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Comparison of Stochastic Soybean Yield Response Functions to Phosphorus Fertilizer

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Abstract

The random parameter approach of fertilizer response model was known better than fixed parameter version for determining optimum doses of fertilizer recommendation. However, the selection of functional forms suitable for certain cropping condition was also critical. The purpose of this study was to know the best model of stochastic soybean yield response function to phosphorus fertilizer. The research was conducted based on multilocation experimental data of soybean yield response to phosphorus fertilizer. The fixed parameter models (M1) of Linear plateau, Spillman-Mitscherlich, Quadratic and Logistic were compared with the random parameter models containing either 1 (M2) or 2 random effects (M3) using -2 log-likelihood, Akaike information criterion, and Bayesian information criterion. Results showed that the AIC values of M1 fixed parameter models sequentially were Linear plateau < Spillmann-Mitscherlich = Logistic < Quadratic. Meanwhile, the AIC values of M2 random parameter models sequentially were Linear plateau < Logistic < Spillmann-Mitscherlich < Quadratic. The AIC values of M3 random parameter models sequentially were Spillmann-Mitscherlich < Logistic < Linear plateau. The best model for soybean yield response function to phosphorus fertilizer was the stochastic Spillmann-Mitscherlich model with location intercept and the increase in yield by applying fertilizer random effects.

Keywords: response functions, fixed effect, random effects, multilocation trials,soybean.

1. Introduction

Regional fertilizer recommendation usually based on a general curve of multilocation fertilizer trials using ordinary least squares quadratic regression with assumption the residuals were normally, independently, and have a constant variances. The model parameters were assumed to have a fixed value. However, this approach was unrealistic ignoring the variability and correlation that probably exist between locations (Wallach, 1995; Makowski et al., 2002; Makowski and Lavielle, 2006).

The alternative model was to estimate parameter of fertilizer response model using mixed model approach of random parameter model. The mixed model approach was possible to consider the random effects that represent the variability between location, the heterogenous variance, and the correlation that probably exist between observation. Some studies showed that the random parameter model approach was better than the fixed parameter version for determining optimum doses of fertilizer recommendation (Makowski et al., 2001; Makowski et al., 2002; Tumusiime et al., 2011; Boyer et al., 2013).

Furthermore, the quadratic function commonly used for fitting fertilizer data was not always the best model. Tumusiime et al. (2011) and Park et al. (2012) showed that the stochastic linear plateau and Mitscherlich’s exponential type functions were better than quadratic function.Boyer et al. (2013) revealed that the stochastic linear plateau functionwas better than quadratic plateau function for corn response to nitrogen fertilizer.

The purpose of this study was to know the best model of stochastic soybean yield response function to phosphorus fertilizer.

2. Methodology

2.1. Data

The study using data of fourteen multilocation trials of phosphorus fertilizationon soybean in Java and Sumatera (Nursyamsi and Sutriadi, 2004; Nursyamsi et al., 2004). Each trial consists of five levels of phosphorus fertilizer treatments. The phosphorus fertilizer levels applied wereas 0, 20, 40, 80 and 160 kg P/ha of SP36. The response measured was soybean grain dry weight (t/ha). The experiments using a randomized complete block design with three replications.The soybean grain yield responses obtained with different phosphorus fertilizer treatments on experiments was shown in Figure 1.

