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Response Surface Methodology for Process Monitoring of Soft Drinks: A Case of Delta Beverages in Zimbabwe

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Abstract

Experimentation plays a crucial role in the manufacturing industry. Observing and gathering information about a process helps define how an input variable transforms into a response variable of particular interest to the company. We applied both the first-order and second-order types of Response Surface Methodology (RSM) to analyze the total quantity of Sparkling Beverages produced, revenue collected and the cost of raw materials. A model for minimizing the variability in the monthly quantities of the Sparkling Beverages was developed. In addition a discussion of a Univariate Statistical process control (USPC) scheme based on general linear profile monitoring of process quality was done. Phase I and II linear profile monitoring schemes were discussed in monitoring the slope and intercepts of the profiles. The control scheme helped to identify out of control profiles and hence the in-control process in the sparkling beverages business. The method of Steepest Ascend was useful in building up a model for maximizing the total quantity of sparkling beverages produced with optimal settings for the revenue and cost of raw materials. A Modified Central Composite Design was used to find operating conditions that minimized the variability in the volumes of sparkling beverages. In addition linear profile monitoring procedures were applied to detect shifts in the slope, intercept and error variance for the volumes of sparkling beverages considering the revenue and cost of raw materials.

Keywords: Response Surface Methodology, steepest ascent and steepest decent, Design of Experiments

and linear profiles.

1. Introduction

The company has shifted much of its attention to process control and the pressure from modern industrial quality movements has seen it adopting statistical process control procedures to eliminate sources of variation in the sparkling beverages manufacturing and distribution processes. The production of sparkling beverages is in accordance with specified international standards that range from mixing required ingredients, packaging, marketing and distribution. Therefore, this paper is applying Response Surface Methodology as a statistical tool to monitor the process of the manufacture of soft drinks. Ideally, the objective is to optimize the production quantity (Q^*) which minimizes total production cost, quantity variability as well as maximizing revenue. Box and Wilson (1951), laid the foundation for the Response Surface Methodology (RSM) and Montgomery (2005) describes the Response Surface Methodology as a collection of statistical and mathematical techniques used to develop, improve and optimize processes in which the response to be optimized is influenced by several input variables. The application of Response Surface Methodology (RSM) to design optimization is aimed at reducing the cost of expensive analysis methods and their associated numerical noise. Response Surface methodology provide superb statistical tool for design and analysis of experiments aimed at process optimization. RSM are powerful optimization tools in the arsenal of statistical design of experiments (DOE). Process engineers take full advantage of DOE to effectively screen the vital few factors from many trivial factors that have no significant impact on the response. One of the main objectives of RSM is the determination of the optimum settings of the control variables that result in a maximum (or minimum) response over a certain region of interest, $\mathfrak{R}$. Optimization techniques used in RSM depend on the nature of the fitted model.

2. Background and Literature

Originally, Response Surface Methodology (RSM) was developed to model experimental responses (Box and Draper, 1987) and then migrated into modeling of numerical experiments. The difference is in the type of error generated by the response. In RSM, errors are assumed to be random. A detail description of the design of experiments theory can be found in Box and Draper (1987), Myers and Montgomery (1995) and Montgomery (1997). Box and Draper (1975) listed several additional design properties that pertain to detection of lack of fit, generalization of satisfactory distribution of information throughout the experimental region, estimation of errors variance, insensitivity to outliers and the errors made in the actual implementation of the settings of the control variables.

Robust parameter design is a well-established engineering technique to increase the quality of a product by making it robust/insensitive to the uncontrollable variations present in the production process. A design is said to be robust if its properties are not severely impacted by failures to satisfy the assumptions made about the model and he error distribution. For the first degree models, the method of steepest ascent (or decent) is a viable technique for sequentially moving toward the optimum response. This method is explain in detail in Myers and Montgomery (1995), Khuri and Cornell (1996) and Box and Draper (2007). Myers and Khuri (1979) developed certain improvements regarding the stopping rule used in the execution of this method. Hoerl (1959) introduced the method of ridge analysis for optimizing the predicted response based on the fitted second degree model. Khuri and Myers (1979) proposed a modification of the method of ridge analysis whereby optimization of $\hat{y}(x)$ is carried out under an added constraint on the size of the prediction variance. Furthermore, Paul and Khuri (2000) extended the modification to liner models where the error variances are heterogeneous and also to generalized liner models. Draper and Hunter (1966) proposed a criterion for the estimation of the unknown parameters in the multi response situation. Their criterion was used for selecting additional experimental runs after a certain number of runs have already been chosen. Lind et al. (1960) developed a graphical approach in which contours of all the responses were superimposed on each other and the region where operation conditions were “near” optimal for all the response was identified.

