation on $\mathbb{R}$, and for each $x \in \mathbb{R}$, $\bar{x}$ = x + Z, the coset of Z in $\mathbb{R}$, both Z and $\mathbb{R}$, being regarded as additive groups. The quotient I-structure $\overline{\mathscr{I}}$ *on* $\frac{\mathbb{R}}{\rho}$ is given by

$$\overline{\mathscr{I}} = \left\{ \frac{\overline{\mathbb{R}}}{\rho}, \Phi \right\} \cup \{(m.i, m.(i+1)) + Z | m \in Z\} \cup \{(n.j, n.(j+2)) + Z | n \in Z\},$$

where for integers *m* and *n*, $(m.i, m.(i+1)) + Z = \{((m+r).i, (m+r).(i+1)) \mid r \in Z\}$, and $(n.j, n.(j+2)) + Z = \{((n+s).j, (n+s).(j+2)) \mid s \in Z\}$.

Hence $\frac{\mathbb{R}}{\rho}$ is a non-trivial anti-Hausdorff I-space.

Corollary 2.1. If X is an anti-Hausdorff I-space and R is an equivalence relation on X, then the quotient I-space $\frac{X}{R}$ is anti-Hausdorff I-space.

Proof: It follows from the definition of quotient I-space that the map f: X $\rightarrow \frac{X}{R}$ given by $f(x)$ = cls x is continuous and onto. The corollary is then follows from Theorem 2.2.

Theorem 2.3. A U-subspace of a non-trivial anti-Hausdorff I-space need not be U-anti-Hausdorff.

Proof: Let us consider the I-space (X, $\mathcal{I}$), where X = {a, b, c, d} and $\mathcal{I}$={X, Φ,{a, b},{a, c}, {a, c, d},{a}}. Then (X, $\mathcal{I}$) is a non- trivial anti-Hausdorff I- space, since there is no pair of disjoint non-empty I-open sets in X. Now let Y = {b, c, d}. Then as a subspace of X, Y has the I- structure, $\mathcal{I}$={Y, Φ, {b},{c},{c, d}}. Obviously, Y is not anti- Hausdorff I-space.

Theorem 2.4. If A and B two non-trivial anti-Hausdorff I-subspaces of an I-space X, then the subspace A $\cap$ B need not be non-trivial anti- Hausdorff I- space.

Proof: Let X = {a, b, c, d, e, f}, $\mathcal{I}$= {X, Φ, {b, c},{a, b, c},{ b, c, d}, {b, c, d, e, f}}.

Clearly (X, $\mathcal{I}$) is a non- trivial I-space. Let A = {a, c, d, f } and B = {a, b, d, f}. Then A and B are I-subspaces of X with $\mathcal{I}_A$ ={A, Φ, {c},{a, c},{c, d}, {c, d, f}}, $\mathcal{I}_B$ = {B, Φ,{b},{a, b},{b, d}, {b, d, f}}. Clearly both A and B are non- trivial anti- Hausdorff I- subspaces of X. Now A $\cap$ B = {a, d, f } and $\mathcal{I}_{A \cap B}$ ={A $\cap$ B, Φ, {a},{d},{d, f}}. Then A $\cap$ B is a non-trivial I-space which is not anti-Hausdorff.

The following provides another example which proves the truth of theorem 2.4.

Example 2.5. Let X = {a, b, c, d, e, f}, $\mathcal{I}$= {X, Φ, {b, c},{a, b, c},{ b, c, d}, {b, c, d, e, f}}. Clearly (X, $\mathcal{I}$) is a non-trivial I- space. Let A = {a, b, c, d, e} and B = {a, b, c, d, f}. Then A and B are I- subspaces of X with $\mathcal{I}_A$ = {A, Φ,{b, c}, {a, b, c}, { b, c, d}, {b, c, d, e}}, $\mathcal{I}_B$ = {B, Φ, {b, c},{a, b, c},{b, c, d}, {b, c, d, f}}. Clearly both A and B are non- trivial anti- Hausdorff I- subspaces of X. Now A $\cap$ B = {a, b, c, d } and $\mathcal{I}_{A \cap B}$ ={A $\cap$ B, Φ, {b, c},{a, b, c},{b, c, d}}. Then A $\cap$ B is a trivial anti-Hausdorff I-space. Thus A $\cap$ B is not a non- trivial anti- Hausdorff I-space.

In the situation of Theorem-2.4, it is also possible that A $\cap$ B is a non-trivial anti-Hausdorff I-space as is shown by the following example.