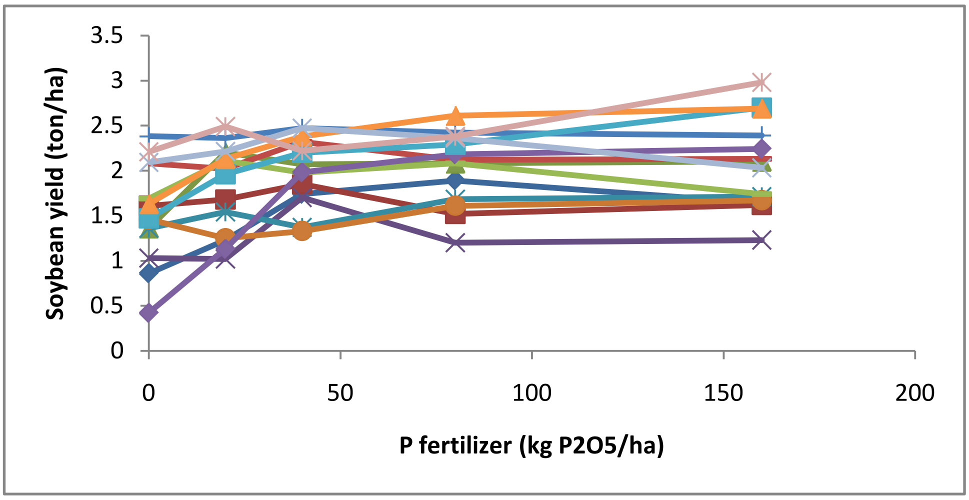

Figure 1. The soybean yield responses to applied phosphorus for fourteen locations

2.2. Methods

The Stochastic Response Model

The stochastic response model generally expressed as

$$Y_{ij} = f(\boldsymbol{\beta}_{ij}, \boldsymbol{u}_{ij}) + \varepsilon_{ij}, \varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2) \quad (1)$$

where Y_{ij} is the i^{th} observation (soybean yield) on the j^{th} location (i=1, 2, ….., n_j); j = 1, 2, …,L); L is the total number of locations and n_j is the number of observations on the j^{th} location; f is the linear or nonlinear function relating soybean yield to phosphorus fertilizer and other possible covariates $\boldsymbol{u}_{ij}$ varying with location; $\boldsymbol{\beta}_{ij}$ is a vector with the parameters of the linear or nonlinear function; ε_{ij} is the residual term; and σ_ε^2 is the variance for the residuals.

The $\boldsymbol{\beta}_{ij}$ vector may be modeled in a second stage as the sum of 2 components: a fixed (population) component, $\boldsymbol{\beta}$, common to all location, and a random component, $\boldsymbol{b}$, specific to each location. Therefore:

$$\boldsymbol{\beta}_{ij} = \boldsymbol{A}_{ij}\boldsymbol{\beta} + \boldsymbol{B}_{ij}\boldsymbol{b}_j, \boldsymbol{b}_j \sim N(0, \sigma_u^2) \quad (2)$$

where $\boldsymbol{A}_{ij}$ and $\boldsymbol{B}_{ij}$ are design matrices for the fixed and random effects, respectively; $\boldsymbol{\beta}$ is a p-dimensional vector of fixed population parameters; $\boldsymbol{b}_j$ is a q-dimensional random effects vector associated with location (not varying with i), and σ_u^2 is the variance of the random effects. It is assumed that observations made on different location are independent, and ε_{ij} is independent of $\boldsymbol{b}_j$ (Lindstrom and Bates, 1990; Davidian and Giltinan, 2003).

The stochastic linear plateau response model was as follows,

$$Y_{ij} = \min\left(\alpha_1 + (\alpha_2 + u_{j3})X_{ij}\,;\, \mu_p + u_{j2}\right) + u_{j1} + \varepsilon_{ij} \tag{3}$$

where Y_{ij} is the soybean yield in i^{th} plot and j^{th} location; X_{ij} is thephosphorus fertilizer level; α_1 is the intercept parameter; α_2 is the linear response coefficient; u_p is the plateau yield; u_{j1} is the (intercept) location random effects; u_{j2} is the plateau location random effects; u_{j3} is the slope random effects; and ε_{ij} is the random error term.

The stochastic Spillman-Mitscherlich response model was as follows,

$$Y_{ij} = \beta_1 - \left(\beta_2 + u_{j2}\right)\exp\left(-\beta_3 + u_{j3}\right)X_{ij} + u_{j1} + \varepsilon_{ij} \tag{4}$$

where β_1 is the maximum or potential yield attainable by applying phosphorus fertilizer in experimental condition; β_2 is the increase in yield by applying phosphorus fertilizer; β_3 is the ratio of successive increment in ouput β_1 to total output *Y*; u_{j1}; u_{j2}; u_{j3} are the random effects; and ε_{ij} is the random error term.