According to Oehlert (2000) response surface Methods work with continuous treatments to find the optimum response by adjusting design variables in order to identify changes in the response in some given direction. Sometimes the quality of a process or product is determined by the relationship between a response variable and one or more predictor variables which is referred to as profile. In other words, the focus would be on monitoring the profile that represents such a relationship, instead of monitoring a single characteristic. Kang and Albin (2000), presented application of profile monitoring and the same work were produced by Mahmoud and Woodall (2004) as well as Mahmoud et al. (2007). Stover and Brill (1998) even proposed Phases in the analysis of profile monitoring of simple linear profiles and similar work was done by Mestek et al. (1994), Mahmoud and Woodall (2004) as well as Mahmoud et al. (2007). Mestek et al. (1994) used a T^2 control chart in combination with principal component analysis (PCA) approach to monitor a simple linear profile in calibration application. Stover and Brill (1998) also proposed two methods for monitoring simple linear profiles, that is, a multivariate T^2 control chart and Principal Component Approach-based control scheme. Mahmoud et al. (2007) were not left out and suggested the

Likelihood Ratio Statistic to monitor linear profiles while Zhu and Lin (2010) did a shewhart-type control chart for monitoring slopes of linear profiles in both Phases I and II. Phase II seeks to detect shift in the process parameters as quickly as possible.

3. Methodology

3.1 Research Data

Monthly data was collected for 48 months from the year 2008 to 2013 in the months from January to September of each year from Delta Corporation since the company enjoyed much of its business in those months. The data collected included the total monthly volumes of sparkling beverages Brewed, the cost of raw materials and total monthly revenue produced.

3.2 The Response Surface Methodology (RSM)

The Response Surface Methodology will be used to build models and evaluate the relative significance of the variables (revenue x_1 and the cost of raw materials x_2) as well as determine the optimum settings for a desirable quantity of sparkling beverages to be produced $\hat{y}(x)$. RSM is a very useful technique of studying the effects of variables on the quantity produced by varying the simultaneously and undertaking a number of experiments. Hence it allows one to find conditions for the optimum (better) response step by step. In applying the Response Surface Methodology the following steps will be followed:

(i) Approximate model function: A suitable approximation to the relationship between the variables and the response will be done starting with low order polynomials, that is, first and second order functions. The method of ordinary Least Squares (OLS) will be employed. The models will be developed to a function of the best quality using goodness of fit tests that determines if the approximate model is satisfactory. This will be done to check if the estimated model adequately describes the behavior of the response (quantity of sparkling beverages) in the current region of experimentation.

(ii) Design of experiments: According to Draper (1987), Design of Experiments are strategies for model fitting applicable to both physical and numerical data with the objective of selecting points where the response should be evaluated. These will be performed to confirm whether the chosen variables have an effect on the response.

(iii) Optimization using the model: The model selected as satisfactory will be used to find the optimum quantity of sparkling beverages to produce per month using optimal revenue and cost of raw materials. A line search in steepest ascent will be performed from centre point of current region of experimentation until no further improvement is observed or if the experimental region gets too small.

3.3 Method of Steepest Ascent

This method will be used to maximize the total monthly quantity of sparkling beverages as described by the first order model fitted which serves as a good local approximation in a small region close to the initial operating conditions. The fitted first order polynomial in two factors will be of the form:

$$\hat{y}(x) = \hat{\beta}_0 + \sum_{i=1}^{2} \hat{\beta}_1 \, x_i \tag{1}$$

This will be facilitated by finding the direction of maximum improvement in the quantity of sparkling beverages. Since we would want to maximize the quantity of sparkling beverages the direction of maximum improvement will be calculated as $\nabla \hat{y}(x)$ and the direction of the gradient will be calculated by the values of the parameter estimates (excluding the intercept), that is,

$$\hat{\beta}' = b' = (b_1, b_2) \tag{2}$$

which is scale dependent just like the scaled parameter estimates. The coordinates of the factor settings on the direction of steepest ascent separated a distance ρ from the origin will be solved from:

Max: $b_1 x_1 + b_2 x_2$

Subject to

$$\sum_{i=1}^{2} x_i^2 \leq \rho^2$$

where ρ is the step size which can be defined by the user. The solution will be arrived at by forming the Lagrangian to give

$$x^* = \rho \left(\frac{b}{\|b\|} \right) \tag{3}$$

Iterations along the direction of steepest ascent until no further increase in the quantity of sparkling beverages will be performed. As soon as we get closer to the optimal point a second order model will be used to model curvature.

3.4 Central Composite Design

This design will be used to construct a second order Response Surface model with the quantitative factors to help increase the precision of the estimated model as soon as we will be close to the optimum quantity of sparkling beverages. This will be used for estimation when the first order model displays significant lack of fit. Since the Response Surface is meant to locate an unknown optimization, a rotatable design will be used to give equal precision of estimation of the surface in all directions with $\alpha = 2.5$. The second order response surface model will be in the form:

$$\hat{y}(x) = \hat{\beta}_0 + \sum_{i=1}^{2} \hat{\beta}_i\, x_i + \sum_{i=1}^{2} \hat{\beta}_{ii}\, x_i^2 + \sum_{i=1}^{2} \sum\nolimits_{i<j}^{2} \hat{\beta}_{ii}\, x_i x_j \tag{4}$$

where $\hat{\beta}$ is the Least Squares Estimate of β. Optimum operating costs and revenue will be estimated analytically from:

$$\hat{x}_1 = \frac{\hat{\beta}_{12}\hat{\beta}_2 - 2\hat{\beta}_{22}\hat{\beta}_1}{4\hat{\beta}_{11}\hat{\beta}_{22} - \hat{\beta}_{12}{}^2} \tag{5}$$

and

$$\hat{x}_2 = \frac{\hat{\beta}_{12}\hat{\beta}_1 - 2\hat{\beta}_{11}\hat{\beta}_2}{4\hat{\beta}_{11}\hat{\beta}_{22} - \hat{\beta}_{12}{}^2} \tag{6}$$

where $\hat{\beta}_1$, $\hat{\beta}_2$,$\hat{\beta}_{12}$, $\hat{\beta}_{11}$ and $\hat{\beta}_{22}$ are least squares regression estimates.