Example 2.6. Let X = {a, b, c, d, e, f, g}, $\mathcal{I}$= {X,Φ, {a, b, c, d},{b, c, d, f},{ b, c, d}, {d, e, f, g}, {d},{d, f}}. Clearly (X, $\mathcal{I}$) is a non- trivial I- space. Let A ={a, b, c, d, e} and B = {a, b, c, d, e, f}. Then A and B are I- subspaces of X with $\mathcal{I}_A$ ={A, Φ, {a, b, c, d},{b, c, d}, {d, e}, {d}}, $\mathcal{I}_B$ = {B, Φ,{a, b, c, d}, {b, c, d, f}, { b, c, d}, {d, e, f},{d},{d,f}}. Clearly both A and B are non-trivial anti- Hausdorff I-subspaces of X. Now A $\cap$ B = {a, b, c, d, e} and $\mathcal{I}_{A \cap B}$ ={A $\cap$ B, Φ, {a, b, c, d},{b, c, d}, {d, e}, {d}}. Then A $\cap$ B is a non-trivial anti-Hausdorff I-space.

Remark 2.1. If A and B are two non-trivial subspaces of a non-trivial I-space X, then the subspace A $\cap$ B may be non-trivial anti-Hausdorff I-space even if neither A nor B is so.

Let X = $\mathbb{R}$, $\mathcal{I}$ = { $\mathbb{R}$, Φ ,{(a,b)$|a,b \in \mathbb{R}$, a< b}}. Clearly (X, $\mathcal{I}$) is a non- trivial I-space.

Let A = $(2,5)\cup\mathbb{Q}^c$ and B = $\left(\sqrt{7},\sqrt{29}\right)\cup\mathbb{Q}$, where $\mathbb{Q}^c = \mathbb{R} - \mathbb{Q}$. Then the I-structure $\mathcal{I}_A$ and $\mathcal{I}_B$ on A and B respectively are $\mathcal{I}_A$ = {A, Φ } $\cup$ {(a,b)$|a,b \in \mathbb{R}, 2\le a< b\le 5$} $\cup$ {(c,d)$\cap$ $\mathbb{Q}^c$ $|c,d \in \mathbb{R}$}.

Since $(3,4)\cap(5,6) \cap \mathbb{Q}^c = \Phi$. (A, $\mathcal{I}_A$) is not anti- Hausdorff I- subspace.

$\mathcal{I}_B$ ={B, Φ } $\cup$ {(a,b)$|a,b \in \mathbb{R}, \sqrt{7} < a< b < \sqrt{29}$ } $\cup$ {(c,d)$\cap\mathbb{Q}$$|c,d \in \mathbb{R}$}. Since $(\sqrt{7},\sqrt{29})\cap(6,7)\cap\mathbb{Q} = \Phi$. (B, $\mathcal{I}_B$) is not anti-Hausdorff I-subspace. Thus both A and B are non-trivial subspaces of an I-space X, neither of which is anti-Hausdorff.

A $\cap$ B = $\left(\sqrt{7},5\right)\cup((2,5)\cap\mathbb{Q})\cup((\sqrt{7},\sqrt{29})\cap\mathbb{Q}^c)$ and $\mathcal{I}_{A\cap B}$={ A $\cap$ B, Φ } $\cup$ { $(\sqrt{7},5)\cap(a,b)$ $|a,b\in\mathbb{R}$, a<b} $\cup$ {$(2,5)\cap\mathbb{Q})\cap(a,b)|a,b\in R, a<b$} $\cup${$(\sqrt{7},\sqrt{29})\cap\mathbb{Q}^c\cap(a,b)$ $|a,b \in \mathbb{R}$, a< b}

Thus $A\cap B$ is a non- trivial anti- Hausdorff I-space.

As in Theorem 2.9 of [6] and Theorem 2.3 of [7], we have

Theorem 2.5. Let A_1 and A_2 be two anti-Hausdorff I-spaces with I-structures $\mathcal{I}_1$ and $\mathcal{I}_2$ respectively. Then ($A_1\cup A_2$, $\langle \mathcal{I}_1\cup\mathcal{I}_2 \rangle$) need not be anti-Hausdorff I-space. Here $\langle \mathcal{I}_1\cup\mathcal{I}_2 \rangle$ is the I-structure generated by $\mathcal{I}_1\cup\mathcal{I}_2$ in $A_1\cup A_2$.

Proof: Let A_1 = {a, c, d, e} $\mathcal{I}_1$= {A_1, Φ, {a, d},{a, e}, {a}}, and A_2 = {a, c, d, f} $\mathcal{I}_2$ = {A_2, Φ, {c, d},{c, f},{c}}. Then (A_1, $\mathcal{I}_1$) and (A_2, $\mathcal{I}_2$) are non-trivial anti-Hausdorff I-spaces.

Then A = $A_1\cup A_2$ = {a, c, d, e, f}. Let $\mathcal{I}$ be the I-structure on A generated by $\mathcal{I}_1\cup\mathcal{I}_2$, i.e., $\mathcal{I}$={A, A_1, A_2, Φ,{a},{c},{d},{c, d},{c, f}, {a, d},{a, e}}}. So, in (X, $\mathcal{I}$), a$\in${a}, d$\in${d} with {a},{d}$\in\mathcal{I}$ and {a}$\cap${d} = Φ.