The stochastic quadratic response model was as follows,

$$Y_{ij} = \gamma_1 + (\gamma_2 + u_{j2})X_i + (\gamma_3 + u_{j3})X_i^2 + u_{j1} + \varepsilon_{ij} \tag{5}$$

where γ_1 is the intercept parameter whose position (values) could shift up or shift down from location to location by location intercept random effect u_{j1}; γ_2 is the linear response coefficient with the random effect u_{j2}; γ_3 is the quadratic response coefficient with the random effect u_{j3}; and ε_{ij} is the random error term.

The stochastic logistic response model was as follows,

$$Y_{ij} = (\delta_3 + u_{j3})/[1 + \exp\left(\delta_1 - (\delta_2 + u_{j2})X_i\right) + u_{j1} + \varepsilon_{ij} \tag{6}$$

where δ_3 is the maximum yield, δ_1 is the intercept parameter; δ_2 is the response coefficient by applying phosphorus fertilizer; u_{j1}; u_{j2}; u_{j3} are the random effects; and ε_{ij} is the random error term (Tembo et al., 2003; Tumusiime et al., 2011; Brorsen, 2013).

If the model was nonstochastic, then the random effects u_{j1}, u_{j2} and u_{j3} would be zero. In the stochastic models the random effects were entered sequentially. The first was the model with one random effect u_{j1} (M2) (Park et al., 2012) and then the model with two random effects u_{j1} and u_{j2} (M3) and the model with three random effects u_{j1}, u_{j2} and u_{j3} (M4). However, the models with three random effects u_{j1}, u_{j2} and u_{j3} (M4) were not convergent. The random parameters u_{j1} and u_{j2} are assumed to have a mean of zero with a 2 × 2 unstructured covariance matrix.

Statistical Analysis

The response model was estimated using nonlinear mixed model procedure (SAS Institute Inc., 2010). Under normality assumption the selection of the best model using criteria of -2log-likelihood (-2LL), Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and residual variance (σ_ε^2). The smaller values of -2LL, AIC, BIC and residual variance indicatinga better model fitting to the data.

3. Results and Discussion

3.1 The Model Parameter Estimate

The parameter estimate of linear plateau model with fixed effect and mixed effects arepresented in Table 1.

There are differences of parameter estimate of intercept, the linear response coefficient and the plateau mean between the fixed model M1 with the random parameter model M2 and M3, although the all three parameter were significant, except for intercept parameter of the random parameter model M3. The SE for residual variance was lowest in M3 following by M2, with the M1 being the greatest. In M2, residual variance was separated into within location variation (σ_ε^2) and between location variation (σ_{u1}^2). The residual variance was reduced by about 75.63 percent in M2 compared with M1 indicating an improvement in the accuracy of model parameter estimation. The log-likelihood, AIC and BIC value of M2 smaller than M1 suggesting a better fit of M2 to the data.

Tabel 1. Parameter estimate of Linear Plateau model

Parameter	Fixed model (M1)	Mixed model (M2)	Mixed model (M3)
	Estimate (SE)	Estimate (SE)	Estimate (SE)
α_1	1.56** (0.11)	1.61** (0.09)	0.10 (0.12)
α_2	0.01** (0.004)	0.01* (0.005)	0.02** (0.003)
u_p	2.05** (0.08)	2.10** (0.07)	0.62** (0.13)
σ_ε^2	0.002**(0.0003)	0.0007**(0.0001)	0.0003**(0.00006)
σ_{u1}^2	-	0.002**(0.0005)	0.01**(0.0017)
σ_{u2}^2	-	-	0.002**(0.0004)
σ_{u1u2}	-	-	3 10-7
-2 LL	86.7	45.0	63.8

AIC	94.7	55.0	75.8
BIC	103.7	58.2	79.7

* significant at 5% level; ** significant at 1% level; σ_{ε}^2= residual variance; $\sigma_{u1}^2, \sigma_{u2}^2$= the individual variance within population; σ_{u1u2} = individual covariance between random effects; M1 = no random effect, M2 = 1 random effect, M3= 2 random effects, SE= standard error

The residual variance of M3 was reduced by about 89.92 and 58.62 percent compared with M1 and M2, respectively. From M3, it appears that the large proportion of between location variation accounted for by location intercept variation (σ_{u1}^2) than the plateau variation (σ_{u2}^2), although the plateau variation also significant at 5 percent level. The covariance between the two random effects relatively small. Based on the model fitting criteria, the stochastic linear plateau M3 was the best model compare with the stochastic linear plateau M2 and the deterministic model M1.