3.5 Robust Parameter Design

The sales of sparkling beverages in millions of dollars were taken as a noise variable in order to find optimal values of the revenue and cost of raw materials that would minimize variability in the quantity of sparkling beverages produced. The mean volume of beverages produced was set between 600 000 hectolitres and 800 000 hectolitres for the eight month period of company business. A first order response model that incorporates the noise variable is expressed as:

$$y(x,z) = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{\beta}_{12} x_1 x_2 + \hat{\gamma}_1 x_1 z_1 + \hat{\delta}_{11} x_1 z_1 + \hat{\delta}_{21} x_2 z_1 + \epsilon \tag{7}$$

where $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_{12}$, $\hat{\gamma}_1$, $\hat{\delta}_{11}$ and $\hat{\delta}_{21}$ are the least squares regression coefficients and ϵ are independent and identically distributed random variables. Using the first order model the mean response is then calculated from:

$$E_z[y(x,z)] = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{\beta}_{12} x_1 x_2 + \hat{\beta}_{11} x_1^2 + \hat{\beta}_{22} x_2^2 \tag{8}$$

and the variance model is given by:

$$Var_z(y) = \sigma_z^2(\gamma_1 + \delta_{11} x_1 + \delta_{22} x_2)^2 + \sigma^2 \tag{9}$$

The desired target of the mean to minimize variability transmitted by the noise variable can be achieved by fixing controllable factors. The final model is expressed as:

$$y(x,z) = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{\beta}_{11} x_1^2 + \hat{\beta}_{12} x_1 x_2 + \hat{\beta}_{22} x_2^2 + \hat{\gamma}_1 x_1 z_1 + \hat{\delta}_{11} x_1 z_1 + \hat{\delta}_{21} x_2 z_1 + \epsilon \quad (10)$$

3.6 Monitoring Linear Profiles

3.6.1 Phase 1 Profile Monitoring

The sparkling beverages volume levels together with the revenue and the cost of raw materials for each eight month period per year constituted linear profiles. 6 profiles were studied altogether. The data for historical profiles was analyzed to evaluate the stability of the process involving the volume of sparkling beverages and revenue as well as that of volume and cost of raw materials. The procedure was also used to detect and get rid of any outliers as well as estimating in-control parameters. Estimates of the slope, intercept and error variance were obtained as below.

$$\hat{\beta}_0 = \frac{\sum_{j=1}^{k} \hat{\beta}_{0j}}{k} \quad (11)$$

and

$$\hat{\beta}_1 = \frac{\sum_{j=1}^{k} \hat{\beta}_{1j}}{k} \quad (12)$$

The estimate of the error variance was obtained from:

$$\hat{\sigma}^2 = \frac{\sum_{j=1}^{k} MSE_j}{k} \quad (13)$$

The control limits for the intercept, $\hat{\beta}_0$ are given by:

Center line (CL): $\hat{\beta}_0$

Lower Control Limit (LCL): $\hat{\beta}_0 - t_{k(n-2),\frac{\alpha}{2}} \sigma \sqrt{\frac{k-1}{kS_{xx}}}$

Upper Control Limit (UCL): $\hat{\beta}_0 + t_{k(n-2),\frac{\alpha}{2}} \sigma \sqrt{\frac{k-1}{kS_{xx}}}$

Similarly the control limits for the slope, $\hat{\beta}_1$ are given by:

Center Line (CL): $\hat{\beta}_1$

Lower Control Limit (LCL): $\hat{\beta}_1 - t_{k(n-2),\frac{\alpha}{2}} \sigma \sqrt{\frac{k-1}{kS_{xx}}}$

Upper Control Limit (UCL): $\hat{\beta}_1 + t_{k(n-2),\frac{\alpha}{2}} \sigma \sqrt{\frac{k-1}{kS_{xx}}}$

where k is the number of linear profiles, and $t_{k(n-2),\frac{\alpha}{2}}$ is a t distribution with $k(n-2)$ degrees of freedom at $\frac{\alpha}{2}$ level of significance. S_{xx} is given by:

$$S_{xx} = \sum_{i=1}^{n}(X_i - \bar{X})^2 \quad (14)$$

Any profile whose estimated value of the slope or intercept that fell outside the control limits was regarded as an outlier by considering the one with the largest deviance from centre first.

3.6.2 Phase 11 Profile Monitoring

Phase II monitoring of linear profiles was also employed since it is important in assessing the performance of control charts in detecting shifts in the parameters of linear profiles. The original values of the revenue and cost of raw materials were coded to make estimates of the intercept and slope independent and separate control charts were used to monitor the intercept and slope using the EWMA 3 chart for monitoring the intercept, the slope and the error variance was used.