Hence (X, $\mathcal{I}$) is not an anti- Hausdorff I-space.

However, we also have

Theorem 2.6. If A and B are two non-trivial anti-Hausdorff I-subspaces of an I- space X, then $A\cup B$ may be non-trivial anti- Hausdorff.

Proof: Let X = {a, b, c, d, e, f, g}, $\mathscr{I}$ = {X, Φ, {a, b, c},{b, c, e, f}, {b, c}, {a, b}, {a, b, d}, {b}}. Clearly (X, $\mathscr{I}$) is a non- trivial anti-Hausdorff I- space. Let A = {a, b, d, e, f}, B = {b, c, d, g}. Then A and B are I- subspaces of X with $\mathscr{I}_A$ ={A, Φ, {a, b},{a, b, d}, {b, e, f}, {b}}, $\mathscr{I}_B$ = {B, Φ,{b, c}, {b}, {b, d}}. Clearly, both A and B are non- trivial anti-Hausdorff I-subspaces of X. Now A ∪ B = {a, b, c, d, e, f, g} and $\mathscr{I}_{A \cup B}$ ={A ∪ B, Φ, {a, b, c},{b, c, e, f}, {b, c}, {a, b}, {a, b, d}, {b}}. Hence, A ∪ B = X too, is a non-trivial anti-Hausdorff I-space.

The following theorems prove the truth of the various possibilities for A ∪ B, where A, B are I-subspaces of X.

Theorem 2.7. If A is a non-trivial anti-Hausdorff I-subspace and B is a non-trivial non anti-Hausdorff I-subspace of X, then A ∪ B may be a non-trivial anti-Hausdorff I- subspace of X.

Proof: Let X = {a, b, c, d, e, f, g}, $\mathscr{I}$ = {X, Φ, {a, b, c},{b, c, e, f}, {b, c}, {a, b}, {a, b, d}, {b}}. Clearly (X, $\mathscr{I}$) is a non- trivial anti-Hausdorff I-space. Let A = {a, b, d, e, f}, B = {a, d, e, f}. Then A and B are I-subspaces of X with $\mathscr{I}_A$ ={A, Φ, {a, b},{a, b, d}, {b, e, f}, {b}}, $\mathscr{I}_B$ = {B, Φ,{a}, {e, f}, { a, d}}. Thus A is a non-trivial anti- Hausdorff I-subspace and B is non-trivial non anti-Hausdorff I-subspace of I-space X. Now A ∪ B = {a, b, d, e, f } and $\mathscr{I}_{A \cup B}$={A ∪ B, Φ, {a, b, d},{b, f}, {b}}.

Here A ∪ B is a non-trivial anti-Hausdorff I-subspace of X.

The following example also proves the truth of Theorem 2.7

Example 2.7. Let X = ℝ, $\mathscr{I}$ = {ℝ, Φ, (1,5), [2,3), (1,2], [2,4], {2}}. Thus X is a non-trivial anti-Hausdorff I-space. Let A = (1, 4], $\mathscr{I}_A$ ={A, Φ, [2,3), (1,2], [2,4], {2}}. A is a non-trivial anti-Hausdorff I- subspace of I-space X. Let B = [1, 2] ∪ [3,6], $\mathscr{I}_B$ = {B, Φ, (1,2],[3,5),{2},[3,4]}. Hence B is a non-trivial non anti-Hausdorff I- subspace of X. Now A ∪ B = [1, 6], $\mathscr{I}_{A \cup B}$ ={A ∪ B, Φ, (1,5), [2,3), (1,2], [2,4], {2}}.

Thus A ∪ B is a non-trivial anti-Hausdorff I-subspace of X.

Theorem 2.8. If A and B are non-trivial non anti-Hausdorff I-subspaces of an I- space X, then A ∪ B may be a non-trivial non anti-Hausdorff I-subspace of X.

Proof: Let X = {a, b, c, d, e, f, g}, $\mathscr{I}$ = {X, Φ, {a, b, c},{b, c, e, f}, {b, c}, {a, b}, {a, b, d}, {b}}. Clearly, (X, $\mathscr{I}$) is a non- trivial anti-Hausdorff I-space. Let A = {a, d, e, f}, B = {c, d, f}. Then A and B are I- subspaces of X with $\mathscr{I}_A$ = {A, Φ,{a}, {e, f}, { a, d}}, $\mathscr{I}_B$ = {B, Φ,{c}, {c, f}, {d}}. Clearly, A and B are non- trivial non anti-Hausdorff I-subspace of I-space X. Now A ∪ B = {a, c, d, e, f} and $\mathscr{I}_{A \cup B}$ ={A ∪ B, Φ, {a, c},{c, e, f},{c}, {a},{a, d}}. Here A ∪ B is a non-trivial non anti-Hausdorff I-subspace of X.