The Spillman-Mitscherlich model parameter estimate with the fixed effect and the mixed effects are given in Table 2.

Tabel 2. Parameter estimate of Spillman-Mitscherlichmodel

Parameter	Fixed model (M1)	Mixed model (M2)	Mixed model (M3)
	Estimate (SE)	Estimate (SE)	Estimate (SE)
β_1	2.06** (0.10)	2.04** (0.10)	2.15** (0.07)
β_2	0.52** (0.15)	0.52** (0.09)	0.46** (0.12)
β_3	0.04 (0.03)	0.04* (0.02)	0.035** (0.007)
σ_{ε}^2	0.002** (0.0003)	0.0007**(0.0001)	0.0003**(0.00008)
σ_{u1}^2	-	0.0016 (0.001)	0.0012**(0.0002)
σ_{u2}^2	-	-	0.0034*(0.001)
σ_{u1u2}	-	-	2 10-8
-2 LL	86.8	46.1	29.6
AIC	94.8	56.1	41.6
BIC	103.8	59.3	45.4

* significant at 5% level, ** significant at 1% level
M1 = no random effect, M2= 1 random effect, M3= 2 random effects

The parameter estimate of β_2 and β_3 forthe fixed parameter model M1 and the random parameter model M2 and M3 were similar, but the β_3 parameter of M1 was not significant (P-value=0.0601). The estimate of potential yield parameter β_1 was differ between the three models. The SE for residual variance was lowest in M3 following by M2, with the M1 being the greatest. In M2, the residual variance reduced by about 73.11 percent compared with M1 indicating the accuracy of model parameter estimation increases. The log-likelihood, AIC and BIC values of M2 smaller than M1 suggesting that M2 was better for fitting the data.

The residual variance of M3 reduced by about 78.15 and 18.75 percent compared with M1 and M2 respectively. In M3, the proportion of between location variability much more explained by location intercept variability (σ_{u1}^2) than the increase in yield by applying fertilizer (σ_{u2}^2). The covariance between the two random effects relatively small. The fitting model criteria indicate that the stochastic Spillman-Mitscherlich modelM3 was the best model compared with the stochastic Spillman-Mitscherlich model M2 and the fixed model M1.

The parameter estimate of quadratic model with the fixed effect and the mixed effectsare given in Table 3. The M3 quadratic model was not convergent.

Tabel 3. Parameter estimate of Quadratic model

Parameter	Fixed model (M1)	Mixed model (M2)
	Estimate (SE)	Estimate (SE)
γ_1	1.60** (0.10)	1.60** (0.11)
γ_2	0.0099** (0.0036)	0.0099** (0.0022)
γ_3	-0.00004* (0.00002)	-0.00004** (0.00001)
σ_ε^2	0.002**(0.0003)	0.0007**(0.00014)
σ_{u1}^2	-	0.0013*(0.00055)
σ_{u2}^2	-	-
σ_{u1u2}	-	-
-2 LL	87.8	48.9
AIC	95.8	58.9
BIC	104.8	62.1

* significant at 5% level, ** significant at 1% level

M1 = no random effect, M2= 1 random effect, M3= 2 random effects

The parameter estimate of intercept, linear responseandquadraticresponse of the fixed parameter model M1 and the random parameter model M2 were similar indicating that in the quadratic model the expected means of mixed effect model as the same as that of the fixed effect model. The SE for residual variance in the M2 was lower than the M1. In M2, the residual variance reduced by about 75.41 percent compared with M1 indicatingan improvement in the accuracy of model parameter estimation. The log-likelihood, AIC and BIC values of M2 was smaller than M1 suggesting a better fittingof M2 to the data.

The parameter estimate of logistic model with the fixed effect and the mixed effects were shown in Table 4.