The EWMA statistics for the Y-intercept was given by:

$$EWMA_I(j) = \theta . b_{0j} + (1-\theta). EWMA_I(j-1) \quad (15)$$

for $j = 1,2,3, \dots$ and $0 < \theta < 1$, is a smoothing constant and the initial value is given by:

$$EWMA_I(0) = \beta_0 \quad (16)$$

The upper and lower control limits are given by:

$$UCL = \beta_0 + L_I . \sigma \left(\sqrt{\frac{\theta}{(2-\theta).n}} \right) \quad (17)$$

and

$$LCL = \beta_0 - L_I . \sigma \left(\sqrt{\frac{\theta}{(2-\theta).n}} \right) \quad (18)$$

respectively. The EWMA chart for monitoring the slope was constructed using the EWMA statistic:

$$EWMA_S(j) = \theta . b_{1j} + (1-\theta). EWMA_S(j-1) \quad (19)$$

where θ is a smoothing constant, b_{0j} and b_{1j} are the least squares estimates of the intercept and slope for each profile j considering coded predictor variables. The initial value given by:

$$EWMA_S(0) = \beta_1 \quad (20)$$

The corresponding upper and lower control limits were given by:

$$UCL = \beta_1 + L_S.\sigma\left(\sqrt{\frac{\theta}{(2-\theta).n}}\right) \tag{21}$$

and

$$UCL = \beta_1 - L_S.\sigma\left(\sqrt{\frac{\theta}{(2-\theta).n}}\right) \tag{22}$$

respectively.

4. Discussion of Results

4.1 Model Building

Table 4.1 shows data used to come up with a model to maximize the total quantity of sparkling beverages produced. Knowledge of the sparkling beverages manufacturing process tells us that reasonable values for the revenue X_1 and cost of raw materials X_2 are $US500million and $US30million respectively. Varying the revenue by $US200 million dollars and the cost by $US20 million dollars gives a reasonable increment. The design centered on $US500 million dollars and $US30 million dollars.

Cost ($m)	Revenue ($m)	Volume (000hls
-1(10)	-1(300)	326
-1(10)	+1(700)	740
+1(50)	-1(300)	157
+1(50)	+1(700)	648
0(30)	0(500)	488
0(30)	0(500)	432
0(30)	0(500)	450

Table 4.1. Response Surface Design

Figure 4.1 is a response surface plot of volume (quantity of sparkling beverages), revenue and cost of raw materials. The plot shows the expected total quantity of sparkling beverages as a function of revenue and the cost of raw materials.

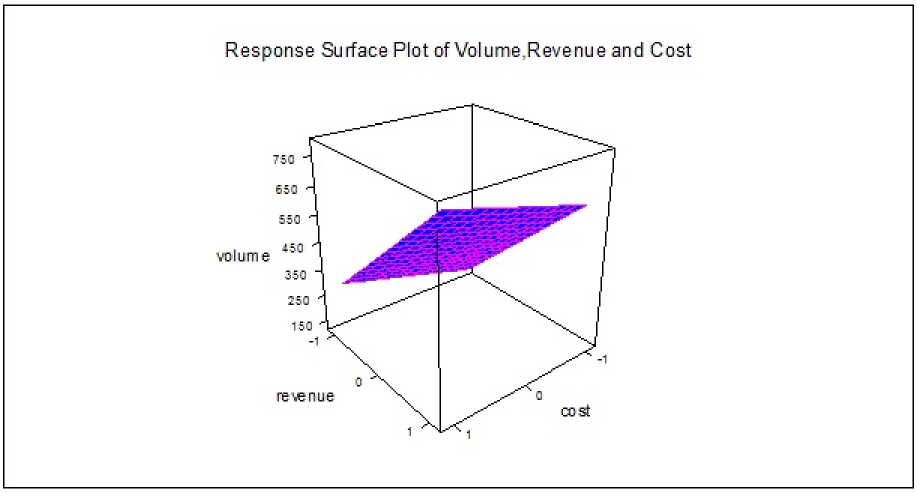

Figure 4.1. Response Surface Plot

Figure 4.2 shows contours of constant volume of sparkling beverages produced for the respective revenue and the cost of raw materials. Approximating the response surface with a plane gave a linear regression problem.

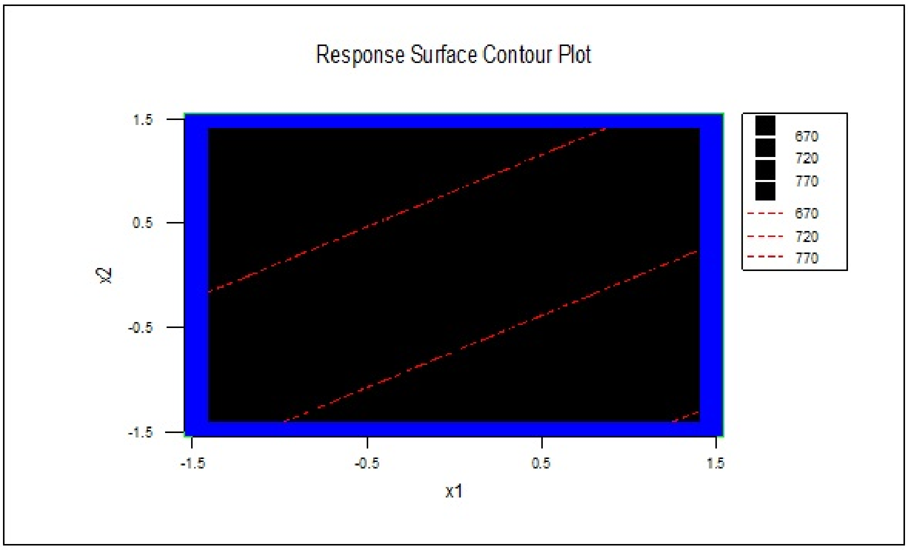

Figure 4.2. Response Surface Contour Plot

Fitting the response surface regression model, we obtained

$$Volume = 463 + 226.25X_1 + 65.25X_2$$

with maximum settings for X_1 and X_2 the company can realize approximately 89 000 hectolitres and 800 000 hectolitres of sparkling beverages in a period of 1 and 9 months respectively which is reasonable in its business operations.