Theorem 2.9. If A and B are non-trivial non anti-Hausdorff I-subspace of an I-space X, then A ∪ B may be a non-trivial anti-Hausdorff I-subspace of X.

The following example proves the truth of the theorem.

Proof: Let X = $\mathbb{R}$, $\mathscr{I}$ = { $\mathbb{R}$, Φ, [2,4], [3,5], [3,4] $\cup$ [7,10]}. Then X is a non-trivial anti-Hausdorff I-space. Let A = ([2,3] $\cap$ $\mathbb{Q}^c$) $\cup$ ([3,4] $\cap$ $\mathbb{Q}$) $\cup$ ([4,5] $\cap$ $\mathbb{Q}^c$) $\cup$ [7,10].Then $\mathscr{I}_A$ ={A, Φ, ([2,3] $\cap$ $\mathbb{Q}^c$), ([3,4] $\cap$ $\mathbb{Q}$), ([4,5] $\cap$ $\mathbb{Q}^c$)}. A is non-trivial non-anti-Hausdorff I-subspace of an I-space X. Let B = ([2,3) $\cap$ $\mathbb{Q}$) $\cup$ ([3,4] $\cap$ $\mathbb{Q}^c$) $\cup$ ((4,5] $\cap$ $\mathbb{Q}$), $\mathscr{I}_B$ = {B, Φ, ([2,3) $\cap$ $\mathbb{Q}$), ([3,4] $\cap$ $\mathbb{Q}^c$), ((4,5] $\cap$ $\mathbb{Q}$)}. B is non-trivial non-anti-Hausdorff I-subspace of an I-space X.

Now A $\cup$ B = [2, 3) $\cup$ [3,4] $\cup$ (4,5] $\cup$ [7,10] = [2,5] $\cup$ [7,10]. Therefore $\mathscr{I}_{A \cup B}$={A $\cup$ B, Φ, [2,4], [3,5], [3,4]}.

Thus A $\cup$ B is a non-trivial anti-Hausdorff I-subspace of X.

Theorem 2.10. If A is a non-trivial anti-Hausdorff I-subspace and B is non-trivial non anti-Hausdorff subspace of an I-space X, then A $\cup$ B may be a non-trivial non anti- Hausdorff I-subspace of X.

Proof: Let X = $\mathbb{R}$, $\mathscr{I}$ = { $\mathbb{R}$, Φ, (1,5), [2,3), (1,2], [2,4], (-3, -2), (-4, -3), {2}}. Then X is non-trivial non anti-Hausdorff I-space. Let A = [1,4), then $\mathscr{I}_A$ ={A, Φ,(1,4), [2,3), (1,2], [2,4],{2}}. A is non-trivial anti-Hausdorff I-subspace of an I-space X.

Let B = (-4, -1), $\mathscr{I}_B$ = {B, Φ, (-3, -2), (-4,-3)}. B is non-trivial non anti-Hausdorff I-subspace of an I-space X.

Now A $\cup$ B = {[1, 4) $\cup$ (- 4,-1)}, $\mathscr{I}_{A \cup B}$ ={A $\cup$ B, Φ, (1,4), [2,3),(1,2], [2,4),{2},(- 3,-2),(- 4, - 3)}.

Here A $\cup$ B is a non-trivial non anti-Hausdorff I-subspace of X.

Theorem 2.11. If A and B two non-trivial anti-Hausdorff I-subspaces of a I-space X, then A $\cup$ B may be a non-trivial non anti-Hausdorff I-subspace of X.

Proof: Let X = $\mathbb{R}$, $\mathscr{I}$ = {$\mathbb{R}$, Φ, [1,5), [2,3), (1,2], [2,4],{2}, [-3, -2], [-2, -1), {-2},}. Then X is non-trivial not anti-Hausdorff I- space. Let A = [1,3), then $\mathscr{I}_A$ ={A, Φ, [2,3), (1,2], {2}}. A is non-trivial anti-Hausdorff I-subspace of an I-space X.

Let B = [-4, -1], $\mathscr{I}_B$ = {B, Φ, [-3, - 2], [-2,-1), {-2}}. B is non-trivial anti-Hausdorff I-subspace of an I-space X.

Now A $\cup$ B = {[1, 3) $\cup$ [-4, -1]}, $\mathscr{I}_{A \cup B}$ ={A $\cup$ B, Φ, [1,3), [2,3), (1,2], {2},[-3, -2], [-2, -1), {-2}}.

Here A $\cup$ B is a non-trivial non anti-Hausdorff I-subspace of X.

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