Tabel 4. Parameter estimate of Logistic model

Parameter	Fixed model (M1)	Mixed model (M2)	Mixed model (M3)
	Estimate (SE)	Estimate (SE)	Estimate (SE)
δ_3	2.06** (0.09)	2.04** (0.099)	2.06** (0.11)
δ_1	-1.087** (0.34)	-1.075** (0.21)	-1.08** (0.21)
δ_2	0.05 (0.03)	0.05* (0.02)	0.05* (0.018)
σ_ε^2	0.002**(0.0003)	0.0007**(0.00014)	0.0007**(0.00014)
σ_{u1}^2	-	0.0016 (0.0010)	0.0012 (0.0008)
σ_{u2}^2	-	-	0.00015 (0.0009)
σ_{u1u2}	-	-	-6 10-7
-2 LL	86.8	46.0	46.1
AIC	94.8	56.0	58.1
BIC	103.8	59.2	61.9

* significant at 5% level, ** significant at 1% level
M1 = no random effect, M2= 1 random effect, M3= 2 random effects

There are differences of parameter estimate of maximum yield, the intercept coefficient and the fertilizer response between the fixed model M1 and the random parameter model M2 and M3. The SE for residual variance in M3 as same as in M2, with the M1 being the greatest. In M2, residual variance was reduced about 73.11 percent compared with the fixed model M1 indicating the accuracy of parameter model estimation improved. The log-likelihood, AIC and BIC value of mixed model M2 smaller than the fixed model M1 suggesting that the mixed model M2 was better for fitting the data.

The residual variance of the mixed model M3 reduced by about 74.79 and 6.25 percent compared with the fixed model M1 and the mixed model M2 respectively. In the mixed model M3, the large proportion of the between location variability explained by the location intercept variability than the response coefficient by applying fertilizer variability. The covariance of the two random effects was small. The fitting model criteria show that the stochastic logistic M2 was the best model compared with the mixed model M3 and the fixed model M1.

3.2 The Model Comparison

The log-likelihood, AIC and BIC values of all the fixed model M1 and the mixed model M2 were shown in Table 5.

Table 5. The log-likelihood, AIC, and BIC values of fixed and mixed effects model

Model	Fixed model (M1)			Mixed model (M2)			Mixed model (M3)		
	-2LL	AIC	BIC	-2LL	AIC	BIC	-2LL	AIC	BIC
LP	86.7	94.7	103.7	45.0	55.0	58.2	63.8	75.8	79.7
SM	86.8	94.8	103.8	46.1	56.1	59.3	29.6	41.6	45.4
Q	87.8	95.8	104.8	48.9	58.9	62.1	-	-	-
LOG	86.8	94.8	103.8	46.0	56.0	59.2	46.1	58.1	61.9

LP = linear plateau; SM = Spillman-Mitscherlich; Q = quadratic; LOG = logistic

In the fixed model M1 the log-likelihood, AIC and BIC values sequentially were linear plateau < Spillmann-Mitscherlich = logistic < quadratic indicating that the linear plateau model was the best fixed model. In the mixed model M2 the log-likelihood, AIC and BIC values sequentially were linear plateau < logistic < Spillmann-Mitscherlich < quadratic indicating that the linear plateau model was also the best M2 mixed model. However, in the mixed model M3 the log-likelihood, AIC and BIC values sequentially were Spillmann-Mitscherlich < logistic < linear plateau. Therefore the Spillmann-Mitscherlich model was the best M3 mixed model.

The log-likelihood, AIC, and BIC values of the mixed model M2 and M3 were smaller than the fixed model M1 in all response functions indicating that the mixed model was better for fitting the data than the fixed model version (Table 5). This phenomena were caused by the fact that in the mixed model decomposing of variance-covariance associating with the random effects make it possible to separate the between location variability from the within location variability. However, the mixed model with the larger number of random effects was not always the best model. Based on the fitting model criteria the stochastic Spillmann-Mitscherlich model with location intercept and the increase in yield by applying fertilizerrandom effects (M3) was the best model for yield response function of soybean to phosphorus fertilizer.

4. Conclusions

The stochastic response model was more accurate than the deterministic version to estimate parameter of fertilizer response model. The best model for soybean yield response function to phosphorus fertilizer was the stochastic Spillmann-Mitscherlich model with location intercept and the increase in yield by applying fertilizer random effects.

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