4.2 Approximating design points for another response surface

Using the first-order response surface model above the direction of steepest ascent is given by (226.25,-65.25) in original units and (1.131, -3.263) in coded units. Iterations along (500, 30) + k(192,-65.25) were performed where (500, 30) is the design centre with k as step size and multiple observations in the centre were used for estimation of the measurement error without relying on any assumptions and for detection of curvature. This led to new data in Table 4.3 below:

Run	X_1 ($\$m$)	X_2 ($\$m$)	Volume (000hls)
1	690	25	696
2	880	20	927
3	1070	15	1159
4	1260	10	1390

Table 4.2. Second first Order Response Surface Design

The process shows that a volume of 696000 hectolitres is near optimum for revenue of $US690 million dollars and $US25 million dollars cost of raw materials. Hence a second order response surface model was necessary with revenue of $US690 million dollars and $US25 million dollars cost of raw materials as centre points.

4.3 Second Order Response Surface Model

In the design additional points on the axis were added to fit a model with additional parameters producing a Rotatable Central Composite Design. The new data is shown in Table 4.3 below:

Run	X_1 ($\$m$)	X_2 ($\$m$)	X_1 $(Coded)$	X_2 $(Coded)$	Volume (000hls)
1	670	15	-1	-1	340
2	710	15	+1	-1	775
3	670	35	-1	+1	350
4	710	35	+1	+1	648
5	662	25	$-\sqrt{2}$	0	300
6	718	25	$\sqrt{2}$	0	750

7	690	11	0	$-\sqrt{2}$	710
8	690	39	0	$\sqrt{2}$	410
9	690	25	0	0	368

Table 4.3. Second Order Response Surface Design

The fitted regression model gives an excellent explanation of the relationship between the response variable and the predictor variables (quantity of sparkling beverages and the revenue as well as the cost of raw materials).

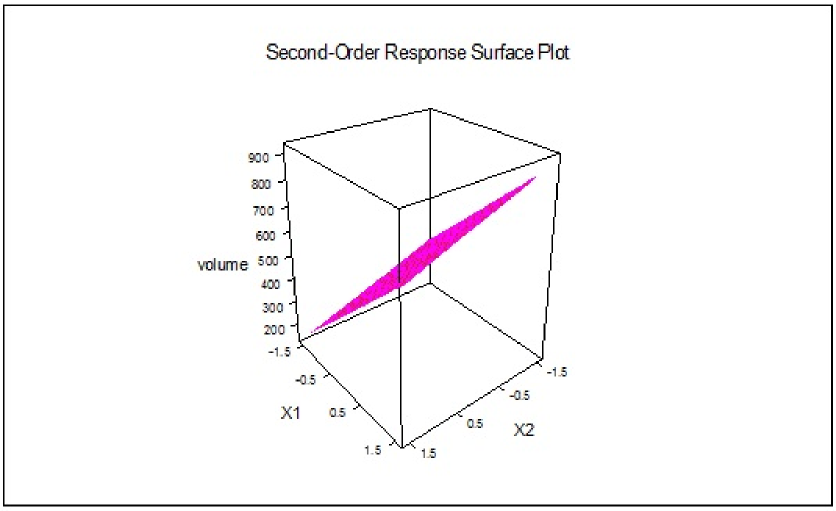

Figure 4.3. Second Order Response Surface Plot

A response surface plot for the volume (quantity) of sparkling beverages produced in figure 4.3 gives the visual impression of the relationship between the predictor variable, revenue X_1 and the cost X_2. From the plot the two variables have remarkable influence on the quantity of sparkling beverages produced. In addition the plot gives a visual representation of the different combinations of the revenue and cost of raw materials that can be attained in the drinks manufacturing process.

The Response Surface Regression model is given by:

$$y = 368.01 + 171.19x_1 - 67.66x_2 + 74.94x_1^2 + 92.45x_1^2 - 34.25x_1x_2 \quad (23)$$

Estimates of the revenue and cost of raw materials (in coded units) that maximizes the total quantity of sparkling beverages in a period of eight months are calculated from (5) and (6) and we obtain $\hat{X}_1 = -1.289$ and $\hat{X}_2 = 0.1612$.

Considering the response surface plot of volume, revenue and cost of raw materials, the maximum value of the quantity of sparkling beverages can be realised for absolute values of X_1 and X_2. Using the model, estimates of the actual values of X_1 and X_2 are $US712 million and $US27 million respectively. With these settings the maximum quantity of sparkling beverages produced will be 637000 hectolitres. If the company could aim to raise approximately $US 79 million dollars per month or $US712 million dollars revenue in nine months and restrict itself to buying raw materials for less than $3 million dollars per month or $27 million dollars in nine months then business success would eminent.

4.4 Process Robustness

Using the sales of sparkling beverages as a noise variable, the idea was to find operating conditions that give a mean response (volume of sparkling beverages) between 600 000 and 800 000 hectolitres while minimizing the variability transmitted from the noise variable. A Modified Central Composite Design for the process is shown in the Table 4.4 below:

$X_1(\$m)$	$X_2(\$m)$	$Z(\$m)$	Volume (000hls)
-1	-1	-1	342
1	-1	-1	775
-1	1	-1	340
1	1	-1	648
-1	-1	1	488
1	-1	1	750
-1	1	1	655
1	1	1	759
-1.68	0	0	368
1.68	0	0	790
0	-1.68	0	535
0	1.68	0	673
0	0	0	690

0	0	0	691
0	0	0	695
0	0	0	687

Table 4.4. Modified Central Composite Design

In an attempt to minimize the variability in the total quantity of sparkling beverages produced a second order-response surface model was fitted using the data above whose response surface contour is shown in Figure 4.5.

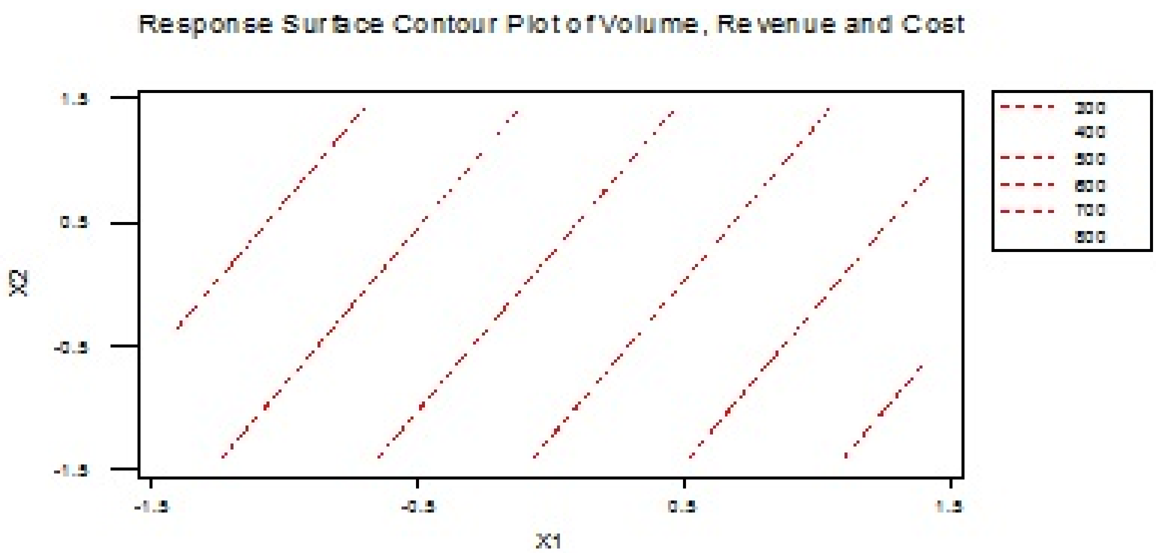

Figure 4.4. Response Surface Contours

The contour plot shows the relationship between the predictor variables (Revenue and Cost of raw materials) and quantity of sparkling beverages produced. In other words the contour plot shows the levels of revenue and cost of raw materials for the respective total quantity of sparkling beverages. The response surface regression was then fitted to see whether there is a perfect relationship between variables. The regression model is then checked for suitability by plotting residuals as shown in figure 4.5.

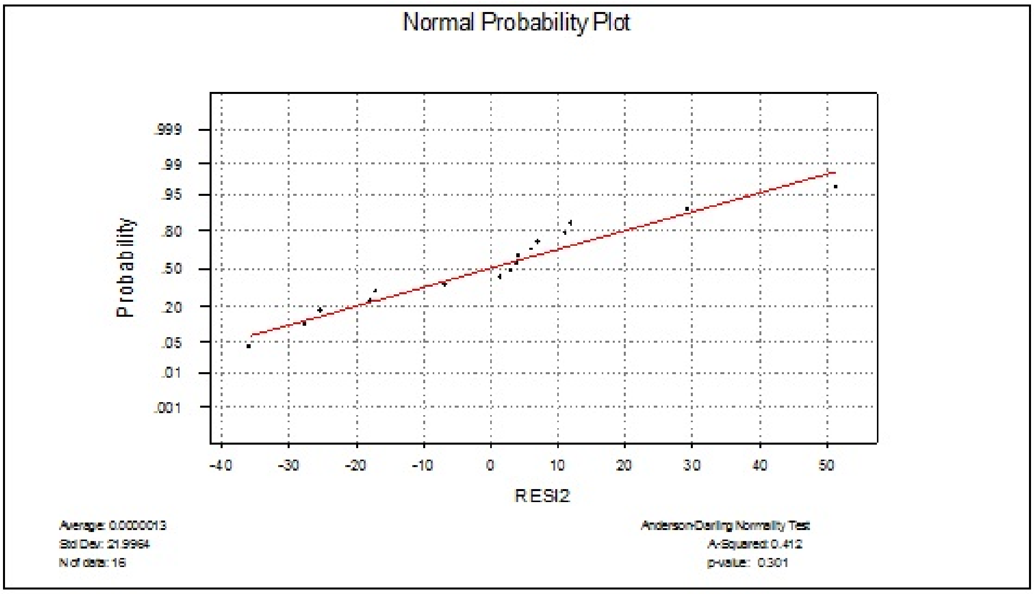

Figure 4.5. Normal probability plot of residuals

There is an indication of pure quadratic effect from the regression model, that is, there is curvature Therefore, the least squares fit is given by:

$$y(x,z) = 683.84 + 133x_1 + 20.44x_2 - 43.10x_1^2 - 34.24x_2^2 - 35.38x_1x_2 - 46.88x_1z + 38.12x_2z \quad (24)$$

and the mean model is given by:

$$E_z[y(x,z)] = 683.84 + 133x_1 + 20.44x_2 - 43.10x_1^2 - 34.24x_2^2 - 35.38x_1x_2 \quad (25)$$

The variance model is given by:

$$Var_z[y(x,z)] = (68.37 + 133x_1 + 20.44x_2)^2 + 10.9 \quad (26)$$

Taking $\sigma_z^2 = 1$ and $\sigma^2 = MSE = 10.9$

$$\frac{\partial E_z}{\partial x_1} = 133 - 86.2x_1 - 35.38x_2 \quad (27)$$

and

$$\frac{\partial E_z}{\partial x_2} = 20.44 - 68.48x_2 - 35.38x_1 \quad (28)$$

Equating equations (27) and (28) to zero and solving them simultaneously, we obtain the optimal values of $US860 million dollars revenue and $US43 million cost of raw materials. With these settings of revenue and cost of raw materials the company has the capacity to produce about 750 000 hectolitres of sparkling beverages in a period of 8 months. This means if the production managers can effectively apply the model that minimizes variability in the quantity of sparkling beverages produced; the unusual levels of production will be avoided. Moreover, the model (24) can be used to fix the settings of revenue expected and the amounts of money to buy raw materials to meet targeted volumes of the drinks.

4.5 Modeling Simple Linear Profiles

The time plot in Figure 4.6 shows the volume of sparkling beverages produced in the 54 months shows that the levels of production have not been stable for the 54 months.

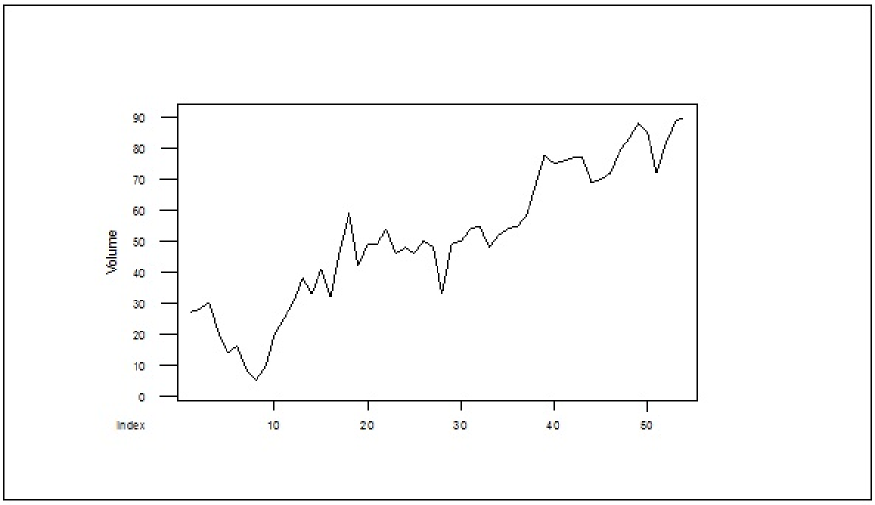

Figure 4.6. Time Series Plot of Volume of sparkling beverages

The relationship between the revenue and the volume of sparkling beverages is displayed in the scatter diagram of Figure 4.7. The scatter plot shows that there is a strong positive linear relationship between the volume of sparkling beverages and the revenue that was collected over the 54 months. As the revenue collection increased the total quantity of sparkling beverages produced increased.

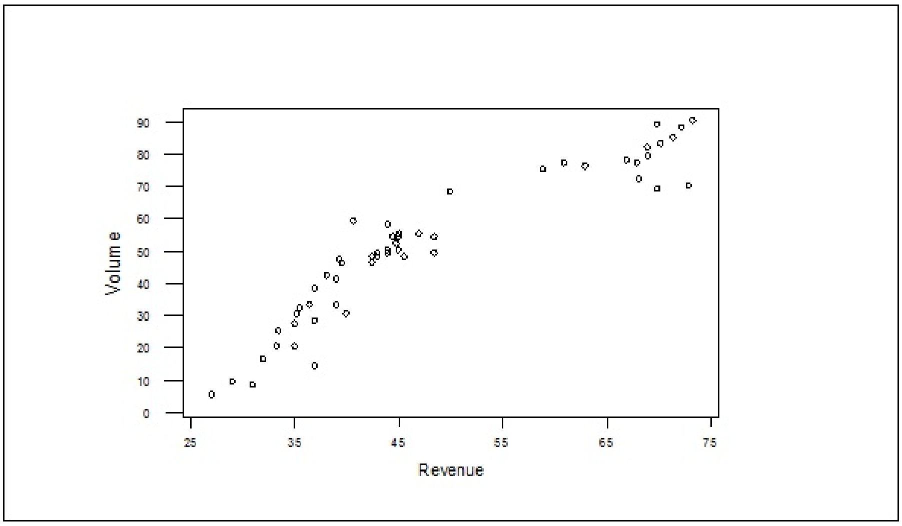

Figure 4.7. Scatter plot of volume against revenue

Tables 4.5 and 4.6 below summarize the values of the intercept and slope for the models on the volume of sparkling beverages and revenue as well as the volume against the cost of raw materials.

Profile	1	2	3	4	5	6
β_{0j}	17.5065	36.258	47.9851	49.938	72.014	82.153
β_{1j}	6.9842	4.4117	1.3402	1.861	0.4199	3.1091

Table 4.5. Slope and Intercept of Volume against Revenue

Profile	1	2	3	4	5	6
β_{0j}	18.095	37.216	48.099	50.086	72.124	81.813
β_{1j}	-146.436	-90.38	-3.896	19.385	55.59	-19.382

Table 4.6. Slope and Intercept of Volume against Cost

The estimates of the intercept slope and error variance obtained for the volume against revenue model considering the six profiles are $\hat{\beta}_0 = 50.98$, $\hat{\beta}_0 = 3.02$ and $\hat{\sigma}^2 = 372.73$ and the estimates of intercept slope and error variance obtained for the volume against cost model considering the six profiles are: $\hat{\beta}_0 = 51.20$, $\hat{\beta}_0 = -31.55$ and $\hat{\sigma}^2 = 398.9$.

4.6 Modeling Simple Linear Profiles

Profile	2	3	4	5	6
$EWMA_I(j)$	53.41	52.33	51.85	55.88	61.14

Table 4.7. Intercept EWMA Statistics of Volume against Revenue

UCL and LCL are 61.45 and 48.40 respectively. Therefore all 5 profiles are in control.

Profile	2	3	4	5	6
$EWMA_S(j)$	2.67	2.40	2.29	1.92	2.16

Table 4.8. Slope EWMA Statistics of Volume against Revenue

UCL and LCL are 2.883 and 1.693 respectively; all 5 profiles are in control.

Profile	1	2	3	4	5
$EWMA_I(j)$	53.68	52.55	50.06	56.07	61.23

Table 4.9. Intercept EWMA Statistics of Volume against Cost

UCL and LCL are 58.88 and 41.12 respectively; profile 5 is out of control.

Profile	2	3	4	5	6
$EWMA_S(j)$	-24.27	-20.19	-20.03	-4.91	-7.80

Table 4.10. Slope EWMA Statistics of Volume against Cost

UCL and LCL are 5.882 and -20.88 respectively; all 5 profiles are in control.

Profile	2	3	4	5	6
$EWMA_E(j)$	1.39	1.96	2.65	3.09	3.59

Table 4.11. Error-Variance EWMA Statistics of Volume against Revenue.

UCL and LCL are 3.859 and 1.213 respectively; all 5 profiles are in control.

Profile	2	3	4	5	6
$EWMA_E(j)$	1.38	1.51	2.33	2.95	3.51

Table 4.12. Error-Variance EWMA Statistics of Volume against Cost.

UCL and LCL are 3.081 and 0.5190 respectively; all 5 profiles are in control.

From the analysis of the EWMA charts the following models were obtained that give in-control processes: The estimates of the intercept, slope and error variance obtained for the volume against revenue model considering the five profiles were: $\hat{\beta}_0 = 57.7,\ \hat{\beta}_1 = 2.23$ and $MSE = 345$ and the estimates of intercept, slope and error variance obtained for the volume against cost model considering the three in-control profiles were: $\hat{\beta}_0 = 56.7,\ \hat{\beta}_1 = 23.7$ and $MSE = 181$ˆ _0 = 56.7.

This means that if the company is to realize stable production levels for the sparkling beverages, monitoring the quantity of the beverages can be made easy by considering a basic value of 57.7 for the intercept and 2.23 value of the slope considering the quantity of sparkling beverages and the revenue collected. Hence changes in the process are observed once a shift from these values is noted. Similarly, any changes noted for intercept and slope from 56.7 and 23.7 respectively considering the quantity of sparkling beverages and the cost of raw materials will signal an unfamiliar production process. A decrease in the variation in total monthly volumes is noted by a decrease in variation of the slope from the target lines.

5. Conclusion

The method of Steepest Ascend proved useful in estimating the maximum quantity of sparkling beverages that can be produced by Delta Beverages given the optimal settings of revenue and cost of raw materials from fitted models. However the method could have given perfect results if the data displayed more curvature. The EWMA charts used were able to detect out of control processes using the slope for both Phase I and Phase II linear profile monitoring applications. It was necessary to eliminate profiles 1 and 6 for easy estimation of in-control profiles. The Response Surface Methodology used in the research did not establish whether revenue collection or cost of raw materials contributed more to the quantity of sparkling beverages produced but managed to give the optimal settings of the predictor variables. The robustness of the sparkling beverages manufacturing process was achieved more precisely through a suitable experimental design. The robustness of the process can be increased by setting reliable tolerances within which the volumes of sparkling beverages could vary.

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